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# LIQUIDITY EFFECTS IN THE GERMAN BOND MARKET: FINDINGS FROM THE JUMBO PFANDBRIEFE SEGMENT

The purpose of this article is to investigate liquidity effects in the German bond market. Using data on Jumbo Pfandbriefe and German government bonds, the author derives an accurate estimate of the term structure of liquidity spreads in the period January 1999 to October 2008. On average, long maturity bonds exhibit a higher liquidity premium than short maturity bonds. In times of crisis, however, the term structure can invert. A principal components analysis shows that 78.70 % of the total variation in liquidity spread changes can be explained by a single component and augmented Dickey-Fuller tests reject the hypothesis of a unit root for liquidity spread changes in all maturity classes under examination. Therefore, an affine one-factor model of the term structure of liquidity spread changes is presented and a factor time series is extracted by the use of a Kalman filter in combination with Maximum Likelihood estimation of the model parameters. Subsequently, the models' empirical performance is analysed using differences between real and model spread changes as well as one-step-ahead prediction errors generated by the Kalman filter. On average the model provides an adequate fit of the term structure of liquidity spreads for medium term maturities.

**Keywords:** term structure, liquidity risk, bond market, Germany, Principal Components Analysis (PCA), affine model, Kalman filter.

## INTRODUCTION

Beside market risk and credit risk, significant risk arises to investors from the lack of liquidity. Liquidity describes the ability to convert an asset into cash in a very short period of time at an expected price, i.e. without loss in value. This ability is of crucial importance to investors especially during times of crisis because it allows them to react and to disengage as quickly as possible. Therefore, investors prefer to hold highly liquid securities particularly during times of great uncertainty – for example, after the default of the Russian government in 1998 "when Treasury bonds suddenly increased in value relative to less liquid debt instruments, causing credit spreads to widen" (Longstaff, 2004, p. 512). Because "bond spreads form the

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basis for a variety of trading strategies" (Koziol and Sauerbier, 2007, p. 81) their changes can result in severe losses, for example at hedge funds that have taken highly levered positions in corporate bonds. Due to these losses academic and practitioner's attention to liquidity risk increased. [See, for example, Goyenko et al., 2008 and the *Wall Street Journal*, "Illiquidity is Crippling the Bond World," (October 19, 1998) p. C1, "Illiquidity means it has become more difficult to buy or sell a given amount of any bond but the most popular Treasury issue. The spread between prices at which investors will buy and sell has widened, and the amounts in which Wall Street firms deal have shrunk across the board for investment grade, high-yield (or junk), emerging market and asset-backed bonds…The sharp reduction in liquidity has preoccupied the Fed because it is the lifeblood of markets."] Investment firms, banks, regulators and science throughout the world have recognized the importance of appropriate methods for identifying, measuring and understanding liquidity risk and its effect on asset prices as well as the associated dynamics of bond market liquidity. In spite of "the importance of understanding liquidity dynamics there remain critical gaps in the literature on bond market liquidity" (Goyenko et al., 2008, p. 3). Unfortunately, most studies with regard to liquidity risk concentrate on the U.S. market. [See, for example, Fisher, 1959; Sarig and Warga, 1989; Amihud and Mendelson, 1991; Warga, 1992; Kamara, 1994; Crabbe and Turner, 1995; Elton and Green, 1998; Elton et al., 2001; Fleming, 2002; Longstaff, 2004; Driessen, 2005.] Not only because of different laws and regulations but also because of different stages of market development the results of these studies cannot be simply transferred to other markets such as the European market. Therefore, further research is needed.

Among the EU Member States which have proceeded to the third stage of the European Economic and Monetary Union (EMU), Germany has the largest bond market. After the U.S. market, it also exhibits the second-largest share of the global bond market (source: ESCB and Bank for international Settlements as in December 2009). Therefore, international market participants show an increasing interest in understanding the factors that determine the prices of German bonds.

Notwithstanding the increasing interest there are only a few studies that analyse liquidity effects in the German market. These studies only span short time periods and some of them date back to pre-euro period. In 2000, Kempf and Uhrig-Homburg proposed a theoretical continuous-time model that explains price differences between liquid and illiquid bonds. To test their model, the authors used data on German government bonds denominated in

Deutsche Mark in the period from January 1992 to December 1994. Their results indicate the existence of significant liquidity effects in the examined market segment during the sample period.

Another paper that analyzes liquidity effects in the German bond market was published by Koziol and Sauerbier in 2007. The authors used data on two types of bonds: German government bonds and Jumbo Pfandbriefe. These bonds are ideally suited to analyze liquidity effects because they are homogenous in almost all respects but liquidity. Based on a model for the valuation of illiquid bonds Koziol and Sauerbier hypothesize a humped-shaped relation between liquidity spreads – i.e. yield differences between liquid and illiquid bonds – and time to maturity but find no fully convincing support for this supposition in the period from January 2000 to December 2001. Unfortunately, their methodology gives rise to impreciseness because they treat coupon bonds as zero bonds. Furthermore, they only span a short time period which hampers the deduction of general conclusions.

