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AN ANALYTICAL METHOD FOR CONSTRUCTION OF A FUNDAMENTAL PORTFOLIO

ANALITYCZNA METODA WYZNACZANIA PORTFELA FUNDAMENTALNEGO

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Summary: The classical models used for the construction of an investment portfolio do not take into account the fundamental values of the companies in question. The model of a fundamental portfolio adds this dimension to the classical criteria of profitability and risk. It is assumed that an investor selects stocks according to their attractiveness measured by some fundamental values of companies. In the paper the authors propose an analytical solution of the optimization problem of constructing a fundamental portfolio and present empirical examples of the calculation of fundamental portfolios of stocks listed on the Warsaw Stock Exchange.

Keywords: portfolio analysis, fundamental value, multicriterial choice, fundamental portfolio.

Streszczenie: Klasyczne metody wyboru portfela inwestycyjnego nie biorą pod uwagę wartości fundamentalnej spółek. Model portfela fundamentalnego dodaje ten wymiar do klasycznych kryteriów zyskowności i ryzyka. Zakłada się w nim, że inwestor wybiera spółki według ich atrakcyjności inwestycyjnej, mierzonej za pomocą pewnych wskaźników fundamentalnych. W artykule przedstawiono propozycję analitycznego rozwiązania problemu optymalizacyjnego konstrukcji portfela fundamentalnego. Zaprezentowano też przykłady wyznaczania tą metodą portfeli fundamentalnych złożonych z akcji spółek notowanych na Giełdzie Papierów Wartościowych w Warszawie.

Słowa kluczowe: analiza portfelowa, wartość fundamentalna, wybór wielokryterialny, portfel fundamentalny.

1. Introduction

The modern portfolio theory was developed in the works of Markowitz [1952; 1959] and Sharpe [1963]. According to their approach the only things that matter in selecting a portfolio are changes in the prices of the considered assets. This assumption is supported by the theory of efficient markets, according to which all significant information concerning a company is immediately reflected in its stocks' prices. Thus, the changes of prices can be used as the sole measure of company performance. In the classical model, potential portfolios of investment are evaluated according to two criteria: profitability and risk. The first criterion is measured with expected rate of return, and the second one with variance or standard deviation of return. No other criteria connected with additional information a about company, are considered.

However, it is well-known in the economic literature that market data are not the only predictors of the returns from stocks. In the real markets the distribution of returns is not stable and depend on some factors that are not contained in the historical data concerning prices. In some studies the importance of some fundamental factors was revealed. Already at the beginning of the 1990s. Fama and French [1993] proposed a three-factor model of asset returns, which takes into account the book-to-market ratio of a company. In the article [Fama, French 2006] the significance of the value of a company was revealed. The same authors [Fama, French 2015] extended their model for two other fundamental factors (profitability and investment strategy). There were also other models which account for other fundamental factors, like the price-to-earnings ratio [Ball 1992] or the book-to-market ratio [Stattman 1980].

Taking into account these anomalies, one can consider an alternative approach to a portfolio selection. In this alternative setup an investor selects shares for his/her portfolio according to their attractiveness, measured by some fundamental values of companies. This allows for the addition of a third dimension for the analysis of portfolio construction. For example, Jacobs and Levy [2013] in their paper take into account the risk associated with leverage. The utility function of an investor includes the costs of margin calls, which can force borrowers to liquidate securities at adverse prices due to illiquidity, losses exceeding the capital invested, and the possibility of bankruptcy.

Tarczyński [1995] applied a synthetically developed measurement to evaluate the economic and financial standing of a company and used this measure as an additional criterion for evaluating possible portfolios. The author called this measure the taxonomic measure of attractiveness of investment (TMAI). A portfolio constructed with the use of this measure is called a fundamental portfolio. In recent years this model was modified, for example by substituting variance as a measure of risk by semi-variance [Rutkowska-Ziarko, Garsztka 2014]. In [Rutkowska-Ziarko 2013] the

Mahalanobis distance was used to determine the TMAI, due to a possible correlation between diagnostic financial variables. This method for constructing a portfolio was tested empirically by Staszak [2017] for companies trading on the Polish stock exchange. The results reveal that accounting for the fundamental value of a portfolio usually gives better results than using the classical Markowitz method. The author considered annually rebalanced portfolios for the period from 2004 to 2016. He found that in 8 out of 13 cases the yearly performance of a portfolio based on TMAI was better than the performance of a minimal variance portfolio (without the TMAI criterion). The total return in the whole period of the former portfolio was two times higher than the latter one.

