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# THE MUSGRAVIAN TRANSFORMATION IN THE TWO-PART TARIFF AND OTHER MONOPOLY MODELS

In this paper, we prove for the first time that a demand ad valorem tax on commodity price is equivalent to the cost payment tax in the two-part tariff model if such a tax is imposed on both admission charge and user's fee. The same result is found in the third-degree price discrimination model. However, this property is not preserved in the rate-of-return-regulated monopoly model. It relates to the importance of the choice of tax policies not only in a pure monopoly, but also in two-part tariff, third-degree price discrimination and regulated monopoly models.

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## 1. INTRODUCTION

Oi (1971) was one of the first economists who formulated a class of pricing policies described by two-part tariffs or two-part pricing. A discriminatory two-part tariff, in which the price (user fee per unit or price of ticket rides) per unit is equated to marginal cost and entire consumer surplus, is appropriated by a lump sum tax (club membership fee or admission charge), is perhaps one of the best pricing strategies for a profit-maximizing monopolist. Actually, already Coase (1946) suggested such a possibility in the presence of declining average cost. Schmalensee's paper (1981b) dealt mainly with the use of two-part pricing arrangements by profit-maximizing firms. Despite the fact that a number of results had been scattered throughout the literature, his work presented them in a unified framework. Generally speaking, the more complex the multipart pricing schemes, the more practical the models, for instance, Sherman and Visscher (1982) demonstrated that multipart price structures might exist in regulated industries not because they contributed to economic efficiency or equity, but rather because such price structures could augment the profits of rate-of-

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return-regulated firms. Braverman et al. (1983) showed that firms endowed with monopoly power might utilize an optional service contract in the form of a guarantee as an instrument for extracting consumer surplus in the two-part tariff. Hayes (1987) demonstrated why monopoly power was not required for the existence of a two-part tariff and also showed the circumstances in which price discrimination using a two-part tariff was preferred by consumers to a single price policy. In addition, Hayes (1987) illustrated why this pricing method could be found in competitive markets as well. Edlin and Epelbaum (1993) explored the interactions among firms with increasing returns and firms that were regulated to break even with two-part pricing. They provided conditions for the existence and efficiency of the general equilibrium with n firms. The two-part tariff is frequently seen in athletic clubs, time-share vacation resorts, golf courses, and an array of membership organizations. We use this model as the reference model with which other monopoly models (rate-of-return-regulated and third-degree price discrimination models) are compared under different taxation schemes. Other related models include: the two-part tariff with bundling and tying practices (Waldman, 2004); zero price and its impact on the two-part tariff by Shampanier et al. (2007); different two-part tariffs by Schlereth et al. (2010); and three-part tariff by Ascarza et al. (2012). Specifically, and the three-part tariff by Ascarza et al. (2012) consists of a monthly fee per line, free minutes within a given usage allowance and users fee (average rate) beyond that limit. It is found that consumers have a positive "affective response" derived from "free minutes". Such a pricing scheme is evidently gaining popularity in the mobile phone industry since customers use significantly more minutes (more revenues) in the three-part tariff (434 minutes) than the two-part tariff (282 minutes) in controlled experimental environments (Ascarza et al., 2012). As a result, total revenue for the service provider was up by 19.7 per cent. However, none of the above papers deals with taxation of either consumers or producers of the two-part tariffs model. In the wake of global financial crises, a zero tax policy is no longer feasible and as such an examination of the ad valorem taxes seems to be most appropriate.

The Musgravian transformation (Musgrave1959) illustrated that a demand ad valorem (percentage) tax $u$ is equivalent to a supply or cost payment ad valorem tax $v$ in terms of price and output in perfect competition and pure monopoly models with $1-u=1/(1+v)$. The proof by Musgrave (1959) via using the profit functions involves straightforward first order conditions. In the case of a demand ad valorem tax in a pure monopoly model, the profit function is $(1-u)PQ-TC(Q)$ as compared to the profit function under cost payment: $PQ-(1+v)TC(Q)$. Musgrave showed that price and quantity remained invariant for every $1-u=1/(1+v)$, i.e. a 10% tax on consumers produces an identical price and output level as those under an 11.111 percent tax on cost payment in a pure monopoly model. The motivation of this paper is three-fold: (1) none of the above papers deal with the Musgravian transformation in the two-part tariffs or other monopoly models (except for the pure monopoly model); (2) pure monopoly models are few and far between and thus are not really empirically relevant. For policy implementations, we need to expand or extend the original transformation to the three other monopoly models; (3) from a different perspective, a demand ad valorem and the corresponding cost payment ad valorem taxes may not be equally implementable, especially in cases where costs are difficult to assess (allocating common overheads) or a tax on gross demand price may not be feasible (e.g. an underground economy). The purpose of this paper is first to examine the existence of the Musgravian transformation in the two-part tariff model. We then show the impacts of (i) the demand ad valorem tax imposed on the price of rides (or green fee per golf game) and/or admission fee (club membership fee) and (ii) cost payment ad valorem tax. We also present the results from third-degree price discrimination and rate-of-return-regulated monopoly models. It should be noted that Yang (1993) and Yang and Fox (1994A) examined the property of the Suits-Musgrave theorem – an equivalent ad valorem tax generates more tax revenue than the per unit tax – in third-degree price discrimination and rate-of-return-regulated monopoly models. Yang and Fox (1994B) also proposed a potential Pareto improvement when a property tax was imposed in the regulated monopoly model. In this paper, however, we focus specifically on ad valorem taxes for the two-part tariffs, third-degree price discrimination and the rate-of-return-regulated models. To the best of our knowledge, the results presented in this paper have not been investigated yet and as such are expected to fill a void in the taxation literature.