The study at hand also uses Jumbo Pfandbriefe and German government bond data because of the special attributes of this market segment that facilitate the analysis of liquidity effects. The results of this analysis, however, are intended to give an indication of liquidity effects in other German market segments as well. In order to avoid the problems Koziol and Sauerbier were facing and to deduce reliable results concerning the relationship between liquidity spreads and time to maturity (i.e. the term structure of liquidity spreads) the study at hand uses modified methods. First, yields of zero bonds are estimated from coupon bonds by the use of the Nelson/Siegel (1987) method which results in spot curves that are smooth by the parsimonious nature of the functional form. Second, a long time-series of bond data is used that spans the time period from January 1999 to October 2008. This is important because it allows for an inclusion of a variety of trade cycle phases and economic events which in turn leads to a higher validity of results. Furthermore, it is a precondition to achieve a further objective of this study: the comprehensive investigation of liquidity spread dynamics, i.e. the analysis of the evolution of the term structure of liquidity spreads in the observed market segment in the course of time. Therefore, the study also includes a correlation analysis and principal component analysis of liquidity spread changes which permits to find common movements and to explain as much of the total variation in liquidity spreads with as few factors as possible. Finally, an affine one factor model of liquidity spreads is presented and tested for its applicability. State variables and model parameters are estimated by means of the Kalman filter in combination with maximum-likelihood estimation.

The remainder of this article is structured as follows. Section 1 starts with a description of the methodology. The data and preliminary results are presented in Section 2. Detailed results are discussed in Section 3.

## 1. METHODOLOGY

When analysing corporate bonds, it is difficult to isolate the effect of liquidity risk, because corporate bond spreads are affected by a number of additional factors like default risk, recovery risk and jumps. In order to avoid these factors, this study uses data on German covered bonds to derive an accurate estimate of the term structure of liquidity spreads. Covered bonds are securities backed by high-quality mortgage loans or public sector loans. Because of their unique safety, which can be explained by the legal framework, covered bonds are popular in Germany where they are called Pfandbriefe. In 1995, the first Jumbo Pfandbriefe were issued. These bonds are based on the same legal framework as ordinary Pfandriefe but currently they exhibit a minimum issue volume of 1 billion euros. One important aspect of this market is its outstanding homogeneity, which is also reflected in the ratings awarded to Jumbo Pfandbriefe. All bonds that are used in this study are AAA-rated by the main rating agencies and therefore have credit standings comparable to government bonds. On the one hand, there is no essential credit-risk in the Pfandbriefe market. [See, for example, Koziol and Sauerbier, 2007, p. 99] Since the introduction of the German Mortgage Bank Act in 1900, no German mortgage bank has defaulted and there has never been a case of principal default over the entire 225 years of history of Pfandbriefe. [Beyond that, Packer et al., 2007, p. 53f, show that “the credit quality of covered bonds can be robust even to very pronounced declines in issuer creditworthiness.”] On the other hand, investors query liquidity in the German Pfandbrief market. According to a poll conducted by the Association of German Pfandbriefe Banks in 2006 among investment banks active in the covered bond market, only 29% of market makers judge liquidity in the Jumbo Pfandbriefe secondary market to be fully satisfactory. 36% of market makers believe that liquidity has to be improved and the rest judges liquidity to be only at times satisfactory. [See Association of German Pfandbriefe Banks, 2006.] The secondary market of German government bonds, however, is considered as a default free market with constantly high liquidity levels. [See German Central Bank, 2000.] Furthermore, both Jumbo Pfandbriefe and German government bonds fall under the same tax legislation. By the use of these two types of bond data it is therefore possible to select two sets of bonds which are homogenous in all respects but liquidity. Differences in yields between these bonds must therefore be due to liquidity effects. This approach is adopted from related studies by Amihud and Mendelson (1991) who compare short-term U.S. Treasury notes and bills in order to investigate liquidity effects in the U.S. market, Longstaff (2004) who compares U.S. Treasury bond prices with the prices of bonds issued by the Resolution Funding Corporation (Refcorp) and Koziol and Sauerbier (2007) who analyze liquidity spreads in the German bond market from January 2000 to December 2001.

In this study, the method introduced by Nelson and Siegel (1987) is used to estimate two spot curves: one for highly liquid German government bonds and another for Jumbo Pfandbriefe. The spot curve estimates are guaranteed to be smooth by the parsimonious nature of the functional form. The difference between both term structures is then defined as the term structure of liquidity spreads. [Of course, the term structure of liquidity spreads could have been also calculated from ordinary Pfandbriefe. But Breger and Stovel, 2004, p. 241, report that because of the extremely low liquidity of conventional Pfandbriefe only a few of them actually trade on a given day. Therefore, most prices of ordinary Pfandbriefe are matrix prices "produced by valuation models". In order to conduct research with traded prices that reflect the pure preferences of market participants, this study uses the prices of Jumbo Pfandbriefe.]

Subsequently, the relation between liquidity spread and the bond's time to maturity can be determined. According to Amihud and Mendelson (1991), the liquidity spread for short-term U.S. Treasury bonds is a decreasing function of the time to maturity. Based on their valuation model, Koziol and Sauerbier (2007) expect a humped-shaped dependence of liquidity spreads on time to maturity which means that liquidity spreads rise with time to maturity for short term bonds and then decline for long term bonds. Because in the study at hand short-term bonds with time to maturity of less than one year are excluded from the sample, a decreasing trend is expected for the term structure of liquidity spreads. Thus, the first research hypothesis is stated as follows:

> *$H_1$: The liquidity spread calculated from Jumbo Pfandbriefe and German government bonds is a decreasing function of time to maturity.*

A correlation analysis can be used to investigate if changes in liquidity spreads on bonds with different maturities are correlated. If changes in liquidity spreads are highly correlated they contain essentially the same information. A principal components analysis (PCA) can then be employed to find common movements in liquidity spreads and to determine factors in order to explain as much of the total variation in the data as possible with as few of these factors as possible. Because increasing liquidity risk is assumed to have a similar effect across bonds with different maturities the second research hypothesis is stated as follows:

*$H_2$: Movements of the term structure of liquidity spreads, calculated from Jumbo Pfandbriefe and German government bonds, are basically due to a single component.*

This hypothesis is consistent with the model assumptions of Kempf and Uhrig-Homburg (2000). The authors model any disadvantage of an illiquid investment in comparison with a liquid one by means of a single state variable. The study at hand pursues their approach by using an affine one-factor model to describe the term-structure dynamics of liquidity spreads.