In this paper we propose a simple algorithm for constructing a fundamental portfolio based on the analytical solutions of the optimization problems. In the empirical part we verify this method by computing the fundamental portfolios of the stocks traded on the Warsaw Stock Exchange. The article is organized as follows. After this introduction, in Section 2 we present the concept of a fundamental portfolio. Section 3 provides analytical solutions to the problems connected with computing fundamental portfolios and an algorithm for constructing such portfolios. Section 4 contains empirical examples from the Warsaw Stock Exchange and Section 5 concludes.

2. Fundamental portfolio

In the article we use a generalization of the classical Markowitz model of portfolio optimization, see [Markowitz 1952; 1959]. We consider an investor who tries to determine optimal composition of his/her portfolio. Assume that there are n risky assets with stochastic rates of returns $R_1, \ldots, R_n$. Let μ_i be the expected return of asset i: $\mu_i = E[R_i]$. By cov_{ij} we denote the covariance between asset i and j, $cov_{ij} = \text{cov}(R_i, R_j)$. By x_i we denote the proportion of wealth invested in asset i. As in the classical Markowitz model, the investor values a portfolio according to the criteria of expected return and risk. The expected return equals

$$\mu_P = \sum_{i=1}^{n} x_i \mu_i \tag{1}$$

and the risk is measured by the variance of return from portfolio, which equals

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j cov_{ij} \; . \tag{2}$$

The concept of the fundamental portfolio proposed by Tarczyński [1995] combines classical measures of market performance with the fundamental analysis. It augments the Markowitz model with a third criterion which describes the financial and economic standing of companies whose shares are in a portfolio.

The third criterion is determined on the basis of several indicators of the fundamental value of a company. In this paper we consider three such indicators.

They describe the financial situation of a company, namely quick ratio (QR), return on assets ratio (ROA) and debt ratio (DR). The indicators were transformed as follows.

The quick ratio indicates the degree of coverage of short-term foreign capital by current assets with a high degree of liquidity. In the literature it is usually assumed that a high level of this indicator means that the company has a good financial standing. Some authors claim that too high values of QR can be a symptom of maintaining too high capital [Gąsiorkiewicz 2011]. However, on the other hand, loss of liquidity can be a serious problem for a company [Sierpińska, Wędzki 1997]. We take this point of view and assume that quick ratio is a stimulator variable – its higher its value, the better the financial condition of the company.

As for the other indicators, we assumed that the ROA is a stimulant and DR is a destimulant. Thus, the second and third diagnostic variables were defined as the values of ROA and the reciprocals of the values of : $w_{2i} = ROA$.

$$w_{3i} = \frac{1}{DR}.$$

Based on the values of the three diagnostic variables, the overall indicators of attractiveness of investment were calculated for each company. As in [Rutkowska-Ziarko, Garsztka 2014], the Mahalanobis distance from the "ideal point" was used to determine the taxonomic measures. All diagnostic variables after transformation are stimulants and for all of them the highest observed values were determined [Hellwig 1968]:

$$w_i^* = \max_i w_{ji},\ j = 1, \dots, 3.$$

An abstract "ideal point" $W^* = (w_1^*, \dots, w_3^*)$ was taken as the reference standard. Its coordinates are equal to the highest values of diagnostic variables after transformation into stimulants. For each company the Mahalanobis distance Q_i was calculated as follows [Mahalanobis 1936]:

$$Q_i = \sqrt{(W_i - W^*)C^{-1}(W_i - W^*)^T},$$

where $W_i = (w_{1i}, \dots, w_{3i})$ is a row vector with the values of the diagnostic variables for company i and C is the covariance matrix of the diagnostic variables. The taxonomic measure of attractiveness of investment in company i was calculated as

$$TMAI_i = 1 - \frac{Q_i}{\max_j Q_j}.$$

For any portfolio of shares, the third criterion of attractiveness of investment in the portfolio is defined as the weighted average of attractiveness of investment in the companies in the portfolio:

