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Soliton pairing of two coaxially co-propagating mutually incoherent 1-D beams in Kerr type media

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In this paper, we have developed a theory (using parabolic equation approach) of coupled propagation of two coaxially co-propagating and mutually incoherent bright 1-D beams in Kerr type media. We have provided a detailed account of the propagation behavior and condition of formation of spatial soliton pairs for various coupling coefficients (κ = 1, 2/3, 2) when wavelengths and widths of the beams are the same/different. We have also identified conditions for a distinct type of coupled propagation. Our simple and straightforward theory presents many features of co-propagating beams which are in agreement with the features reported earlier using coupled nonlinear Schrödinger equation (NLSE). The paper adds to the understanding of coupled propagation by revealing many additional features not reported earlier.

Keywords: Kerr media, self-focusing, spatial solitons, soliton pairing.

1. Introduction

Formation of optical spatial soliton has attracted a lot of attention following the progress on photorefractive solitons [1], quadratic solitons [2] and solitons in saturable nonlinear media [3]. Investigation of soliton formation, interaction and soliton induced waveguide is of high interest due to their potential applications in all-optical switching and all-optical interconnects [4, 5], as well as waveguide applications [6–8]. Coupled spatial soliton pairs obtained using two co-propagating beams in nonlinear media are a special case of multicomponent solitons being studied starting from the early 1970's [9] and are important in all-optical switching devices (see, for example, references [10–13]) and therefore, such pairing has always been an intriguing issue among spatial soliton interactions. Interaction of two spatial/temporal co-propagating solitons in bulk/waveguide media, and the possibility of formation of bright and/or dark soliton pairs have already been discussed in many papers, for example, in [14–20].

In the present paper, we have extended the work of [19, 21] to obtain general coupled propagation equations for two co-propagating 1-D beams. The theory presented is based on WKB and paraxial ray approximation and the assumption that the beams maintain their Gaussian shape while the widths vary along propagation length. Using these equations, we have provided a detailed explanation of the coupled propagation of two bright beams in Kerr media. We have identified conditions for distinct types of coupled propagation. Considering propagation of beams in all possible physical situations and parameters, we have obtained solitonic solutions for various coupling coefficients ($\kappa = 1, 2/3, 2$) when wavelengths and widths of the beams are the same/different.

It is worth mentioning here that a similar theory can be found in [19] for two co-propagating 2-D beams (whereas the present theory is for 1-D beams). However, in that theory, the choice of constants ε_{ij} is not obvious and becomes very difficult particularly when coupling coefficient is not equal to unity. All results given in [19] could be reproduced by considering 2-D beams and coupling coefficient equal to unity in the present theory. In addition, the present theory is also capable of dealing with other possible cases like coupling coefficient other than unity and different/same beam widths. Therefore, the present theory is more versatile and simple.

The chief aim of this paper is to provide a simple and straightforward theory of coaxially co-propagating 1-D beams in nonlinear media. Another goal is to provide a physically intuitive understanding of the coupled propagation of coaxially co-propagating 1-D beams in all possible physical situations and parameters.

2. Theory of coupled propagation

2.1. One dimensional (1-D) bright Gaussian beams

A Gaussian beam of elliptical cross-section could, ingeneral, be expressed as

$$A_1^2(z) = \frac{E_{01}^2}{f_x f_y} \exp\left(-\frac{x^2}{r_x^2 f_x^2} - \frac{y^2}{r_y^2 f_y^2} \right)$$

where A_1 is the real amplitude of the electric vector of the beam, f_x and f_y are the dimensionless beam width parameters with the initial value 1, and r_x and r_y are the initial widths of the beam along the x- and y-directions, respectively.

A 1-D optical beam could be viewed as a beam of elliptical cross-section with finite minor axis and infinite major axis, *i.e.*, $r_y \to \infty$, therefore

$$A_1^2(z) = \frac{E_{01}^2}{f_x f_y} \exp\left(-\frac{x^2}{r_x^2 f_x^2} \right)$$

In the above equation, the dimensionless beam width parameter f_y remains constant at its initial value 1 as the beam does not diffract along the y-axis (as $r_y \to \infty$), therefore,

$$A_1^2(z) = \frac{E_{01}^2}{f_x}\exp\left(-\frac{x^2}{r_x^2 f_x^2}\right)$$

we rename x as r in the above equation and replace the subscript x by 1, *i.e.*,

$$A_1^2(z) = \frac{E_{01}^2}{f_1}\exp\left(-\frac{r^2}{r_1^2 f_1^2}\right)$$

Similarly, the second 1-D beam may be expressed as:

$$A_2^2(z) = \frac{E_{02}^2}{f_2}\exp\left(-\frac{r^2}{r_2^2 f_2^2}\right)$$

or

$$A_j^2(z) = \frac{E_{0j}^2}{f_j}\exp\left(-\frac{r^2}{r_j^2 f_j^2}\right), \qquad j = 1, 2. \tag{1}$$