## 2. TWO-PART TARIFF MODEL

### 2.1. Representative Consumers of Identical Income and Taste

Consider a case in which all consumers have identical utility functions (the assumption is relaxed later) and incomes (Oi, 1971, p.79). The two-part tariffs monopolist may be a golf club or an amusement park or an appliance with a service contract. A single price strategy is practiced over n consumers of identical taste. The profit function of a monopolist can take the following form:

$$\pi = \sum x_i P + \sum T_i - C\left(\sum x_i\right), \tag{1}$$

where $P$ – price per ride or per golf game, $M$ – income, $T_i$ – a lump sum admission tariff for customer $i$ (e.g. the admission fee of an amusement park or a membership fee of a golf club), $x_i =$ #(number) of rides or #(number) of golf games played for customer $i$, $x_i = x_j$ for all $i \neq j$, $\sum T_i = T,\ T_i = T_j$, $\sum x_i = X$, $X = D\left(P, M_i - T_i\right)$ is the demand function of the two-part tariff model, $\frac{dx_i}{dM_i} = -\frac{dx_i}{dT_i}$

As shown by Oi (1971), differentiation with respect to $T_i$ yields

$$\frac{d\pi}{dT_i} = P\frac{dx_i}{dT_i} + 1 - C'\left(\frac{dx_i}{dT_i}\right) = 1 + \left(P - C'\right)\left(\frac{dx_i}{dT_i}\right) = 1 - \left(P - C'\right)\left(\frac{dx_i}{dM_i}\right) \tag{2}$$

Taking the first derivative of the budget constraint $XP + Y = M - T$ or $\sum x_i P + \sum y_i = \sum M_i - \sum T_i$ with respect to $M_i$, we have $P\frac{dx_i}{dM_i} + \frac{dy_i}{dM_i} = 1$. Substituting it to equation (2) yields

$$\frac{d\pi}{dT_i} = \frac{dy_i}{dM_i} + C'\frac{dx_i}{dM_i}, \tag{3}$$

where $y_i$ (other goods consumed for customer $i$ and $\sum y_i = Y$ ) and $x_i$ are normal goods and hence $d\pi / dT_i > 0$ if a representative consumer stay in Disneyland. The dilemma is that even though $d\pi / dT_i > 0$, the amusement park owner cannot charge an unlimited price $T_i$ as the admission fee. This is because if tariff $T_i$ is too high, the consumer would be better off to withdraw from the park and as such the optimum tariff $T^*$ has a limit on the demand function evaluated at an optimal price or $\psi\left(P\right)$. Following Oi (1971) and from Roy's identity we have for each customer the following:

$$\frac{d\left(T^* = \int \psi(P)\,dp\right)}{dP} = -\psi(P) = -x_i \tag{4}$$

The optimum tariff $T^*$ is the consumer surplus in such a way that a representative consumer is indifferent between withdrawing from and staying in the amusement park. Once the maximum value of $T^*$ is set, a monopoly must determine the price to maximize their profit via the first-order condition:

$$\pi = \sum x_i P + \sum T^* - C\left(\sum x_i\right),$$

$$\frac{d\pi}{dP} = \sum x_i + P\frac{d\sum x_i}{dP} + \sum \frac{dT^*}{dP} - C'\left(\frac{d\sum x_i}{dP}\right)$$

$$= \sum x_i + (P - C')\sum \frac{dx_i}{dP} - \sum x_i = 0$$

$$= (P - C')\sum \frac{dx_i}{dP} = 0$$

$$\text{or } P - C' = 0\,. \tag{5}$$

The result indicates that the optimum price of ride $P^*$ should be set at marginal cost $C'$ and admission fee $T^*$ should equal the amount of consumer surplus above $P^*$ for the two-part tariffs monopoly.

### 2.2. Demand Ad Valorem Tax on Both Price of Rides and Admission Tariff

If percentage tax rate $u$ is imposed on both prices of rides ($P$) and admission tariff ($T$), the profit function and its first-order condition take the following forms:

$$\pi = (1-u)\sum x_i P + (1-u)\sum T_i - C\left(\sum x_i\right), \tag{6}$$

$$\frac{d\pi}{dP} = (1-u)\left(\sum x_i + P\frac{d\sum x_i}{dP}\right) + (1-u)\sum \frac{dT_i}{dP} - C'\left(\sum \frac{dx_i}{dP}\right) \tag{7}$$

$$= (1-u)\sum x_i + \left[(1-u)P - C'\right]\sum \frac{dx_i}{dP} - (1-u)\sum x_i = 0\,.$$

It follows immediately that

$$C' = (1-u)P^*, \tag{8}$$

where marginal cost $C'$ depends on the quantity of rides that the consumers took ($X$).