$$TMAI_P = \sum_{i=1}^{n} x_i TMAI_i. \tag{3}$$

With the introduction of $TMAI_p$ we have three criteria for assessing an investment: profitability, risk and fundamental value. The formulas for calculating these criteria are given in Equations 1 and 2 and Equation 3. The portfolio which is efficient with respect to the set of all three criteria is called a fundamental portfolio. One of the methods for obtaining such a portfolio is solving the problem of minimizing the variance of a portfolio with constraints on the two other criteria. This leads to the following optimization problem:

$$minimize \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j cov_{ij} \tag{4}$$

with respect to

$$\sum_{i=1}^{n} x_i \mu_i \geq \gamma \tag{5}$$

and

$$\sum_{i=1}^{n} x_i TMAI_i \geq TMAI_\gamma, \tag{6}$$

where γ is the target rate of return and $TMAI_\gamma$ is the fundamental value of portfolio required by the investor. Of course, there is an additional condition that $\sum_{i=1}^{n} x_i = 1$.

3. The analytical solution

Let Σ be a covariance matrix of returns, i.e.

$$\Sigma = \begin{bmatrix} cov_{11} & \cdots & cov_{1n} \\ \vdots & \ddots & \vdots \\ cov_{1n} & \cdots & cov_{nn} \end{bmatrix}.$$

Define the following vectors: $\mu = (\mu_1, \dots, \mu_n)^T$, $z = (TMAI_1, \dots, TMAI_n)^T$ and let e be a column vector of length n: $e = (1, \dots, 1)^T$. Using the vector notation, the optimization problem (4)-(6) can be formulated as follows:

$$minimize \ \frac{1}{2} x^T \Sigma x \tag{7}$$

with respect to

$$x^T e = 1, \tag{8}$$

$$x^T \mu \geq \gamma \tag{9}$$

and

$$x^T z \geq TMAI_\gamma. \tag{10}$$

The Kuhn-Tucker conditions for the problem (7)-(10) are as follows

$$\Sigma x = \lambda_1 e + \lambda_2 \mu + \lambda_3 z, \tag{11}$$

where $\lambda_2, \lambda_3 \geq 0$, with the complementary conditions

$$\lambda_2(x^T\mu - \gamma) = 0, \quad (12)$$

$$\lambda_3\left(x^T z - TMAI_\gamma\right) = 0. \quad (13)$$

Assuming that covariance matrix is nondegenerate (as it is in practical usages), it follows from equation (11) that the solution has the form:

$$x = \lambda_1\Sigma^{-1}e + \lambda_2\Sigma^{-1}\mu + \lambda_3\Sigma^{-1}z. \quad (14)$$

We have to consider several cases. To simplify the notation, we define the following symbols. Let

$$g_{11} = e^T v^1, g_{12} = e^T v^2, g_{13} = e^T v^3, g_{22} = \mu^T v^2, \quad (15)$$
$$g_{23} = \mu^T v^3, g_{33} = z^T v^3,$$

where

$$v^1 = \Sigma^{-1}e, v^2 = \Sigma^{-1}\mu, v^3 = \Sigma^{-1}z. \quad (16)$$

Firstly, assume that $\lambda_2 = \lambda_3 = 0$. From complimentary condition (12) and (13) it turns out that only condition (8) must be satisfied as equality. Putting solution (14) with $\lambda_2 = \lambda_3 = 0$ into (8) we obtain the solution

$$\tilde{x}^1 = \frac{1}{g_{11}}\Sigma^{-1}e. \quad (17)$$

The minimal variance in this case equals

$$\sigma_1^2 = \frac{1}{g_{11}^2}. \quad (18)$$

When $\lambda_2 > 0$, $\lambda_3 = 0$, in the optimal solution condition (8) is fulfilled and condition (9) holds as an equation, which gives a system of equations

$$\lambda_1 g_{11} + \lambda_2 g_{12} = 1,$$

$$\lambda_1 g_{12} + \lambda_2 g_{22} = \gamma.$$

After some manipulations one can write the solution of this system as

$$\tilde{x}^{12} = (1 - \alpha_{12})\,\tilde{x}^1 + \alpha_{12}\frac{v^2}{g_{12}}, \quad (19)$$

where

$$\alpha_{12} = \frac{g_{12}(\gamma g_{11} - g_{12})}{g_{11}g_{22} - g_{12}^2}.$$