2.2. Dielectric constant of the medium

We consider coaxial co-propagation of the above-mentioned two 1-D Gaussian beams of frequencies ω_1 and ω_2, respectively, along the z-axis. These beams modify dielectric constant of the medium as [19, 21, 22]:

$$\varepsilon(\omega_1) = \varepsilon_{10} + \varphi_1(A_1, A_2) \tag{2}$$

$$\varepsilon(\omega_2) = \varepsilon_{20} + \varphi_2(A_1, A_2) \tag{3}$$

where ε_{10} and ε_{20} are the dielectric constants at frequencies ω_1 and ω_2, respectively, and φ_1 and φ_2 are the nonlinear dielectric constants. For the Kerr type nonlinear medium, φ_1 and φ_2 may be expressed as:

$$\varphi_1 = \alpha_1 A_1^2 + \kappa\alpha_2 A_2^2 \tag{4}$$

$$\varphi_2 = \kappa\alpha_1 A_1^2 + \alpha_2 A_2^2 \tag{5}$$

In equations (4) and (5), α_1 and α_2 are nonlinear coefficients of the medium at frequencies ω_1 and ω_2, respectively, κ is the coupling coefficient of the two beams that depends on the experimental conditions, $\alpha_j A_j^2$ (for $j = 1, 2$) is the dimensionless axial electric field intensity.

In paraxial ray approximation, one can, ingeneral, expand φ_1 and φ_2 around their value at $r = 0$. Employing A_j from Eq. (1), φ_1 can be expanded by Taylor's expansion and terms except square ones can be neglected, so

$$\varphi_1(A_1, A_2) = \varphi_1\left[\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa\alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\alpha_1 E_{01}^2}{r_1^2 f_1^3} + \frac{\kappa\alpha_2 E_{02}^2}{r_2^2 f_2^3}\right) r^2\right]$$

Here $\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa\alpha_2 E_{02}^2}{f_2}\right) \gg \left(\frac{\alpha_1 E_{01}^2}{r_1^2 f_1^3} + \frac{\kappa\alpha_2 E_{02}^2}{r_2^2 f_2^3}\right) r^2$, therefore, one can write:

$$\varphi_1 = \varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa\alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\alpha_1 E_{01}^2 r^2}{r_1^2 f_1^3} + \frac{\kappa\alpha_2 E_{02}^2 r^2}{r_2^2 f_2^3}\right) \varphi_1'\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa\alpha_2 E_{02}^2}{f_2}\right)$$

where prime over φ_1 denotes derivative with respect to the argument. On simplifying the above equation, we get

$$\varphi_1 = \varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa\alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\alpha_1 E_{01}^2 r^2}{r_1^2 f_1^3} + \frac{\kappa\alpha_2 E_{02}^2 r^2}{r_2^2 f_2^3}\right) \tag{6}$$

since $\varphi_1'\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa\alpha_2 E_{02}^2}{f_2}\right) = 1$.

Similarly, one can obtain

$$\varphi_2 = \varphi_2\left(\frac{\kappa\alpha_1 E_{01}^2}{f_1} + \frac{\alpha_2 E_{02}^2}{f_2}\right) - \left(\frac{\kappa\alpha_1 E_{01}^2 r^2}{r_1^2 f_1^3} + \frac{\alpha_2 E_{02}^2 r^2}{r_2^2 f_2^3}\right) \tag{7}$$

2.3. Coupled propagation of beams

In a medium described by Eqs. (2) and (3), the electric vector of the waves are governed by Maxwell's equations, which in WKB approximation reduce to the wave equation

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = 0 \tag{8}$$

where $D = \varepsilon E$ is the electric displacement vector. For slowly converging or slowly diverging beams, Eq. (8) can be satisfied by the following solutions

$$E = E_1 \exp\left[i(\omega_1 t - k_1 z)\right] + E_2 \exp\left[i(\omega_2 t - k_2 z)\right] \tag{9}$$

where E_1 and E_2 are the space dependent complex amplitudes and $k_1 = \frac{\omega_1}{c}\sqrt{\varepsilon_{10}}$ and $k_2 = \frac{\omega_2}{c}\sqrt{\varepsilon_{20}}$ are the propagation constants.

Eikonals can be introduced to describe E_1 and E_2 as

$$E_1 = A_1 \exp\left[-ik_1 S_1\right] \tag{10}$$

$$E_2 = A_2 \exp\left[-ik_2 S_2\right] \tag{11}$$

Here, A_1 and A_2 are the space dependent real amplitudes. On substituting Eqs. (9)–(11) in the wave equation, we get the following set of equations:

$$2\frac{\partial S_j}{\partial z} + \left(\frac{\partial S_j}{\partial r}\right)^2 = \frac{\varphi_j}{\varepsilon_{j0}} + \frac{1}{k_j^2 A_j}\frac{\partial^2 A_j}{\partial r^2} \tag{12}$$

$$\frac{\partial A_j^2}{\partial z} + A_j^2\frac{\partial^2 S_j}{\partial r^2} + \frac{\partial S_j}{\partial r}\frac{\partial A_j^2}{\partial r} = 0 \tag{13}$$

With subscript 1 or 2 in the above equations, we get the relevant equations for the first or the second beam. To solve Eqs. (12) and (13) we assume that the nonlinear part of the dielectric constant is much smaller than the linear part, and therefore, nonlinearity may be treated as perturbation. One may, therefore, assume generalized spherical wave solution for Eqs. (12) and (13):

$$S_j = \frac{r^2}{2}\beta_j(z) + \eta_j(z) \tag{14}$$

$$A_j^2 = \frac{E_{0j}^2}{f_j}\exp\left(-\frac{r^2}{r_j^2 f_j^2}\right) \tag{15}$$

$$\beta_j = \frac{1}{f_j}\frac{\partial f_j}{\partial z} \tag{16}$$

It can be noted that β_j represents the inverse of radius of curvature of the beams' fronts, and $r_j f_j$ stand for the widths of the beams.