### 2.3. Production Ad Valorem Tax on Cost Function

An ad valorem tax $v$ on cost payments is equivalent to shifting the cost function $C(\sum x_i)$ up. Its profit function and the first-order condition are:

$$\pi = \sum x_i P + \sum T_i - (1+v)C\left(\sum x_i\right), \tag{9}$$

$$\frac{d\pi}{dP} = \sum x_i + P\sum\frac{dx_i}{dP} - \sum x_i - (1+v)C'\left(\sum\frac{dx_i}{dP}\right),$$

$$= \left[P - (1+v)C'\right]\sum\frac{dx_i}{dP} = 0, \tag{10}$$

which leads to

$$C' = \frac{P^*}{1+v}. \tag{11}$$

At a given output $x_i$ or $X$ under both taxes, the demand price remains the same. Thus a comparison of equations (8) and (11) immediately establishes $(1-u) = 1/(1+v)$: a 20% tax on gross price ($u =$ 0.2) is equivalent to a 25% tax on cost payments ($v =$ 0.25). If the ad valorem tax $u$ is imposed on either the price of rides ($P$) or the admission tariff ($T_i$) only, the equivalence relation will not hold because the first and the last terms in (7) fail to cancel each other out.

### 2.4. Uniform Tariff with N Consumers of Different Income and Preference

Representative consumers like those in perfect competition are few and far between in the real world. In this section we relax the assumption. Consider $n$ different customers whose numbers of rides dictated by their preferences follow $x_1 < x_2 < \cdots < x_n$ for a uniform admission tariff ($T$), which is in turn determined by the a uniform price of rides ($P$). When both the price and the tariff are to be determined, we have to make the assumption that the monopoly would retain every customer ($N$) and as such the optimum tariff is the consumer surplus of the consumer with the smallest number of rides $x_1$. As such we have $dT/dP = -x_1$ via Roy's identity. Following Oi (1971), the profit and the first order condition are

$$\max_P \pi = \sum x_i P + NT - C\left(\sum x_i\right)$$

$$\frac{d\pi}{dP} = \sum x_i + P\left(\frac{d\sum x_i}{dP} + \frac{d\sum x_i}{dT}\frac{dT}{dP}\right) + N\frac{dT}{dP} - C'\left(\frac{d\sum x_i}{dP} + \frac{d\sum x_i}{dT}\frac{dT}{dP}\right)$$

$$= \sum x_i + \left(P - C'\right)\left(\sum \frac{dx_i}{dP} + \sum \frac{dx_i}{dM_i}x_1\right) + N\left(-x_1\right)$$

$$= \sum x_i\left(1 - N\frac{x_1}{\sum x_i}\right) + \left(P - C'\right)\left(\sum \frac{dx_i}{dP} + x_1\frac{dx_i}{dM_i}\right). \tag{13}$$

Note that subscript $i$ denotes customer $i$ and $dx_i / dT = -dx_i / dM_i$ as shown before.

Let

$$E = \frac{P}{\sum x_i}\left(\sum \frac{dx_i}{dP} + x_1 \sum \frac{dx_i}{dM_i}\right) \tag{14}$$

and hence, the term in the parentheses of (13) becomes

$$\sum \frac{dx_i}{dP} + x_1 \sum \frac{dx_i}{dM_i} = E\frac{\sum x_i}{P}. \tag{15}$$

Equation (13) can now be simplified to

$$\frac{d\pi}{dP} = \sum x_i\left(1 - N\frac{x_1}{\sum x_i}\right) + \left(E\frac{\sum x_i}{P}\right)\left(P - C'\right) = 0$$

or

$$C' = P + \frac{P}{E}\left(1 - N\frac{x_1}{\sum x_i}\right) = \mathrm{P}\left(1 + \frac{1 - N\frac{x_1}{\sum x_i}}{E}\right). \tag{16}$$

Note that the marginal cost is a monotonic function of the number of rides ($\sum x_i$).

### 2.5. Demand Ad Valorem Tax on both Price of Rides and Admission Tariff

When an ad valorem tax ($u$) is imposed on both rides and admission fees, the profit function and its first-order condition are:

$$\pi = (1-u)P\sum x_i + (1-u)NT - C\left(\sum x_i\right) \tag{17}$$

$$\frac{d\pi}{dP} = (1-u)\left[\sum x_i + P\left(\sum \frac{dx_i}{dP} + \sum \frac{dx_i}{dT}\frac{dT}{dP}\right)\right]$$

$$-C'\left(\sum \frac{dx_i}{dP} + x_1 \sum \frac{dx_i}{dM_i}\right) + N\left(\frac{dT}{dP}\right)(1-u)$$

$$= (1-u)\left[\sum x_i + P(\cdot) - N(x_1)\right] - C'(\cdot) = 0\,, \tag{18}$$

where $(\cdot) = \left(\sum \frac{dx_i}{dP} + \sum \frac{dx_i}{dT}\frac{dT}{dP}\right) = \left(\sum \frac{dx_i}{dP} + \frac{dx_i}{dM_i}x_1\right)$.