The variance of the portfolio in this solution equals $\sigma_{12}^2 = \sigma_1^2 + \Delta\sigma_{12}^2$, where

$$\Delta\sigma_{12}^2 = \frac{(\gamma g_{11} - g_{12})^2}{g_{11}}. \quad (20)$$

The case $\lambda_2 = 0$, $\lambda_3 > 0$, is analogic to the last one. The optimal solution satisfies conditions (8) and (10) as equations. The solution can be expressed as

$$\tilde{x}^{13} = (1 - \alpha_{13})\, \tilde{x}^1 + \alpha_{13} \frac{v^3}{g_{13}}, \tag{21}$$

where

$$\alpha_{13} = \frac{g_{13}(TMAI_\gamma g_{11} - g_{13})}{g_{11} g_{33} - g_{13}^2}.$$

The growth of variance when switching from $\tilde{x}^1$ to $\tilde{x}^{13}$ equals

$$\Delta\sigma_{13}^2 = \frac{\left(TMAI_\gamma g_{11} - g_{13}\right)^2}{g_{11}}. \tag{22}$$

The last remaining case is when $\lambda_2 > 0$, $\lambda_3 > 0$. In this case all conditions (8)-(10) must be fulfilled as equalities, which brings us to the following system of equations

$$\lambda G = a,$$

where λ is the vector of multipliers $\lambda = (\lambda_1, \lambda_2, \lambda_3)^T$, $a = \left(1, \gamma, TMAI_\gamma\right)^T$ and matrix G is

$$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{12} & g_{22} & g_{23} \\ g_{13} & g_{23} & g_{33} \end{bmatrix}.$$

The optimal solution to problem (7)-(10) with all conditions fulfilled as an equation can be thus formulated as follows:

$$\tilde{x}^{123} = \lambda_1 v^1 + \lambda_2 v^2 + \lambda_3 v^3, \text{where } \lambda = G^{-1} a. \tag{23}$$

These derivations lead to the following algorithm for finding the optimal solution of problem (4)-(6):

1. Compute the vector $\tilde{x}^1$ and check if it fulfills conditions (5) and (6). If so, it is the optimal solution.
2. Otherwise, calculate $\Delta\sigma_{12}^2$ and $\Delta\sigma_{13}^2$. Choose the smaller value: let it be $\Delta\sigma_{1k}^2$.
3. Compute the vector $\tilde{x}^{1k}$ (for $k = 2,3$) and check if it fulfills conditions (5) and (6). If so, it is the optimal solution. Otherwise the optimal solution is the vector $\tilde{x}^{123}$.

4. Empirical results

In the empirical part we analyze 14 of the largest and most liquid companies listed on the Warsaw Stock Exchange. The sample includes all the companies from the WIG20 index except the financial institutions, as they use different methods of financial reporting. The computations are based on quarterly returns calculated on daily closing prices in the period starting from the beginning of 2015 and ending at 28 March 2018. The quarterly investment period was assumed because it is the reporting frequency of public companies trading on the stock market. As for the length of estimation period, it should be long enough to allow for reliable estimators. On the other hand, it should be not too long because historical data from the too distant past do not convey information about the current market conditions. We decided that data from the previous three years (approximately) allows to obtain reliable estimations of parameters and the estimations are up-to-date.

Returns are computed as relative increases of prices according to the formula:

$$r_{it} = \frac{P_{i,t+s} - P_{i,t}}{P_{i,t}} \cdot 100\%, \tag{24}$$

where r_{it} is the rate of return on security i at time t, s is the length of investment horizon (in our case one quarter) expressed in days and $P_{i,t}$ is the quoted price of security i at time t.

Financial indicators for each company were calculated based on annual financial reports for 2017. For each company in the sample we computed the taxonomic measure of attractiveness of investment. Mean return and standard deviations were calculated based on the time series of returns. Table 1 contains information concerning profitability, risk and taxonomic measures of attractiveness of investment for all companies. The last column contains a ranking of companies according to their TMAIs.