Because of the higher risk, investors will ask for a higher yield if they are investing in illiquid bonds. In order to take account of this kind of yield spread we introduce a liquidity discount factor $D_t(T)$ from time $t$ to time $T$. With the price of a risk-free liquid zero-coupon bond $P_t(T)$ which pays one euro at maturity $T$ the price of an illiquid zero-coupon bond with time to maturity $\tau = T - t$ is then given by

$$P_t^*(T) = P_t(T) \cdot D_t(T).$$

Let $l_t(\tau)$ denote the yield spread due to liquidity risk and consider dates $t_i$ and $t_{i+1}$. The liquidity discount factor can then be expressed as

$$D_{t_i} = \exp\left(-l_{t_i}(\tau) \cdot \tau\right).$$

With

$$\delta D_{t_i} = \exp\left(-\left(l_{t_{i+1}}(\tau) - l_{t_i}(\tau)\right) \cdot \tau\right) = \exp\left(-\Delta l_{t_i}(\tau) \cdot \tau\right)$$

we get

$$D_{t_{i+1}} = D_{t_i} \cdot \delta D_{t_i} = exp\left(-l_{t_{i+1}}(\tau) \cdot \tau\right).$$

Now let $x$ denote a state variable that accounts for changes in liquidity spreads and assume that $x$ has the following stochastic differential

$$dx_t = -\alpha x_t dt + \sigma dW_t, \tag{1}$$

where $\alpha > 0$ and $\sigma$ are positive constants and $W$ is a Wiener process under the empirical probability measure $P$. In this specification the drift depends on $x_t$ negatively through the parameter $\alpha$. Therefore, $x$ fluctuates around the long-run mean of zero and the parameter $\alpha$ controls how long excursions away from zero will take. Given these dynamics for the state variable, results by Vasicek (1977) in combination with the assumption of a long-run mean of zero make it straightforward to derive an analytically tractable affine function $f$ for liquidity spread changes

$$\Delta l_t(x_t, \tau) = f(x_t).$$

The affine relationship between liquidity spread changes and the state variable allows for an estimation of the most likely realization of this unobservable variable with the Kalman filter.

Affine models are particularly suited for estimating using the Kalman filter because of their linear structure. [See, for example, Geyer and Pichler, 1999.] To use the Kalman filter the liquidity model has to be written in the state space form. [See Harvey (2001) for technical details.]

If we consider the dates $t_i$ and $t_{i-1}$ with $\Delta t \equiv t_i - t_{i-1}$ for all $i = 1, 2, \ldots, m$ the transition equation is given by

$$x_{t_i} = e^{-\alpha \cdot \Delta t} \cdot x_{t_{i-1}} + \eta_{t_i}$$

with

$$\eta_{t_i} \sim N\left(0, \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha\Delta t})\right).$$

Let $\Delta l_{t_i}(\tau_j)$ be the theoretical liquidity spread change of an illiquid zero-coupon bond with time to maturity $\tau_j$ $(j = 1, 2, \ldots, n)$ at time $t_i$ $(i = 1, 2, \ldots, m)$ and $\Delta \bar{l}_{t_i}(\tau_j)$ the corresponding real spread change. The liquidity model will not exactly explain the real spread changes, so a normally distributed error term $\varepsilon_{t_i}(\tau_j)$ is added

$$\Delta\bar{l}_{t_i}(\tau) = \Delta l_{t_i}(\tau) + \varepsilon_{t_i}(\tau)$$

where

$$\Delta\bar{l}_{t_i}(\tau) \equiv \left(\Delta\bar{l}_{t_i}(\tau_1), \Delta\bar{l}_{t_i}(\tau_2), \cdots, \Delta\bar{l}_{t_i}(\tau_n)\right)'$$

$$\Delta l_{t_i}(\tau) \equiv \left(\Delta l_{t_i}(\tau_1), \Delta l_{t_i}(\tau_2), \cdots, \Delta l_{t_i}(\tau_n)\right)'$$

$$\varepsilon_{t_i}(\tau) \equiv \left(\varepsilon_{t_i}(\tau_1), \varepsilon_{t_i}(\tau_2), \cdots, \varepsilon_{t_i}(\tau_n)\right)'$$

Furthermore, the measurement equation is

$$\Delta\bar{l}_{t_i}(\tau) = bx_{t_i} - a + \varepsilon_{t_i}(\tau)$$

where $a$ and $b$ are given by the Vasicek (1977) model. For the error term we have $\varepsilon_{t_i}(\tau) \sim N_n(\mathbf{0}, \mathbf{\Sigma})$. $\mathbf{\Sigma}$ is the variance-covariance matrix of $\varepsilon_{t_i}(\tau)$. $\mathbf{\Sigma}$ has constant dimensions $n \times n$ and is assumed to be a diagonal matrix

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_{\varepsilon_1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{\varepsilon_n}^2 \end{pmatrix},$$

The state-space representation has the advantage of allowing for panel data, i.e. combined time series and cross sectional data, in establishing dependency of observed series on latent factors. Given a parametric process form for the latent factor, the most likely realization of the factor series can subsequently be estimated in combination with Maximum Likelihood estimation of the model parameters. For this purpose, the previously calculated empirical liquidity spread changes can be employed. In addition, the state space representation has the potential to offer insights which can be used to ensure that the dynamics of the affine term structure model are reasonable from an empirical perspective.