Following the same procedure and letting $E = \frac{P}{\sum x_i}(\cdot)$ we have

$$\frac{d\pi}{dP} = (1-u)(-Nx_1) + (1-u)\sum x_i + (1-u)P\frac{E\sum x_i}{P} - C'\frac{E\sum x_i}{P}$$

$$= (1-u)\left[-Nx_1 + \sum x_i + P\frac{E\sum x_i}{P}\right] - C'\frac{E\sum x_i}{P} = 0\,,$$

which leads to

$$C' = (1-u)P\left(1 + \frac{1 - N\frac{x_1}{\sum x_i}}{E}\right). \tag{19}$$

### 2.6. Production Ad Valorem Tax on Cost Payments

A production ad valorem tax $v$ on a cost function is expected to reduce profit. We present the profit and the first-order condition as below

$$\pi = \sum x_i P + NT - (1+v) C\left(\sum x_i\right) \tag{20}$$

$$\frac{d\pi}{dP} = \sum x_i + P\left(\sum \frac{dx_i}{dP} + \sum \frac{dx_i}{dT}\frac{dT}{dP}\right)$$

$$+N\left(\frac{dT}{dP}\right) - (1+v)C'\left(\sum \frac{dx_i}{dP} + \sum \frac{dx_i}{dT}\frac{dT}{dP}\right) = 0$$

$$= \sum x_i - Nx_1 + \left[P - (1+v)C'\right]\left(\sum \frac{dx_i}{dP} + \frac{d\sum x_i}{dM_i} x_1\right) \tag{21}$$

$$= \sum x_i \left(1 - N\frac{x_1}{\sum x_i}\right) + \left[P - (1+v)C'\right]\left(\sum \frac{dx_i}{dP} + x_1 \sum \frac{dx_i}{dM_i}\right).$$

By following the same procedure, we have

$$C' = \frac{1}{(1+v)} P \left(1 + \frac{1 - N\frac{x_1}{\sum x_i}}{E}\right). \tag{22}$$

A comparison of equations (19) and (22) indicates that at a given output $\sum x_i = X$ (hence the same price $P$), the Musgravian transformation must hold: $(1-u) = 1/(1+v)$. Note that if the demand ad valorem tax is imposed on admission tariff only, by following the similar steps, we can derive

$$C' = P\left(1 + \frac{1 - (1-u)N\frac{x_1}{\sum x_i}}{E}\right), \tag{23}$$

which differs from equation (22), i.e., the transformation does not hold. On the other hand, if the demand ad valorem tax is imposed on the price of rides only, by following the similar procedures, we can show that

$$C' = P\left(1-u+\frac{(1-u)-N\dfrac{x_1}{\sum x_i}}{E}\right), \tag{24}$$

which differs from equation (22). This is to say, the Musgravian transformation between the demand ad valorem tax $u$ (on the price of rides only) and cost payment ad valorem tax $v$ does not hold, i.e. $u =$ 0.20 may well produce different outputs and prices from those at $v =$ 0.25.

## 3. MUSGRAVIAN TRANSFORMATION IN THIRD-DEGREE PRICE DISCRIMINATION MODEL

The third-degree price discrimination model has a long history dating back to Pigou (1920) and Robinson (1933). Since then it has earned a prominent place in microeconomics. Subsequent works on the output effect can be found in Silberberg (1970), Battalio and Ekelund Jr. (1972), Finn (1974), Greenhut and Ohta (1976), Smith and Formby (1981), Shih et al.(1988). The literature on the welfare effect can be found in Yamey (1974), Schmalensee (1981a), Varian (1985), Nahata et al. (1990) and Yang (1993). Despite the prolific literature on the model the taxation effects still evade the literature.

Consider a case in which a demand ad valorem tax $u < 1$ is imposed in all markets of a price-discriminating monopolist with the following profit function and its first-order condition:

$$\max \pi_i = (1-u)\sum_i P_i Q_i - TC\left(\sum Q_i = Q\right)$$
$$(1-u)MR_i = MC\left(\sum Q_i = Q\right). \tag{25}$$

This is a familiar first order condition as that in a pure monopoly model.

Similarly, a cost payment ad valorem tax $v$ $(v > 0)$ gives rise to the following profit function and the first-order condition

$$\pi_2 = \sum p_i Q_i - (1+v)TC\left(\sum Q_i = Q\right) \tag{26}$$

$$MR_i = (1+v)MC\left(\sum Q_i = Q\right). \tag{27}$$

Given that $0 < u < 1$ and $v > 0$, there exists an output level $Q$ that can be generated by both a set of $u$ and $v$. Let us assume that this output level $Q^*$ exists and compare equations (25) and (27). A brief inspection indicates

$$MC\left(\sum Q_i = Q\right) = MR_i / (1+v) = (1-\mu) MR_i . \tag{28}$$

At a given output $Q_i$ of market $i$, hence total output $Q$, the marginal cost is constant, which establishes $(1-u) = 1/(1+v)$, the Musgravian transformation. Numerical simulations using two linear demand functions from Hirschey (2008) readily verifies that prices and outputs are identical for $u = 0.1$ and $v = 0.11111$.