Table 1. Profitability, risk and taxonomic measures of attractiveness of investment

Company	Mean (%)	Standard deviation (%)	TMAI	Rank
ACP	2.06	7.86	0.3964	3
CCC	–2.35	13.81	0.2747	6
CPS	0.08	8.10	0.0915	13
ENA	4.59	13.79	0.2100	9
ENG	8.03	19.28	0.2793	5
EUR	4.21	16.58	0.1826	11
KHG	3.81	21.85	0.2382	8
LPP	–0.51	15.80	0.5985	1
LTS	–4.77	13.16	0.1927	10
OPL	4.94	13.17	0.0000	14
PGE	5.88	12.87	0.2412	7
PGN	–0.91	13.72	0.5782	2
PKN	–2.26	14.57	0.2836	4
TPE	6.70	15.37	0.1043	12

Source: own study.

During the period under research, the company LPP had the highest TMAI. This company was also in the first quartile of the companies with the highest risk and its expected return was below the mean. The lowest standard deviation was for ACP and this company was ranked very high according to attractiveness of investment. The TMAIs of companies was negatively correlated with the mean returns (correlation coefficient -0.345) and there was no correlation between standard deviation of return and TMAI (correlation coefficient 0.0642). The companies with a higher mean return tended to have higher risk measured with variance (correlation coefficient 0.331).

The problem of portfolio choice in this situation is the trade-off between risk and two other criteria. We look for a portfolio which minimizes risk, but low-risk portfolios tend to have a lower expected return. On the other hand, if we assume higher requirements on the mean return of a portfolio, the solution will have a lower TMAI. Figure 1 depicts the efficient frontier for the three-criteria portfolio choice. Each point on the graph represents a solution for problem (4)-(6) for different combination of the required mean return and the required level of TMAI.

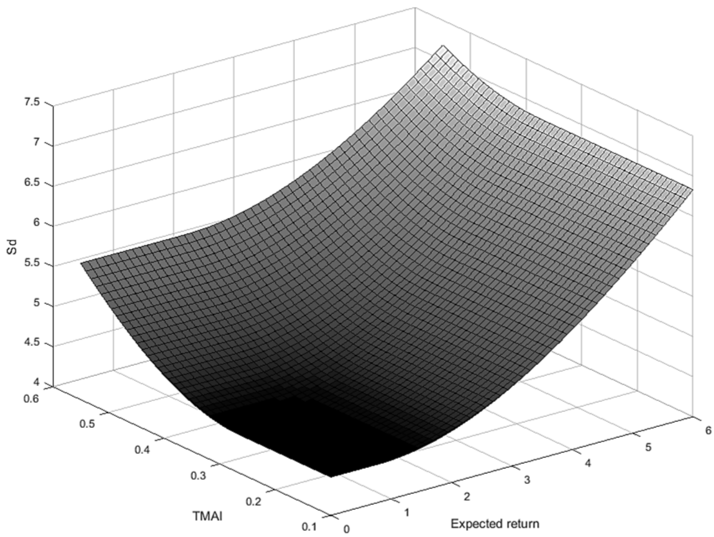

Fig. 1. Effective frontier for problem (1)-(3)

Source: own study.

To analyze the impact of fundamental values of companies in which one is willing to invest on the trade-off between profitability and risk, we determined the shape of efficient frontiers for various values of the required TMAI. We calculated, using the algorithm proposed in Section 3, the effective portfolios for $TMAI_\gamma$ at the levels of 0.1, 0.4 and 0.5. The results are depicted in Figure 2. As one can see, the higher levels of $TMAI_\gamma$ move the effective frontier upwards. As the required profitability of expected return grows, the restriction connected with TMAI (given by Equation 6) is less important and for the highest levels of expected rate of return it is not binding. This phenomenon is expressed in the figure by the convergence of effective frontiers on the right side of the plot.

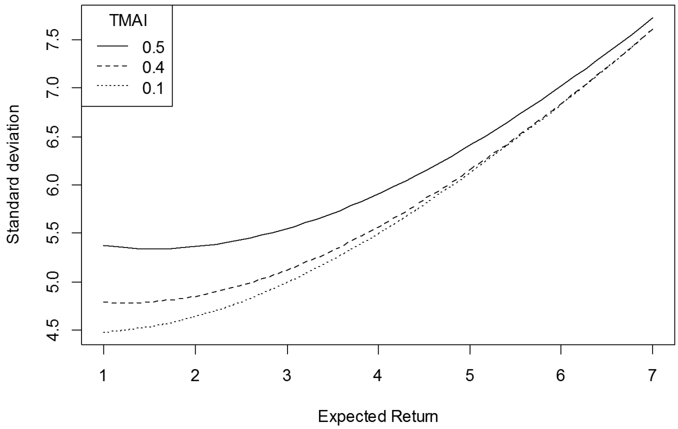

Fig. 2. Effective frontiers for different values of TMAI

Source: own study.