## 2. DATA DESCRIPTION AND PRELIMINARY RESULTS

The bond data used in this study are obtained from Bloomberg. The sample comprises 471 Jumbo Pfandbriefe and 156 German government bonds denominated in EUR. Weekly closing bid prices were collected on every Wednesday from January 6, 1999, to October 8, 2008. Thus, this study includes price data on Jumbos and Bunds for 510 days during the sample period. For the most part collected prices are transaction prices. If there are no transaction prices an actual bid quote for Bunds and Jumbos can almost always be obtained. The data set is restricted to fixed-rate, AAA-rated straight bonds with annual coupon payments. Beside coupons, further descriptive data, i.e. issuer, coupons, maturity dates, issued amount and ratings are downloaded. As in Duffee (1999), all bonds have at least one year remaining to maturity. Furthermore, the bonds are included in Merrill Lynch's Pan-European Broad Market Index, which tracks the performance of the major investment grade bond markets in the Pan-European region. In addition, Jumbos are currently required to have a minimum issue volume of 1 billion euros, but Pfandbriefe which have been issued before April 28, 2004 can keep the status of a Jumbo even if they have an issue volume of less than 1 billion euros. The average size of Jumbos included in the sample set is approximately 1.5 billion euros. Some Jumbos have a volume of up to 5 billion euros. Coupon rates on Jumbos in the sample range between 2.5% and 6.75% and average out at 4.43%. The government bonds included in the sample have average and maximum sizes of 12 and 27 billion euros respectively. Their coupon rates are 4.78% on average and range between 2% and 9%.

On each Wednesday during the sample period, the method proposed by Nelson and Siegel (1987) is used to estimate spot rates for 9 equally spaced points on the term structure, i.e. 2 to 10 years to maturity, for both government bonds and Jumbo Pfandbriefe. [A comparison with the results obtained by the Svensson method for German government bonds, published by the German Bundesbank reveals only small differences. The corresponding results for Pfandbriefe, however can differ because of different base data.] The liquidity spread is then defined as the difference between the spot rate on Jumbo Pfandbriefe and the spot rate on government bonds of the same maturity. Figure 1 plots the time series of liquidity spreads on zero bonds with 2, 6 and 10 years to maturity over the period January 1999 – October 2008.

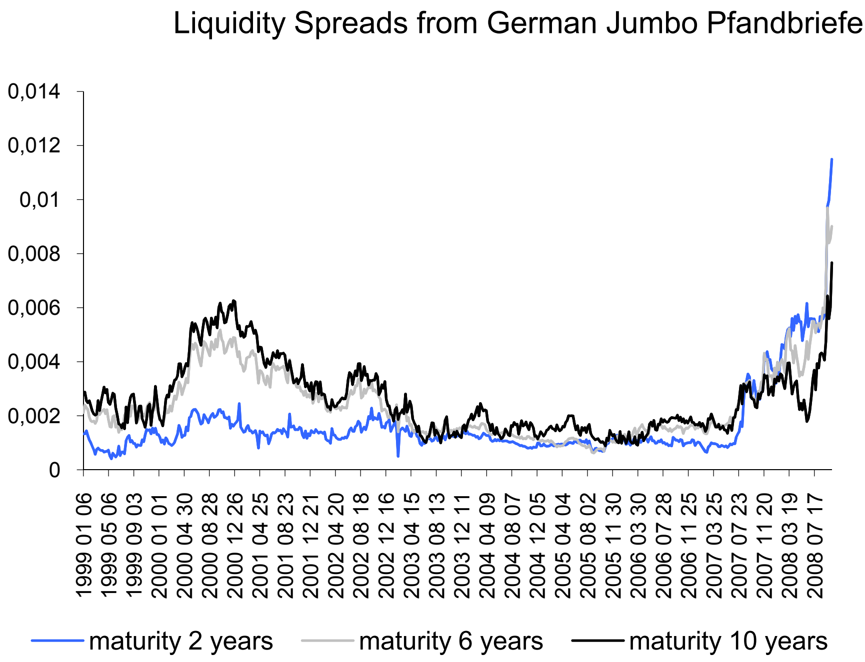


Figure 1. Liquidity spread series

Source: own calculations with the use of Excel and Matlab

The figure above displays the time series of liquidity spreads calculated from German government bonds an Jumbo Pfandbriefe on 2-, 6- and 10-year spot payments for the almost ten year period from January 6, 1999 to October 8, 2008.

## 3. ANALYSES AND DETAILED RESULTS

Table 1 shows that on average, long maturity bonds exhibit a higher liquidity premium than short maturity bonds. Hypothesis 1 is therefore not supported. On the one hand, this result can be due to the fact that Amihud and Mendelson (1991) compare only short-term U.S. Treasury notes and bills in order to investigate liquidity effects, whereas this study uses long term bonds. On the other hand, it can be explained by the following consideration: in the case of illiquid bonds with rather short time to maturity, the nominal value will be paid back within a short period of time. So even if it is not possible to sell this bond, the liquidity risk is rather small in comparison with bonds that exhibit a considerable remaining time to maturity. Because at maturity there is no difference between liquid and illiquid bonds, liquidity spreads must decrease when time to maturity approaches zero. [See, for example, Koziol and Sauerbier, 2007, p. 82 or Kempf and Uhrig-Homburg, 2000, p. 36f.] If a bond is hard to sell and does not mature in the near future, the liquidity risk is high and therefore investors will demand a higher yield. The logical consequence is an upward sloping term structure of liquidity spreads. [Kempf and Uhrig-Homburg (2000, p. 36f) show for their sample that the mean price difference between liquid and illiquid bonds increases with bonds' maturity. Although this is not necessarily proof of a positive slope of the term structure of liquidity spreads in their sample, it supports the findings of the study at hand.]