## 4. MUSGRAVIAN TRANSFORMATION IN THE RATE-OF-RETURN-REGULATED MONOPOLY

The rate-of-return-regulated monopoly model pioneered by Averch and Johnson (1962) or A-J, has been an important topic in microeconomics, and since then there has been a vast amount of literature devoted to either theoretical advances or empirical estimations. The results of the empirical tests on the A-J hypothesis are of a mixed bag: Petersen (1975), Hayashi and Trapani (1976) and Cowing (1978) supported the model, whereas Moore (1970), Boyes (1976), and Barron and Taggert (1977) rejected the model. Recent applications may be found in Silverman (1982, 1985), Atkinson and Halvorsen (1984) and Hsu and Chen (1990). The A-J model still remains one of the major monopoly models, especially in the wake of the ongoing trend to regulate the utility industry again. The taxation effects of the model may be found in Yang and Fox (1994A; 1994B) regarding property tax and the Suits-Musgrave theorem. In this section, we analyze the two different taxation effects. Following the convention of Baumol and Klevorick (1970) and Bailey (1973), we can express the objective function of a regulated monopolist facing a demand ad valorem tax rate $u$ as

$$\max \pi = (1-u) PQ - wL - rcK \tag{29}$$

$$\text{subject to}: (1-u) PQ - wL - scK \le 0 \tag{30}$$

$$K \ge 0,\ L \ge 0,\ Q \ge 0,\ P \ge 0 \tag{31}$$

where $Q = f(L, K)$ is a production function, $P$ – price, $L$ – labor, $w$ – wage rate, $r$ – cost of financial capital, $c$ – cost of physical capital, $K$ – capital, $s$ – allowed rate of return.

The first-order condition from the Lagrangian function $X$ is

$$X_L = (1-\alpha)\left[(1-u)R_L - w\right] = 0$$

or

$$(1-u)R_L = w \text{ for } \alpha \neq 1 \tag{32}$$

$$X_K = (1-\alpha)\left[(1-u)R_K - rc + \alpha sc\right] = 0$$

$$R_K = (rc - \alpha sc)/(1-\alpha)(1-u), \tag{33}$$

where $R_L$ and $R_K$ are marginal revenue products of $L$ and $K$, and $\alpha$ is the Lagrange multiplier. Similarly, a supply ad valorem tax rate $v$ can be applied to the total cost payment $wL + rcK$ (on both $L$ and $K$) with the following profit function and the constraints:

$$\max \pi = PQ - (1+v)(wL + rcK) \tag{34}$$

subject to:

$$PQ - (1+v)(wL - scK) \leq 0 \tag{35}$$

$$K \geq 0,\ L \geq 0,\ Q \geq 0,\ P \geq 0. \tag{36}$$

Hence, the first-order conditions of the Lagrangian function $Y$ are

$$Y_L = R_L - (1+v)w - \beta R_L + (1+v)\beta w = 0$$

or

$$R_L = (1+v)w \text{ for } \alpha \neq 1 \tag{37}$$

$$Y_K = R_K(1-\beta) - (1+v)rc - \beta sc = 0$$

or

$$R_K = \left[(1+v)rc - \beta sc\right]/(1-\beta) \tag{38}$$

where $\beta$ is the Lagrange multiplier under the supply ad valorem tax. For a given output level, the marginal revenue product of labor is identical because there exists a unique one-to-one correspondence between total output and marginal product of labor. By equating equations (32) and (37), we have $1/(1-u) = (1+v)$ just for the marginal revenue product of labor. However, by comparing equations (33) and (38), it does not suggest any equivalence for that of capital. In particular, the two Lagrange multipliers ($\alpha$ and $\beta$) reflecting marginal profits under two taxation systems when the allowed rate of return is infinitesimally changed, are not likely to be the same. As shown by Musgrave (1959), the two profit levels are not going to be the same even in the case of an unregulated (pure) monopoly model. As such, it is highly unlikely that $\alpha = \beta$ . Note that equations (33) and (38) are equivalent only when both multipliers are zero: an unregulated pure monopoly case as was proved by Musgrave (1959). Consequently, the Musgravian transformation does not hold since both equilibrium conditions are not likely to be satisfied simultaneously. We illustrate the non-equivalence property via a simulation in which a linear demand function $P = 1 - 0.001Q$, a CES production function $\mathrm{Q} = 0.5\left[\left(0.25L^{-3} + 0.75K^{-3}\right)\right]^{-1/3}$ and $w$ = 0.025, $s = 0.20$, $r$ = 0.15, and $c = 1$ are assumed. The numerical results are shown in Table 1. As mentioned above, the Musgravian transformation does not hold in the rate-of-return-regulated monopoly. That is, a demand ad valorem tax on consumers may well generate rather different impacts from those of the cost ad valorem tax on the corresponding utility plants.

Table 1

The Non-Equivalence Taxation Property of the Rate-of-Return -Regulated Monopoly*

| **Tax rate / Solution** | **P** | **Q** | **L** | **K** | **π** |
|---|---|---|---|---|---|
| Zero tax | 0.531* | 468.95 | 700.146 | 1157.662 | 57.883 |
| *u=0.2* | 0.583 | 417.319 | 756.322 | 878.115 | 43.906 |
| *v=0.25* | 0.626 | 374.308 | 3136.171 | 608.981 | 28.113 |

* The simulation is performed using LINGO (1998)

Source: authors' own.