Using equations (14)–(16) in equation (12) and using paraxial ray approximation, *i.e.*, $(r/r_j f_j)^4 \ll 1$, we obtain

$$r^2\left(\frac{1}{f_j}\frac{\partial^2 f_j}{\partial z^2}\right) + 2\frac{\partial \eta_j}{\partial z} = \frac{1}{k_j^2 A_j}\left(-\frac{A_j}{r_j^2 f_j^2} + \frac{A_j r^2}{r_j^4 f_j^4}\right) + \frac{\varphi_j(A_1, A_2)}{\varepsilon_{j0}} \tag{17}$$

On substituting $\varphi_1(A_1, A_2)$ from equation (6), equation (17) takes the form (for the first beam)

$$r^2\left(\frac{1}{f_1}\frac{\partial^2 f_1}{\partial z^2}\right) + 2\frac{\partial \eta_1}{\partial z} = \frac{1}{k_1^2 A_1}\left(-\frac{A_1}{r_1^2 f_1^2} + \frac{A_1 r^2}{r_1^4 f_1^4}\right) + \frac{1}{\varepsilon_{10}}\varphi_1\left(\frac{\alpha_1 E_{01}^2}{f_1} + \frac{\kappa\alpha_2 E_{02}^2}{f_2}\right)$$
$$-\frac{1}{\varepsilon_{10}}\left(\frac{\alpha_1 E_{01}^2 r^2}{r_1^2 f_1^3} + \frac{\kappa\alpha_2 E_{02}^2 r^2}{r_2^2 f_2^3}\right)$$

Equating the coefficients of r^2 on both sides of the above equation, one obtains the following propagation equation that governs the beam width parameter of the first beam with the propagation distance

$$\frac{\partial^2 f_1}{\partial z^2} = \frac{1}{k_1^2 r_1^4 f_1^3} - \frac{C}{\varepsilon_{10} r_1^2 f_1^2} - \frac{\kappa D f_1}{\varepsilon_{10} r_2^2 f_2^3} \tag{18}$$

where: $C = \alpha_1 E_{01}^2$ and $D = \alpha_2 E_{02}^2$.

Similarly, the propagation equation for the second beam could be obtained as

$$\frac{\partial^2 f_2}{\partial z^2} = \frac{1}{k_2^2 r_2^4 f_2^3} - \frac{D}{\varepsilon_{20} r_2^2 f_2^2} - \frac{\kappa C f_2}{\varepsilon_{20} r_1^2 f_1^3} \tag{19}$$

The set of coupled Eqs. (18) and (19) governs the evolution of widths of the two beams with the propagation distance.

For self-trapped beams (spatial solitons), we must have $\partial f_j/\partial z = \partial^2 f_j/\partial z^2 = 0$. One can assume $\partial f_j/\partial z = 0$ as the initial condition of the beams. To have $\partial^2 f_j/\partial z^2 = 0$, we correspondingly need:

$$D = \frac{\varepsilon_{10} r_2^2}{\kappa k_1^2 r_1^4} - \frac{C r_2^2}{\kappa r_1^2} \tag{20}$$

$$D = \frac{\varepsilon_{20}}{k_2^2 r_2^2} - \frac{\kappa C r_2^2}{r_1^2} \tag{21}$$

3. Numerical appreciation and discussion

It is worth mentioning here that coupling coefficient κ depends on the experimental conditions. In the present paper, we have investigated coupled beam propagation for coupling coefficients used in earlier literature, *i.e.*, $\kappa = 2$, $\kappa = 2/3$ (see, *e.g.*, [14]) and $\kappa = 1$ [10–12].

3.1. Coupled propagation when coupling coefficient $\kappa = 2$

3.1.1. Case I: beams of the same frequency and the same widths

For the purpose of numerical evaluation of Eqs. (18)–(21), we choose the following set of parameters: $\omega_1 = \omega_2 = 2.7148\times10^{15}$ rad/s, $\varepsilon_{10} = \varepsilon_{20} = (1.6276)^2$, and $r_1 = r_2 = 10$ μm. The approach given here is valid for any other set of parameters.

In Figure 1, we plot D with C using equations (20) and (21) for $\kappa = 2$ and for the above mentioned parameters. In the figure, the solid line represents the solution of Eq. (20) while dashed line is the solution of Eq. (21). Point P_1 is also a solution of Eq. (20), which corresponds to $D = 0$ (zero power) of the second beam, therefore, the power of the first beam corresponding to P_1 is its self-traped power. Similarly, the power corresponding to point P_2 is the self-trapped power of the second beam. The point of intersection S is the common solution of Eqs. (20) and (21). Therefore, values of C and D at point S correspond to the powers of the two beams for mutual self-trapping. In other words, if two beams are coaxially propagated in the nonlinear medium with their power corresponding to point S, both will simultaneously be self-trapped or they will form a spatial soliton pair. To verify mutual trapping, we choose C and D from the point of intersection S, *i.e.*, $C = D = 4.0743\times10^{-5}$ and obtain the evolution of the beams' width with the propagation distance using Eqs. (18) and (19), as shown in Fig. 2. We have plotted $0.9\times f_2$ just to resolve f_1 and f_2. It is clearly observable that both beams are mutually self-trapped or they form a spatial soliton pair. It can be observed in Fig. 1 that the power of each beam (corresponding to point S) required for soliton pair is one third of the self-trapped power (corresponding to points P_1, P_2). It is also obvious that there exists only one solution for soliton pairing.