In times of crisis, however, the situation can change because investors may shift funds into short-term government bonds [see Goyenko et al. 2008, p. 5] which in turn might cause prices (yields) of government bonds to increase (fall). Because liquidity spreads are determined as the difference between yields of Jumbo Pfandbriefe and government bonds, the liquidity spread increases when the yields on government bonds fall – other things being equal.

Table 1

Average liquidity spreads

This table shows the average liquidity spreads in the period January 6, 1999 to October 8, 2008. The results reported are for spreads on zero bonds with 2 to 10 years to maturity.

| Maturity in years | 2 years | 3 years | 4 years | 5 years | 6 years | 7 years | 8 years | 9 years | 10 years |
|---|---|---|---|---|---|---|---|---|---|
| average liquidity spread | 0,0017 | 0,0020 | 0,0022 | 0,0023 | 0,0024 | 0,0025 | 0,0025 | 0,0026 | 0,0026 |

Source: own calculations with the use of Excel and Matlab

Furthermore, table 2 shows that there is considerable correlation between the changes in liquidity spreads on zero-coupon bonds with different maturities. The correlations are the highest for close maturity dates. As we move to bonds of distant maturity dates the correlations decline to a lowest of 0.35.

Table 2

Correlation of liquidity spread changes

This table shows the correlations of weekly liquidity spread changes from January 6, 1999 to October 8, 2008. The results reported are for spreads on zero bonds with 2 to 10 years to maturity.

| | 2 years | 3 years | 4 years | 5 years | 6 years | 7 years | 8 years | 9 years | 10 years |
|---|---|---|---|---|---|---|---|---|---|
| 2 years | 1,0000 | | | | | | | | |
| 3 years | 0,9090 | 1,0000 | | | | | | | |
| 4 years | 0,8324 | 0,9768 | 1,0000 | | | | | | |
| 5 years | 0,7666 | 0,9181 | 0,9786 | 1,0000 | | | | | |
| 6 years | 0,6978 | 0,8296 | 0,9161 | 0,9776 | 1,0000 | | | | |
| 7 years | 0,6244 | 0,7184 | 0,8176 | 0,9109 | 0,9762 | 1,0000 | | | |
| 8 years | 0,5476 | 0,5936 | 0,6916 | 0,8046 | 0,9051 | 0,9748 | 1,0000 | | |
| 9 years | 0,4706 | 0,4666 | 0,5519 | 0,6713 | 0,7953 | 0,9031 | 0,9755 | 1,0000 | |
| 10 years | 0,3987 | 0,3503 | 0,4160 | 0,5307 | 0,6660 | 0,8002 | 0,9108 | 0,9790 | 1,0000 |

Source: own calculations with the use of Excel

Because of their high correlation, liquidity spreads of different maturities essentially contain the same information. For this reason, the number of variables can be reduced while maintaining a bigger part of the original information. This is where principal components analysis comes in. PCA is a technique for finding common movements in liquidity spreads. The purpose of PCA is to determine factors (i.e. principal components) in order to explain as much of the total variation in the data as possible with as few of these factors as possible. [See Dillon and Goldstein (1984) for technical details.] The principal components are extracted from the covariance matrix of liquidity spread changes so that the first principal component accounts for the largest amount of the total variation in the data. As shown in Table 3, the first three principal components account for 99.48 % of the total variation in liquidity spread changes. Furthermore, the results reveal that 78.70 % of the variation in liquidity spreads on zero bonds is due to the first component.

Table 3

Results of Principal Components Analysis

This table reports the results of a principal components analysis of liquidity spread changes. The eigenvectors and eigenvalues are extracted from the covariance matrix of liquidity spread changes. The first principal component accounts for 78.70 % of the total variation in liquidity spread changes on zero bonds. The second and third components account for almost all the remaining variation in the data.

| | | | |
|---|---|---|---|
| Type: | weekly absolute liquidity spread changes | | |
| Period: | 1/6/1999 - 10/8/2008 | | |
| Trading Days: | 510 | | |
| Included nodes: | 2, 3, 4, 5, 6, 7, 8, 9, 10 years | | |
| Number of element in eigenvector | Principal components Values of elements in eigenvectors | | |
| 1 | 0,2693 | -0,3109 | 0,7271 |
| 2 | 0,3317 | -0,4223 | 0,1856 |
| 3 | 0,3599 | -0,3477 | -0,1439 |
| 4 | 0,3664 | -0,2011 | -0,3057 |
| 5 | 0,3605 | -0,0339 | -0,3311 |
| 6 | 0,3482 | 0,1307 | -0,2509 |
| 7 | 0,3331 | 0,2827 | -0,0939 |
| 8 | 0,3172 | 0,4187 | 0,1155 |
| 9 | 0,3014 | 0,5387 | 0,3577 |
| Eigenvalues | 5,52E-07 | 1,20E-07 | 2,57E-08 |
| Proportion of total variance | 0,7870 | 0,1712 | 0,0366 |

Source: own calculations with the use of Excel

Therefore, PCA implies that movements of the term structure of liquidity spreads can for the most part be explained by a single component and hypothesis 2 is confirmed. As can be seen from the values of elements in eigenvectors, the first component represents a shift of the liquidity spread curves, whereas the second and third components can be interpreted as twist and change in curvature, respectively.