## 5. ASSUMPTIONS, EMPIRICAL APPLICATIONS AND NEW DIRECTIONS

In the literature of economics methodology, the validity of assumptions plays indeed a key role. In natural sciences, a counter-example can falsify a theory but empirical evidence can never prove a theory. Such a Popperian falsification doctrine can be rigorous but many times is at odds with logical positivism, i.e. the validity of assumptions of a theory is not that important as long as the theory predicts well the outcome, especially in social sciences with oversimplified assumptions (Friedman, 1953). It is within the framework of logical positivism that we examine the assumptions of the three monopoly models.

The major assumption of the two-part tariff model is the identical preference of each consumer facing a demand schedule from a (local) monopolist, i.e. a group of identical consumers can be modeled easily. However, several groups of customers are quite plausible. In this case the two-part tariff model can be altered slightly. First, admission fee ($T$ or consumer surplus) is determined on the demand curve of the weakest group from price $P_1$ which is greater than marginal cost. Second, one may then calculate the profit at $P_1$: $(n_1+n_2)T+(n_1+n_2)\sum x_i(P_1-MC)$, where $n_1$ and $n_2$ are the number of customers in groups 1 and 2, respectively. Third, try another price $P_2>MC$ and repeat the process until a profit-maximizing price $P^*$ is found. Once $P^*$ is located, the optimum admission fee $T^*$ can be determined as well. The rest of the taxation analyses follow exactly the same procedure.

Empirical applications of the two-part model are abundant: Verizon Wireless, Sprint, T-Mobile, AT&T wireless, Minutewatch and CellKnight all have one type or the other characteristic of two-part tariff plans. Verizon offers a \$60 monthly access fee with an average rate of \$15 per 500 MB. Other applications include credit cards, printers, Polaroid cameras, shopping clubs (e.g. Sam's Club, Costco), golf clubs, theme parks, landline telephones, cell phones, sports clubs, and time shares. Not surprisingly, it has become part of everyday life. Nonetheless, the taxation practice on the two-part tariff industry is not definitive: an ad valorem tax is generally imposed on the gross receipt (admission charge plus user fee): it is equivalent to the demand ad valorem tax in our paper. In the cell phone industry, US consumers pay an average of 17.05% in taxes and fees on their cell phone bill including 11.36 percent in state and local charges. The demand ad valorem tax $u=0.1705$ can be converted into cost ad valorem tax $v=0.2055$ via $1-u=1/(1+v)$. The top five states in terms of federal, state and local taxes are Washington ($u=24.42\%$ or $v=32.31\%$), Nebraska ($u=24.31\%$ or $v=32.12\%$), New York ($u=23.56\%$ or $v=30.82\%$), Florida ($u=22.38\%$ or $v=28.83\%$), and Illinois ($u=21.63\%$ or $v=27.60\%$), respectively (Mackey and Henchman, 2014). In Taiwan, consumers pay a 5% business tax on their cell phone bill. The 5% demand ad valorem tax is equivalent to 5.26% cost ad valorem tax (Ministry of Finance, 2014). It is to be noted that cell phone taxes in the US are mostly imposed as an ad valorem tax, but some of the taxes can be levied at a flat rate. For instance, a 911-emergency tax is levied at \$4 per line per month (Baltimore, Maryland), which can be readily computed into a percentage-based ad valorem tax. This is a regressive tax on the poor who live in a neighborhood where 911 calls are needed. A family share plan of four lines would easily cost $15 per month on a bill of $100 (15% ad valorem tax) in Chicago for emergency call(s) only.

Most golf clubs charge a two-part fee that includes initiation fee (or annual membership fee) and green fee per time. In addition, consumers pay an Amusement Park Tax that is imposed on gross receipts and is a demand ad valorem tax in Taiwan (Taiwan Ministry of Finance, 2014). The Amusement Park Tax rates are 2.5% in Taipei ($u = 2.5\%$ or $v = 2.56\%$), 10% in New Taipei City ($u = 10\%$ or $v = 11.11\%$) and 5% in Taichung ($u = 5\%$ or $v = 5.26\%$). We convert these demand ad valorem taxes ($u$) into cost ad valorem taxes ($v$) easily via the Musgravian transformation developed in this paper.

A residential natural gas bill in the US has two components: a monthly fee (customer's share of fixed costs) and volumetric charge, which was set at 130% of the marginal cost. Taxes may be imposed on gross receipts (demand side) disguised as gas cost recovery adjustment (monthly fee plus volumetric charges). On the supply side, natural gas taxes are known as severance tax once gas is extracted (e.g. 1.8% in Ohio; 2.6% in Louisiana; 4.6% in Texas; 7.2% in West Virginia). Note that $v = 0.018$ can be converted into $u = 0.0177$ via $1 - u = 1/\left(1 + v\right)$ for Ohio. The corresponding demand ad valorem tax rates are 2.53%, 4.40% and 6.72% for Louisiana, Texas and West Virginia, respectively. The combined demand ($u$) and supply ($v$) ad valorem taxes can be calculated by the Musgravian transformation: $\left(1 - u\right) = 1/\left(1 + v\right)$. For instance, a 6% demand ad valorem tax ($u = 6\%$) along with a 25% severance tax on natural gas ($v =$ 25% can be transformed to $u =$ 20%) makes a total tax rate of 26% ($6\% + 20\%$).