To check the ex-ante performance of the calculated portfolios and compare it with the standard approach, we calculated returns of the portfolios that solves the three-criteria optimization problem, based on the market returns of the stocks in the quarters after the estimation period. We also calculated the returns of the Markowitz portfolios (i.e. portfolios that minimize variation for the given expected return and do not account for additional, fundamental criterion). Table 2 contains the differences between rates of returns of the fundamental portfolios and rates of returns of the Markowitz portfolios. Table 3 contains the differences between the rates of returns of the fundamental portfolios and the rate of return of the market portfolio. As one can see, in the analysed period the returns of the fundamental portfolio were never lower than the returns of the Markowitz portfolio. In some cases they were higher, albeit the differences were not very large. Taking into account the fundamental values of the companies this could improve the portfolio performance, compared with a situation in which one accounts only for the expected return and variation. It is worth noting that the differences were greater for higher conditions on the fundamental value. The conclusions from comparing the fundamental portfolio with the market portfolio (Table 3) are not so obvious. In some cases the return from the market portfolio was higher (negative values in the table). However, as in the comparison with the Markowitz portfolio, the higher requirements on the fundamental value (TMAI) lead to better performance.

Table 2. The differences between rates of return of the fundamental portfolio and the Markowitz portfolio

		TMAI					
		0.10	**0.20**	**0.30**	**0.40**	**0.50**	**0.55**
Expected return	**0.0**	0.00	0.00	0.01	0.04	0.06	0.08
	0.7	0.00	0.00	0.01	0.04	0.06	0.07
	1.3	0.00	0.00	0.01	0.03	0.06	0.07
	2.0	0.00	0.00	0.00	0.03	0.05	0.07
	2.7	0.00	0.00	0.00	0.02	0.05	0.06
	3.3	0.00	0.00	0.00	0.02	0.05	0.06
	4.0	0.00	0.00	0.00	0.02	0.04	0.06
	4.7	0.00	0.00	0.00	0.01	0.04	0.05
	5.3	0.00	0.00	0.00	0.01	0.04	0.05
	6.0	0.00	0.00	0.00	0.01	0.03	0.04

Source: own study.

Table 3. The differences between rates of return of the fundamental portfolio and the market portfolio

		TMAI					
		0.10	**0.20**	**0.30**	**0.40**	**0.50**	**0.55**
Expected return	**0.0**	–0.02	–0.02	–0.01	0.02	0.04	0.06
	0.7	–0.02	–0.02	–0.01	0.02	0.04	0.06
	1.3	–0.01	–0.01	–0.01	0.02	0.04	0.06
	2.0	–0.01	–0.01	–0.01	0.02	0.05	0.06
	2.7	0.00	0.00	0.00	0.02	0.05	0.06
	3.3	0.00	0.00	0.00	0.02	0.05	0.06
	4.0	0.00	0.00	0.00	0.02	0.05	0.06
	4.7	0.01	0.01	0.01	0.02	0.05	0.06
	5.3	0.01	0.01	0.01	0.02	0.05	0.06
	6.0	0.02	0.02	0.02	0.02	0.05	0.06

Source: own study.

5. Conclusions

In the paper we propose a method for constructing a fundamental portfolio of assets, for which three criteria are considered: profitability (measured with expected return), risk (measured with variance of returns) and fundamental values of companies in the portfolio (measured with the taxonomic measure of attractiveness of investments). The proposed algorithm for finding effective portfolios with respect to all three criteria is based on analytical solutions of the optimization problem in which one tries to minimize the variance of the portfolio return with restrictions on its mean return and fundamental value. We have shown that the algorithm is effective and allows one to construct fundamental portfolios with minimal computational effort.

The proposed algorithm allows us to determine the effective frontier (i.e. the tradeoff between all three criteria) for several levels of requirements concerning the fundamental values of companies whose stocks are in the portfolio. The results from empirical research for the major companies trading on the Warsaw Stock Exchange reveal that if an investor requires a high expected return from his/her portfolio, then the constraint on fundamental value is not binding.

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