In order to use the proposed one-factor affine model, one needs to verify the stationarity of liquidity spread changes. Therefore, figure 2 plots the time series of liquidity spread changes on 2-, 6-, and 10-year spot payments in the sample period and the associated sample autocorrelation function (SACF).

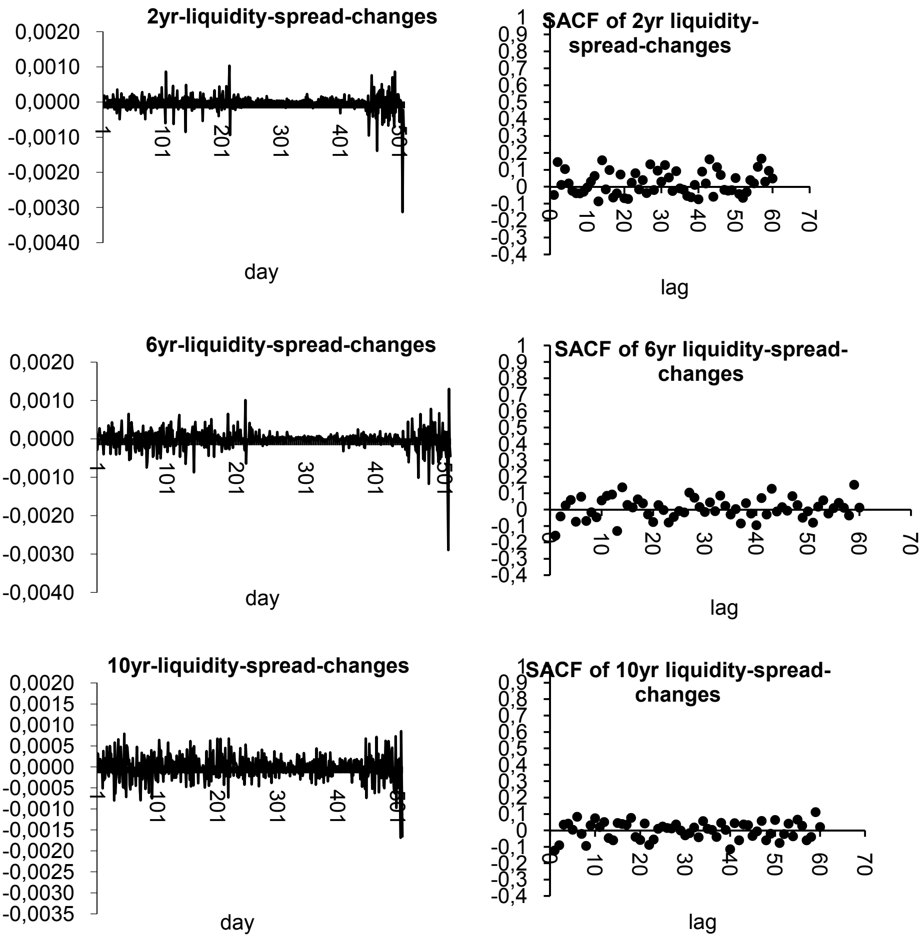


Figure 2. Time series of liquidity spread changes and SACF

Source: own calculations with the use of Excel and Matlab

The figure shows the time series plots of liquidity spread changes on 2-, 6-, and 10-year spot payments for the almost ten year period from January 6, 1999 to October 8, 2008, and the associated sample autocorrelation function (SACF).

The spread changes are stationary. Although for low-maturity bonds, changes seem to exhibit non-constant volatility, the augmented Dickey-Fuller test rejects the hypothesis of a unit root for all maturity classes under consideration (2, 3, 4,..., 10 years).

Table 4

Results of augmented Dickey-Fuller test

This test assumes that the true underlying process is a unit root process. The hypothesis of a unit root is rejected in all maturity classes under consideration. The more negative the test statistic, the stronger the rejection.

| Maturity | 2yr | 3yr | 4yr | 5yr | 6yr | 7yr | 8yr | 9yr | 10yr |
|---|---|---|---|---|---|---|---|---|---|
| TestStat | -23,79 | -27,12 | -27,75 | -27,43 | -26,54 | -25,47 | -24,68 | -24,46 | -24,83 |
| CriticalValue | -3,42 | -3,42 | -3,42 | -3,42 | -3,42 | -3,42 | -3,42 | -3,42 | -3,42 |

Source: own calculations with the use of Matlab

Therefore, the affine model presented in section 1 can be used and the unobservable process of the state variable $x$ that accounts for liquidity spread changes can be estimated by means of the Kalman filter in combination with Maximum-Likelihood estimation of the model parameters. Liquidity spread changes of zero-coupon bonds with time to maturity of 2, 3, …, 10 years are included in the estimation procedure. There are 509 weekly observations for liquidity spread changes from January 6, 1999, to October 8, 2008.

The resulting process is presented in figure 3. It ranges between 10,39 bp and -23,06 bp with an average of -0,56 bp and a standard deviation of 2,67 bp. The maximum value is attained on September 24, 2008 while the minimum occurs on September 17, 2008. Therefore, the highest volatility of the state variable occurs during the last days of the observation period.