Same features have been revealed in investigation of co-propagating beams using coupled nonlinear Schrödinger equations (NLSE) [14], however, one can note that

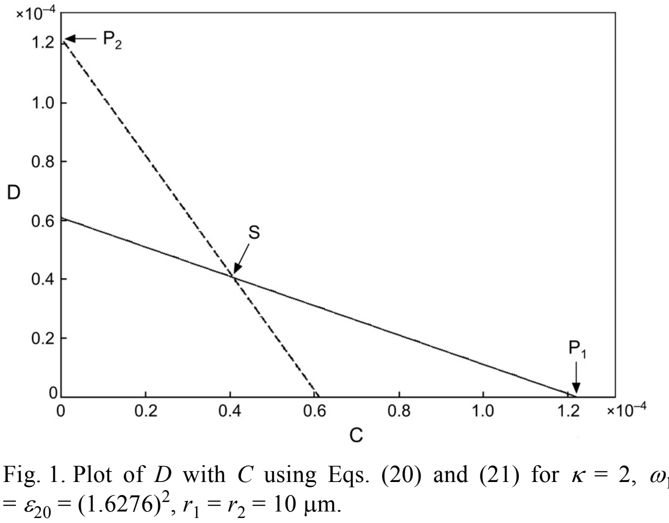

Fig. 1. Plot of D with C using Eqs. (20) and (21) for $\kappa = 2$, $\omega_1 = \omega_2 = 2.7148\times10^{15}$ rad/s, $\varepsilon_{10} = \varepsilon_{20} = (1.6276)^2$, $r_1 = r_2 = 10$ μm.

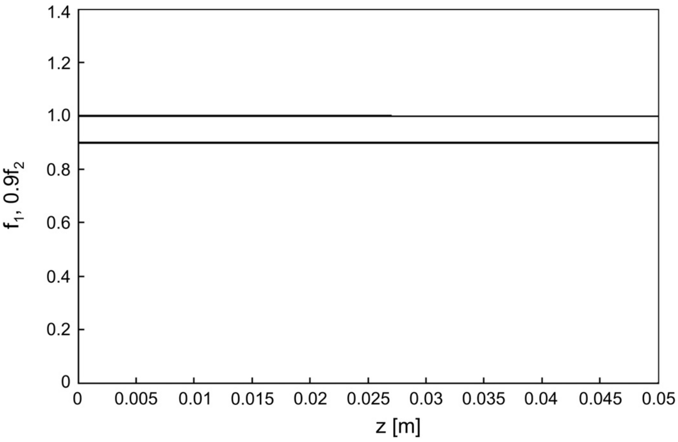

Fig. 2. Evolution of the widths of beams with the propagation distance is obtained with the parameters of Fig. 1, $C = D = 4.0743\times10^{-5}$ and using Eqs. (18) and (19). The chosen widths of beams are the same, however, $0.9\times f_2$ has been plotted just to resolve f_1 and f_2. The figure shows a soliton pair formation.

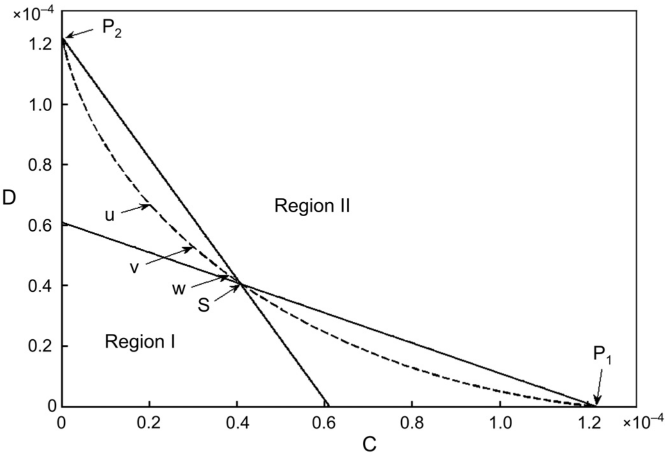

Fig. 3. An arc of the circle that passes through P_1, S and P_2 is drawn intuitively. This arc has been identified as the existence curve of rhythmic breather pair.

the present treatment is much simpler and provides an intuitive picture of solitonic solution.

We go further and, to the best of our knowledge, we are the first to report conditions for distinct types of coupled propagation. We intuitively draw an arc of the circle that passes through P_1, S and P_2, as shown in Fig. 3. We have identified this arc as the existence curve of rhythmic breather pair (out-of phase width oscillations of the two beams). To confirm our claim, breather pairs are obtained in Figs. 4–6 using beam powers corresponding to points u ($C = 0.69\times10^{-4}$, $D = 0.2\times10^{-4}$), v ($C = 0.53\times10^{-4}$,

$D = 0.3\times10^{-4}$) and w ($C = 3.7705\times10^{-5}$, $D = 4.3705\times10^{-5}$), respectively. One can notice an out-of-phase rhythm in beam width oscillations in all these figures. Similar breather pairs could be obtained from the entire arc. It can be seen that amplitude of width oscillations of the breather pair is smaller if chosen point on the arc is nearer to S. In fact, at point S, the amplitude of width oscillation becomes zero and soliton pair is formed.