As is well-known in the literature, assumptions of third-degree price discrimination are the ability to set prices from different consumer groups based on price elasticity, segmentation of different markets, and the prevention of arbitrage. Due to the advent of threshold regression techniques (Tong, 1990), it is rather straightforward for a monopolist to estimate several statistically significant price elasticities (Huang and Yang, 2006). The regime-switching model has the advantage of locating breaking points endogenously in a given data set. Segmenting different markets and preventing arbitrage present no major obstacles. For instance, a senior's discount at restaurants can be verified via driver license or other ID; a student subscription to the Wall Street Journal requires a student ID. The resale of such goods is highly unlikely even though reselling prescription drugs from Mexico or Canada in the US was quite profitable. However, the legal restrictions have tightened as doctors' prescriptions from the US and adjacent countries are required at the port of entry. This has in effect stopped, to a large extent, the arbitrage behavior in the prescription drug industry.

There is a plethora of applications of third-degree price discrimination models: seasonal airfares, hotel discounts, advance purchase of train or airline tickets, student vs. alumni ticket prices at college football games, college textbooks sold domestically or abroad, prescription drugs, and others. The patent in itself is a license to set different prices to different groups of customers. Nearly all taxes on prescription drugs are ad valorem in the US: Colorado ($u = 2.9\%$ or $v = 2.99\%$); Georgia ($u = 4\%$ or $v = 4.17\%$ ); Idaho ($u = 6\%$ or $v = 6.38\%$). Illinois ($u = 1\%$ or $v = 1.01\%$), Tennessee ($u = 7\%$ or $v = 7.53\%$) etc. The tax rate on gross receipts is generally higher in developing countries: Bolivia ($u = 13\%$ or $v = 14.94\%$), Brazil ($u = 24\%$ or $v = 31.58$), Congo ($u = 19\%$ or $v = 23.46\%$), and China ($u = 20\%$ or $v = 25\%$). Customers in Asia are normally charged an airport terminal tariff per airline ticket. The tariffs are different between countries: NTD300 in Taipei (Taiwan Taoyuan International Airport, 2014), JPY2,570 in Tokyo (Tokyo International Air Terminal Corporation, 2014), THB700 in Bangkok (Suvarnabhumi Bangkok Airport, 2014) and RMB90 in Shanghai (Shanghai Airport Authority, 2014) per ticket. All of the airport tariffs are demand specific taxes and can be converted into regressive ad valorem taxes. Suppose the price of an airline ticket is NTD10,000 from Taipei to Tokyo and the exchange rate is 0.25NTD/JPY. These terminal tariffs are equivalent to 3% (300/10,000=3%) and 6.43% (2570×0.25/10,000=6.43%) demand ad valorem tax for Taiwan and Japan, respectively. It is easy to calculate the cost-based supply ad valorem tax ($v = 3.09\%$ for Taiwan and $v = 6.87\%$ for Japan) in this example. These demand ad valorem taxes can easily be calculated into cost-based supply ad valorem taxes via the Musgravian transformation developed in this paper.

In Taiwan, there are student discount plans for those who buy motorcycles frequently. The commodity tax which is a kind of cost ad valorem tax for producing motorcycles is 17%. In addition, the business tax which is a demand ad valorem tax for selling motorcycles is 5% (Taiwan Ministry of Finance, 2014) and can be converted into cost ad valorem tax $v = 5.26\%$ via $1 - u = 1/(1 + v)$. Thus it is easy to calculate the total ad valorem tax for a motorcycle (17%+5.26%=22.26%) in Taiwan.

Regulations on utility companies have come a long way since the seminal paper by Averch and Johnson (1962). The fundamental assumption is that a utility company is a natural monopoly with declining average and marginal cost curves. The well-known estimated cost structure of utility firms by Christensen et al. (1973) and Hsu and Chen (1990) confirmed it. Since the fair rate return on which price is based encourages "padding" of the cost or overcapitalization, the duty of a state public utility commission is to monitor the potential bias. There is also the capture theory of regulation, in which the utility company takes over the regulatory agencies. Regardless of the problems, the rate-of-return-regulated monopoly model has been used for nearly 50 years, especially in the industries supplying water, natural gas, utility and cable TV.

Nearly every state in the US levies a utility tax (electricity power): Alabama ($u = 3.87\%$ or $v = 4.03\%$), Delaware ($u = 4.25\%$ or $v = 4.44\%$), Minnesota ($u = 2.3\%$ or $v = 2.35\%$), and Washington ($u = 3.87\%$ or $v = 4.03\%$). These are demand ad valorem (percentage) taxes on gross receipts (utility bills). The cost sides of taxes are carbon taxes on burning fossil fuels. For instance, British Columbia in Canada imposed a carbon tax of \$25 per ton of $CO_2$; Australia's carbon tax is levied at \$19.60 and Chile's carbon tax is only \$5. These are equivalent to unit taxes on the cost of generating electricity. Suppose the total quantity of output from emitting a ton of $CO_2$ is 100 units (unit cost is \$5), the carbon tax on cost is \$25, hence the cost ad valorem tax rate is 25/(100×5)=5%. Note that the cost ad valorem tax rates cannot be converted to the corresponding demand ad valorem taxes, because the Musgravian transformation between demand and cost sides does not apply.