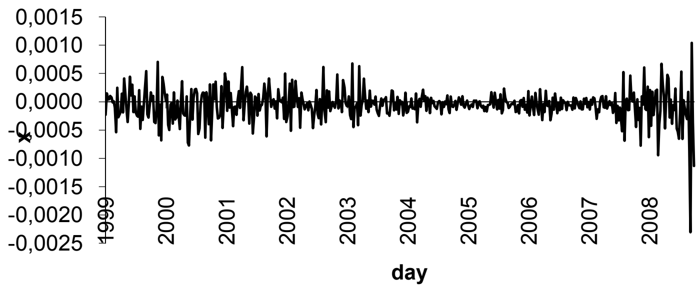


Figure 3. Estimated process of the state variable $x$

Source: own calculations with the use of Excel and Matlab

The figure shows the filtered values of the unobserved process of the state variable $x$. To infer this process the Kalman filter uses spread changes of illiquid zero-coupon bonds with time to maturity of 2, 3, …, 10 years.

Furthermore, the calculated liquidity spread changes are used to estimate the parameters of the stochastic process (1) and the standard deviations of errors $\varepsilon_{t_i}\left(\tau_j\right)$. Results are given in table 5. In all cases plausible parameter estimates are obtained.

Table 5

Estimated parameters and standard errors (in parentheses)

Using Kalman filter ML, the parameters of the stochastic process (1) and the standard deviations of errors are estimated.

| Parameter | Estimate |
|---|---|
| Theta^Q | 0,001270983 (0,000100099) |
| Alpha | 0,024079504 (0,000380321) |
| Sigma | 0,003088653 (8,81148E-05) |
| **Standard deviation of errors** | **Estimate** |
| se1 | 0,000214284 (6,72503E-06) |
| se2 | 0,000167794 (5,26253E-06) |
| se3 | 0,000118805 (3,72438E-06) |
| se4 | 6,01051E-05 (1,88389E-06) |
| se5 | 0 (3,63727E-06) |
| se6 | 5,87955E-05 (1,84293E-06) |
| se7 | 0,000116559 (3,65431E-06) |
| se8 | 0,000174145 (5,46150E-06) |
| se9 | 0,000232021 (7,27969E-06) |

Source: own calculations with the use of Matlab

The estimated process of the state variable $x$ and parameter estimates can then be used to determine the theoretical liquidity spread changes $\Delta l_{t_i}(\tau)$ of illiquid zero-coupon bonds. Subsequently, we can compare these theoretical changes to real liquidity spread changes.

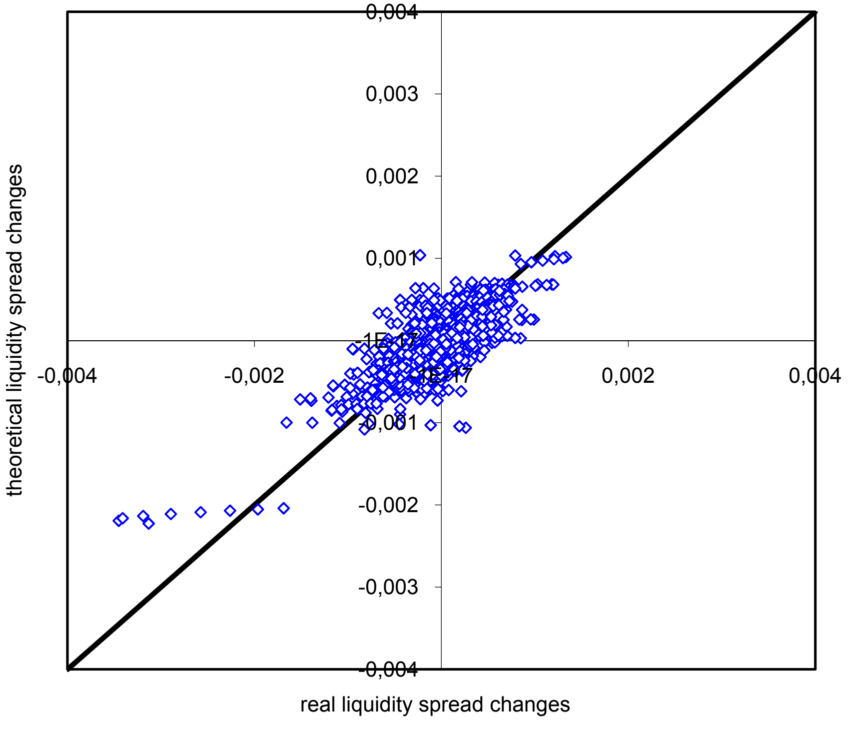


Figure 4. Real liquidity spread changes versus theoretical liquidity spread changes

Source: own calculations with the use of Excel

The figure plots real liquidity spread changes versus theoretical liquidity spread changes. The model does not exactly explain the data, however, deviations remain passable in the majority of cases.

Figure 4 plots real changes versus theoretical changes. Although the model values do not exactly coincide with the data, deviations are not too large in the majority of cases. Furthermore, residuals – defined as the difference between real and theoretical liquidity spread changes – can be constructed.

Table 6

Differences between real and theoretical liquidity spread changes

This table reports descriptive statistics (sample mean, standard deviation, and autocorrelation) of the differences between real and theoretical liquidity spread changes.

| Maturity | 2 yr | 3 yr | 4 yr | 5 yr | 6 yr | 7 yr | 8 yr | 9 yr | 10 yr |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 1,074E-05 | 5,484E-06 | 1,789E-06 | 8,043E-07 | 2,499E-06 | 6,652E-06 | 1,305E-05 | 2,155E-05 | 3,201E-05 |
| Std. D. | 0,0002466 | 0,0002485 | 0,0002191 | 0,0001710 | 0,0001165 | 6,108E-05 | 2,411E-05 | 6,833E-05 | 0,0001279 |
| $\rho_1$ | -0,024631 | -0,117827 | -0,170728 | -0,212200 | -0,246635 | -0,268982 | -0,101505 | -0,233475 | -0,271646 |
| $\rho_{12}$ | -0,047545 | 0,012927 | 0,036850 | 0,0528666 | 0,0680374 | 0,0874941 | 0,0701220 | 0,0594773 | 0,0698734 |