On the basis of our investigations, we have divided Fig. 3 into two regions, identifying them with two distinct types of coupled propagation: region I (below the dashed arc) and region II (above the dashed arc).

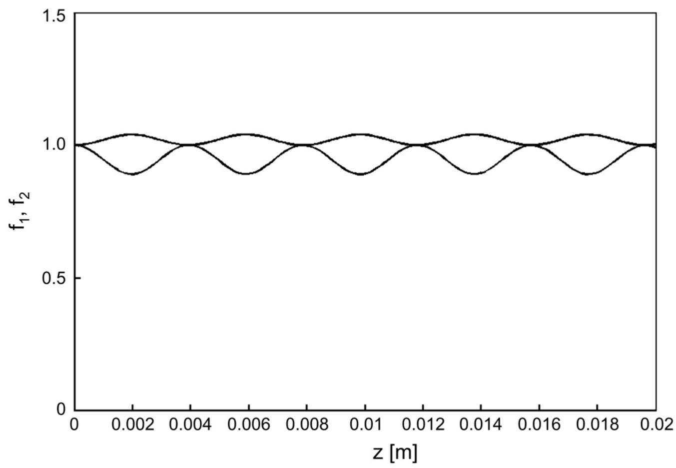

Fig. 4. Rhythmic breather pair is obtained using beam powers corresponding to point u ($C = 0.755\times10^{-5}$, $D = 1.0\times10^{-4}$) of Fig. 3.

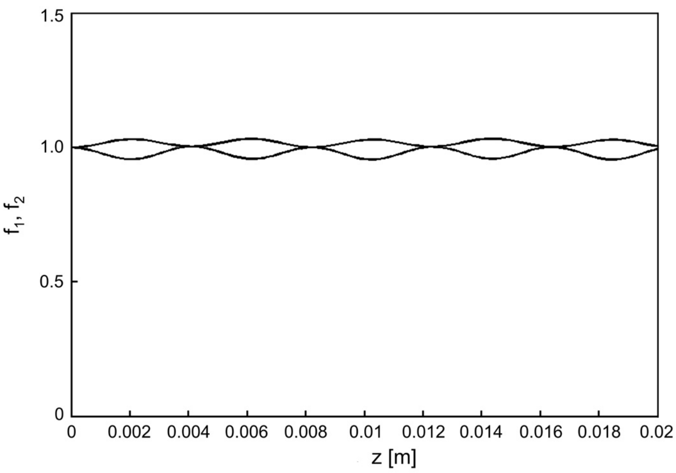

Fig. 5. Rhythmic breather pair is obtained using beam powers corresponding to point v ($C = 2.111\times10^{-5}$, $D = 6.9853\times10^{-5}$) of Fig. 3.

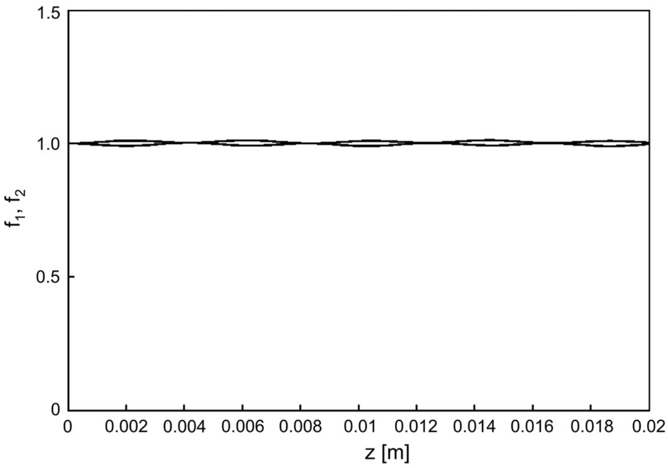

Fig. 6. Rhythmic breather pair is obtained using beam powers corresponding to point w ($C = 3.7705\times10^{-5}$, $D = 4.3705\times10^{-5}$) of Fig. 3.

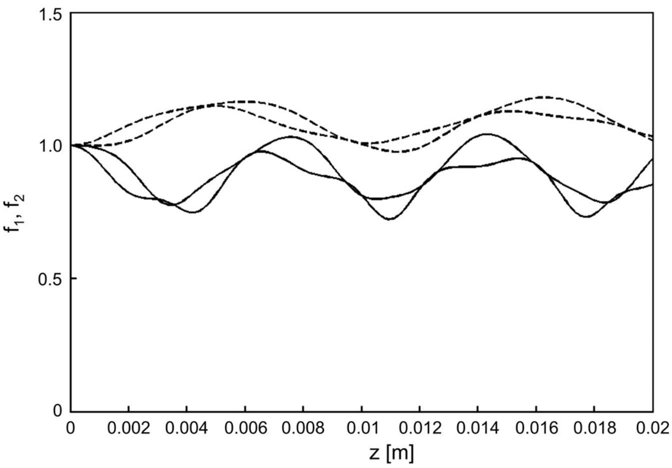

Fig. 7. Breather pair obtained from region I of Fig. 3 ($C = 0.47\times10^{-5}$, $D = 0.3\times10^{-5}$) is shown by dashed line and the same obtained from region II ($C = 0.67\times10^{-5}$, $D = 0.3\times10^{-5}$) is shown by solid line.