In Taiwan, the tax schemes of utility industries are similar to the US. Taiwan imposes a 5% business tax, a demand ad valorem tax, on utility industries which can be converted into a 5.26% cost ad valorem tax. The cost sides of taxes are a commodity tax on producing or importing fossil fuels. For instance, the taxes are NTD6,830 for gasoline, NTD3,990 for diesel fuel and NTD690 for gas per kiloliter (Ministry of Finance, 2014). They can readily be converted into an ad valorem tax.

The traditional two-part tariff model is now being rapidly expanded to the three-part tariff model, especially in the mobile phone industry. This consists of a fixed monthly fee, free minutes allowance and overage charge (charge for the calls above and beyond the allowance during peak hours, usually from 6:00 AM to 9:00 PM). For example, for a fixed monthly fee, 500 peak minutes and 5000 weekend and weekly night minutes are allowed. Overage rates apply if one uses above and beyond the limits. The average rate during peak hours can be 40 cents or 50 cents per minute depending on the different mobile phone companies. Recently, new findings have indicated that customers value free minutes more in such a way that cannot be explained by the income effect in classical price theory. The taxation theory developed in this paper can be easily applied to a three-part tariff because the tax on free minutes is zero.

The third-degree price discrimination model is gaining momentum in the era of big data as more and more consumers effect their shopping habits and personal information via on-line shopping. Customized e-commerce is the trend in the future and as such taxation theories developed in this paper apply nicely: imposing ad valorem tax either on customers or manufacturing companies makes no difference. Thus, one may have the choice to levy tax on the party that has the least tendency to evade taxes.

Table 2

Summary of the Monopoly Models

| **Model** / **Properties** | **Two-Part Tariff** | **Third-Degree Price Discrimination** | **Rate-of-Return-Regulated Monopoly** |
|---|---|---|---|
| Model Assumption | It can accommodate several groups of customers with heterogeneous preferences | Price elasticity can now be readily calculated by regime-switching econometrics. Prevention of reselling is gaining momentum as checking and inspection at port of entry or interstate commerce law enforcement has tightened. | Natural monopoly assumption for utility companies was statistically confirmed by Christensen et al. (1973) and by Hsu and Chen (1990). Overcapitalization and cheating can be effectively checked by proper measures of reward and punishment. |
| Application | Mobile phone, credit cards, printer, Taxicab fare, shopping club, sports club, amusement park, time shares, landline telephone. | Senior's discount at restaurants, movie theaters, college textbooks sold in different countries, student subscriptions to *Wall Street Journal*, college football game prices for students and alumni, advance train and airline tickets, airfare, prescription drugs in different countries, subway rides in Taiwan. | Water companies, natural gas companies, utility companies, cable TV. |
| Taxation Theory | $(1-u)=1/(1+v)$ applies if tax is levied on both admission fee and usage charge (gross receipt). | $(1-u)=1/(1+v)$ applies in all regions. | $(1-u)=1/(1+v)$ does not apply so there is no corres-ponddence between demand ad valorem tax $\mu$ and cost ad valorem tax $v$. Case by case simulations are required. |

Source: authors' own.

The rate-of-return-regulated monopoly was once suspect because of its problems. However, the deregulation at a wholesale level (power generation) in California was disastrous: power companies did not build a single generating plant and hence, the rate skyrocketed. In many states of the US, regulation on utility price is still the norm. Cheating and overcapitalization may be effectively diminished by rewards on cost saving and punishment for padding (Tirole, 1988). The taxation impacts, however, need to be evaluated by simulations case by case since the Musgravian transformation does not hold true in the model.

## CONCLUSION

It has been over half a century since Musgrave proposed and proved the equivalence of demand ad valorem and the corresponding cost payment ad valorem taxes in both perfect competition and pure monopoly models. In a world of zero tax evasion, the two tax systems are identical in price-output decision in a pure monopoly. However, this property should not be taken for granted as other monopoly models, which are more empirically relevant, may not preserve this equivalence property. As shown in this paper, when both user price and admission fees are taxed at the same rate, the Musgravian transformation indeed holds. However, this does not hold if only one of them is taxed. In the case of third-degree price discrimination, it holds if a uniform tax rate (national or federal) is imposed across all regions (markets). In the case of a rate-of-return-regulated natural monopoly, it was shown that the Musgravian transformation does not hold even in the case of a linear demand function and a homogeneous of degree 1 CES production function. It is recommended that a cost payment tax may be appropriate in the presence of prevalent tax evasion at retail level, i.e. sidewalk peddlers selling sunglasses. On the other hand, a tax on consumers may be preferred for a firm with multiple products where overheads are difficult to allocate among different products.

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