Source: own calculations with the use of Excel and Matlab

Table 6 reports the average residuals and standard deviations of residuals as well as their first and 12$^{th}$ order autocorrelations. Average residuals are low for maturities between 3 and 7 years and increase for lower and higher maturities. Standard deviations of residuals, however, are higher for low maturities. The first order autocorrelations are close to zero for low maturities and between -0,1 and -0,28 for higher maturities. The 12$^{th}$ order autocorrelations fluctuate between -0,06 and +0,1. Figure 5 plots the average of theoretical liquidity spread changes versus the corresponding real spread changes. This reflects the pattern that can also be observed in average residuals, i.e. an appropriate fit for maturities between 3 and 7 years.

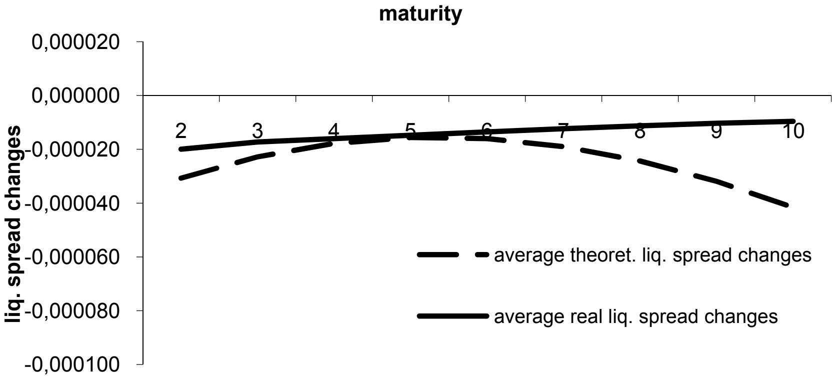


Figure 5. Average theoretical liquidity spread changes versus average real liquidity spread changes

Source: own calculations with the use of Excel

The figure plots average theoretical liquidity spread changes versus real liquidity spread changes. The model fits the data best for maturities between 3 and 7 years. For maturities of 2, 8, 9 and 10 years model spreads and real spreads on average diverge.

The Kalman filter can also be used to predict the state variable $x$ and thus liquidity spread changes. [See, for example, Harvey (2001).] These predicted values can then be employed to judge the quality of the model. First, the differences between real liquidity spread changes $\Delta l_{t_i}(\tau)$ and predicted liquidity spread changes $\Delta l_{t_i|t_{i-1}}(\tau)$ have to be calculated. The differences are equal to the one-step-ahead prediction errors of the Kalman filter. In a well specified model, the time-series average of differences should be close to zero for all maturities and they should be serially uncorrelated. [See De Jong, 2000, p. 306.] Summary statistics are given in table 7.

Table 7

Differences between real and predicted liquidity spread changes

This table reports descriptive statistics (sample mean, standard deviation, and autocorrelation) of the one-step-ahead prediction errors generated by the Kalman filter.

| Maturity | 2 yr | 3 yr | 4 yr | 5 yr | 6 yr | 7 yr | 8 yr | 9 yr | 10 yr |
|---|---|---|---|---|---|---|---|---|---|
| Mean | 0,00001 | 0 | 0 | 0 | 0 | 0 | 0,00001 | 0,00002 | 0,00003 |
| Std. D. | 0,00039 | 0,00043 | 0,00044 | 0,00043 | 0,00041 | 0,00040 | 0,00040 | 0,00040 | 0,00040 |
| $\rho_1$ | -0,48814 | -0,54424 | -0,55889 | -0,56196 | -0,55059 | -0,52514 | -0,48852 | -0,44537 | -0,40167 |
| $\rho_{12}$ | 0,06803 | 0,08724 | 0,09343 | 0,09651 | 0,09632 | 0,09250 | 0,08555 | 0,07678 | 0,06791 |

Source: own calculations with the use of Excel and Matlab

The results reveal that time-series averages are indeed very close to zero. First order autocorrelations are around 0,5 and 12$^{th}$ order autocorrelations are all below 0,1. Summing up, most test statistics indicate an adequate fit of liquidity spread dynamics by the one-factor affine model. Nevertheless, the model fails to fit the term structure at the long end and first order autocorrelations are slightly too high.

## CONCLUSION

This paper investigates the term structure of liquidity spreads calculated from Jumbo Pfandbriefe and German government bonds in the period January 6, 1999 to October 8, 2008. Using the two types of bond data, an accurate estimate of this term structure and its dynamics can be derived. On average an upward sloping term structure of liquidity spreads is found. In times of crisis, however, the situation can change to an inverted term structure.

The changes in liquidity spreads on zero-coupon bonds with different maturities exhibit considerable correlation and a principal components analysis reveals that movements of the term structure can for the most part be explained by a single component. Furthermore, liquidity spread changes are tested for stationarity and augmented Dickey-Fuller tests reject the hypothesis of a unit root for liquidity spread changes in all maturity classes under consideration. Therefore, an affine one-factor model of the term structure of liquidity spreads is presented and a factor time series is extracted by the use of the Kalman filter in combination with Maximum Likelihood estimation of the model parameters. Subsequently, factor series, parameters and one-step-ahead prediction errors generated by the Kalman filter can be used to analyse the models' empirical performance. Most test statistics indicate an adequate fit of the term structure of liquidity spreads and its dynamics for maturities between 3 and 7 years.

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