Breather pair obtained from region I ($C = 0.47\times10^{-4}$, $D = 0.3\times10^{-4}$) is shown in Fig. 7 by dashed lines. One can notice that the pair first defocuses and then focuses while breathing and widths of beams oscillate about a width larger than the initial one. We confirmed that all breather pairs obtained from region I exhibit the same features. Breather pair obtained from region II ($C = 0.67\times10^{-4}$, $D = 0.3\times10^{-4}$) is shown by solid lines. One can notice that the pair first focuses and then defocuses while breathing and widths of beams oscillate about a width smaller than the initial one.

One can notice a remarkable difference in the rhythmic breather pairs obtained from the arc, breather pairs of region I and breather pairs of region II.

3.1.2. Case II: beams of different frequencies but the same width

We choose different frequencies than in the earlier set of parameters, *i.e.*, $\omega_1 =$ $= 2.7148\times10^{15}$ rad/s and $\omega_2 = 2.5148\times10^{15}$ rad/s, keeping the rest of the parameters unchanged. For those parameters, Fig. 1 gets modified to Fig. 8. The solitonic solution shifts towards the x-axis with $C = 0.542\times10^{-4}$, $D = 0.339\times10^{-4}$ (for $\omega_1 < \omega_2$, solitonic solution shifts towards the y-axis). It can be easily seen from Eqs. (20) and (21) that solitonic solution exists in the $+x$, $+y$ quadrant only for the frequency

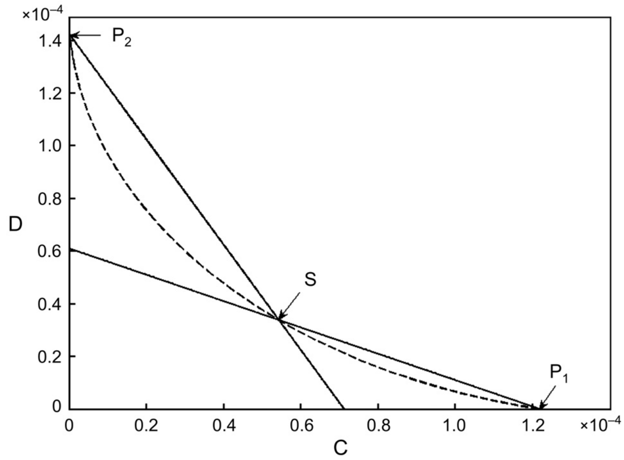

Fig. 8. With beams of the same width but different frequencies, *i.e.*, $\omega_1 = 2.7148\times10^{15}$ rad/s, $\omega_2 = 2.5148\times10^{15}$ rad/s, and $r_1 = r_2 = 10$ μm, Fig. 3 is modified as shown. The solitonic solution shifts towards the x-axis with $C = 0.542\times10^{-4}$, $D = 0.339\times10^{-4}$.

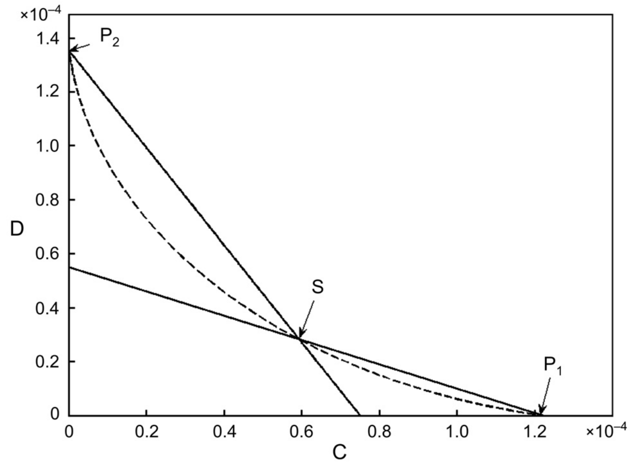

Fig. 9. With beams of different widths but the same frequency, *i.e.*, $\omega_1 = \omega_2 = 2.7148\times10^{15}$ rad/s, $r_1 = 10$ μm and $r_2 = 9.5$ μm, Fig. 3 is modified as shown. The solitonic solution shifts towards the x-axis with $C = 0.592\times10^{-4}$, $D = 0.284\times10^{-4}$.

ratio within the range $\sqrt{1/\kappa} < \omega_1/\omega_2 < \sqrt{\kappa}$. Beyond this range, solitonic solution goes beyond the $+x$, $+y$ quadrant. Physical interpretation of the above is that no soliton pair of bright beams exists for the beam frequency ratio beyond the aforementioned range. The same results have been obtained in [14] using comparatively complex NLSE. Through numerical investigations, we confirmed here also that one can have regions I and II by drawing an arc of the circle that passes through P_1, S and P_2 (see Fig. 8). The behavior of coupled propagation in this case in different regions is similar to that of the previous case.

3.1.3. Case III: beams of the same frequency but different widths

Before discussing this case it is worthwhile to mention here that in coupled NLSE equations, the widths of two beams must be postulated to be identical to have solitonic solution, whereas in reality, those are often far from being equal [18]. The present theory provides solitonic solutions for coupled beams of unequal widths.

We choose the same frequency $\omega_1 = \omega_2 = 2.7148\times10^{15}$ rad/s and different widths $r_1 = 10$ μm and $r_2 = 9.5$ μm of the two beams, the other parameters being the same as in of Section 3.1.1. For the parameters chosen, Fig. 1 gets modified to Fig. 9, *i.e.*, solitonic solution shifts towards the x-axis with $C = 0.592\times10^{-4}$, $D = 0.284\times10^{-4}$. For $r_2 > r_1$, solitonic solution shifts towards the y-axis. It could be easily shown using Eqs. (20) and (21) that solitonic solution exists in the $+x$ $+y$ quadrant only if the beam width ratio lies within the range $(1/\kappa)^{1/4} < r_2/r_1 < \kappa^{1/4}$, in other words, no solitonic pair of two bright beams exists for the beam width ratio beyond this range.

3.2. Coupled propagation when coupling coefficient $\kappa = 2/3$

We choose $\kappa = 2/3$ keeping other parameters the same as those of Section 3.1.1. For the parameters chosen, Fig. 3 is modified to Fig. 10. In this case, the power of each beam required for soliton pair is 60% of the self-trapped power of the individual beam, moreover, only one solution exists for soliton pairing.

We have also confirmed here through numerical investigation that the existence curve of rhythmic breather pair is the arc of the circle that passes through P_1, S and P_2, as shown in Fig. 10, and no soliton pair of the same beam widths exists for the frequency ratio beyond the range $\sqrt{1/\kappa} < \omega_1/\omega_2 < \sqrt{\kappa}$. Propagation characteristics of the co-propagating beams in this case in regions I and II are the exactly the same as in the case of $\kappa = 2$.

3.3. Coupled propagation when coupling coefficient $\kappa = 1$

3.3.1. Case I: two beams of the same frequency

For the set of parameters used in Section 3.1.1, an interesting and important situation arises when the coupling coefficient is unity, *i.e.*, $\kappa = 1$. The solutions of Eqs. (20) and

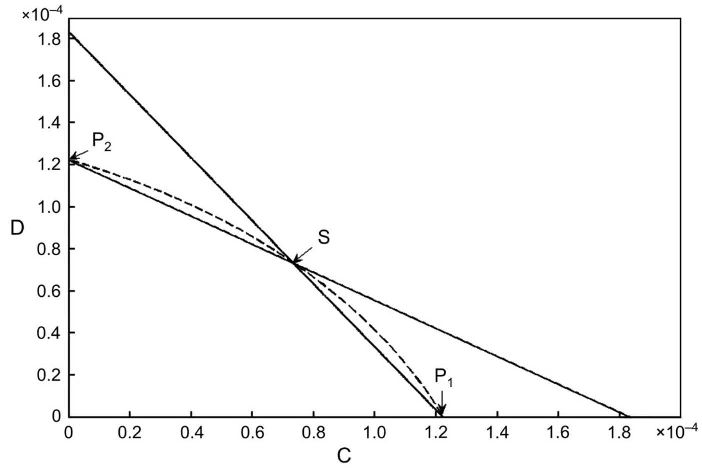

Fig. 10. For parameters of Fig. 3 and κ = 2/3, Fig. 3 is modified as shown. In this case, the power of each beam required for soliton pair is 60% of the power of the beam required to form a single soliton. Only one solution exists for soliton pairing.

(21) merge and form a single line, as shown in Fig. 11. Every point of this line corresponds to the powers of the two beams required to from one soliton pair as every point is the common solution of Eqs. (20) and (21). One such soliton pair is shown by solid lines in Fig. 12, where $0.95\times f_2$ has been plotted just to resolve f_1 and f_2. If beam powers are chosen from a point below the existence line of Fig. 11, both beams mutually defocus and then focus as shown by dotted lines in Fig. 12, and if those are

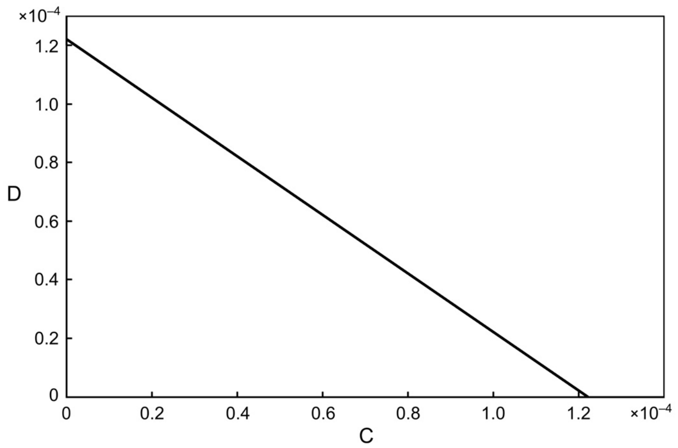

Fig. 11. An interesting situation arises for κ = 1. For the parameters of Fig. 3 and κ = 1, solutions of Eqs. (20) and (21) merge and form a single existence line of soliton pair as shown. Every point of this line corresponds to the powers of beams of one soliton pair.

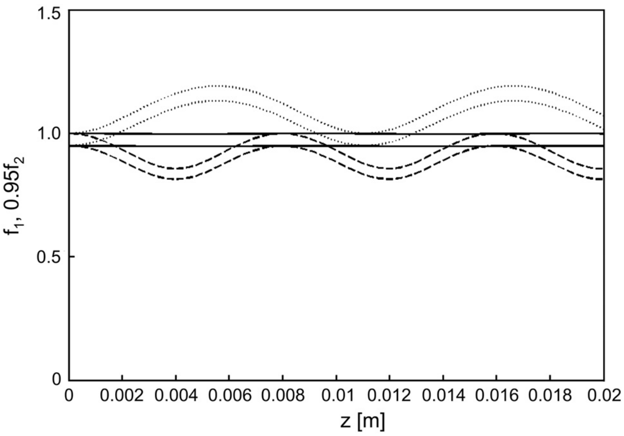

Fig. 12. If the powers of beams are chosen from a point on the existence line (of Fig. 11), both beams form a soliton pair as shown by solid lines. Here, $0.95\times f_2$ has been plotted to resolve f_1 and f_2. If those are chosen from a point below the existence line, both beams mutually defocus and then focus, as shown by dotted lines, and if those are chosen from a point above the existence line, they mutually focus and then defocus, as shown by dashed lines.

chosen from a point above the existence line of Fig. 11, they mutually focus and then defocus as shown by dashed lines in Fig. 12.

3.3.2. Case II: two beams of different frequencies

We choose different frequencies of the two beams $\omega_1 = 2.7148\times10^{15}$ rad/s and $\omega_2 = 2.5148\times10^{15}$ rad/s, the other parameters being the same as in Section 3.3.1. For the parameters chosen, solutions of Eqs. (20) and (21) form two parallel lines, as shown in Fig. 13, in other words, no solitonic pair exists of different frequencies and the same width (when $\kappa = 1$). However, if we equate Eqs. (20) and (21) with $\kappa = 1$, we get a condition for the beam widths to have soliton pairs, which is $r_2/r_1 = (\omega_1/\omega_2)^{1/2}$. If we use this beam width ratio in Eqs. (20) and (21), both solutions merge, as shown in Fig. 14. We have confirmed that every point of this line provides powers of beams for one soliton pair. In summary, when the coupling coefficient is equal to unity, soliton pairs of different frequencies and the same width do not exist. Pair formation becomes possible only if the width ratio is chosen as mentioned above.

4. Conclusions

Using parabolic equation approach, we have developed a theory of coupled propagation of two coaxially co-propagating and mutually incoherent bright 1-D beams in Kerr media. Propagation behavior and condition for the spatial soliton pairs to be formed have been investigated in detail for all possible situations and parameters,

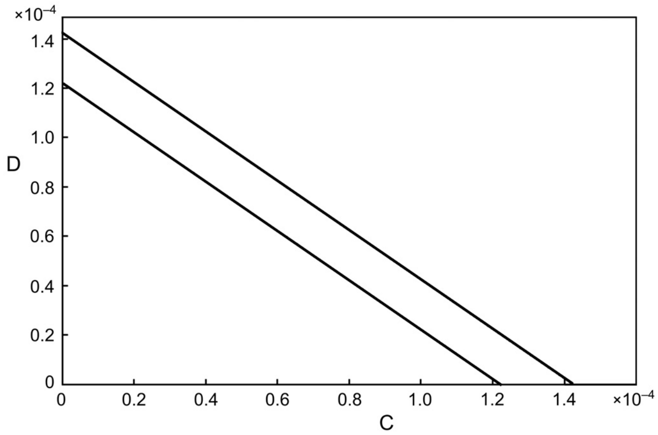

Fig. 13. For κ = 1 and the two beams of different frequencies and the same width, for example, $\omega_1 = 2.7148\times10^{15}$ rad/s, $\omega_2 = 2.5148\times10^{15}$ rad/s, and $r_1 = r_2 = 10$ μm, the solutions of Eqs. (20) and (21) form two parallel lines as shown, *i.e.*, no solitonic pair exists of different frequency and the same width when $\kappa = 1$.

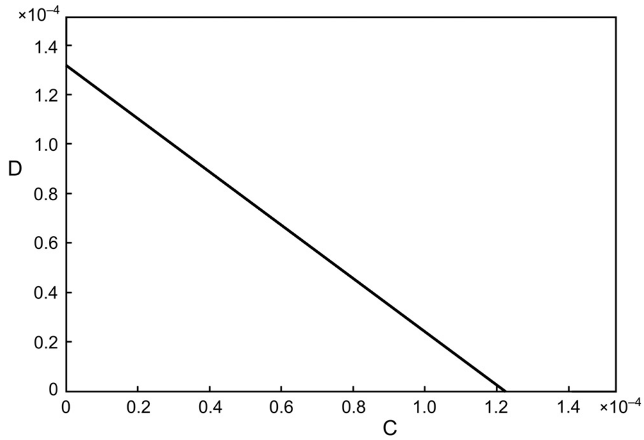

Fig. 14. Soliton pairs can be formed by having the ratio of the widths of beams as per $r_2 = r_1(\omega_1/\omega_2)^{1/2}$. With this width ratio of beams and parameters of Fig. 13, solutions of Eqs. (20) and (21) merge and every point of the merged line provides one soliton pair.

and conditions for distinct types of coupled propagation have been identified. It is shown that:

– only one solution exists for solitonic pairing when coupling coefficient is different from unity;

– if coupling coefficient is different from unity, solitonic pairing is possible with same/different beam widths and/or same/different frequency of the beams, while, in case of coupling coefficient equal to unity, solitonic pairing with different frequencies is only possible with different beams' widths;

– infinite solutions for solitonic pairing exist when coupling coefficient is unity and beam widths and frequencies are the same.

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