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Theoretical and numerical analysis of double-negative slab

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Reflection and transmission analysis due to the interaction of electromagnetic waves with a frequency dispersive double-negative slab are investigated in detail. In particular, the reflection and the transmission coefficients are found and defined. The incident field is assumed to be a plane monochromatic wave of transverse magnetic polarization. Imposing the boundary conditions at the interfaces, the transmission and the reflection coefficients at each interface can be obtained. Numerical results are presented for both cases of transverse magnetic and electric waves to show the effects of the incidence angle, the frequency, and the structure parameters on the reflection and the transmission coefficients.

Keywords: double-negative slab, reflection and transmission coefficients, electromagnetic wave interaction.

1. Introduction

The idea of double negative (DNG) media whose permittivity and permeability are negative real values has recently achieved a significant importance in electromagnetics' community due to the possibility of the application of them in the microwave, millimeter-wave and optical frequencies [1–6]. Several terminologies are recommended for DNG media such as left-handed media (LHM), backward-wave media (BW media), double negative (DNG) metamaterials, negative index media (NIM), media with negative refractive index, *etc*. The concept was first introduced by Veselago in 1968. In his study, DNG substances were defined, the propagation of waves and the refraction of a ray in such medium were presented [1]. The DNG medium was analyzed and simulated by PENDRY [2]. He showed that a slab of the DNG material forms unconventional lenses such as superlenses. SMITH *et al.* [3] manufactured a composite material that formed a DNG medium, and did several microwave experiments to validate the properties of the medium introduced in [1]. ZIOLKOWSKI and HEYMAN [4] studied the electromagnetic wave propagation in DNG analytically and numerically to observe the characteristic features of propagation and scattering in DNG and to apply the results to the concept of perfect lens. Conceptual and speculative ideas for potential applications of the DNG materials were suggested, and physical remarks and intuitive comments were provided in [5]. KONG [6] investigated the electromagnetic wave interaction with stratified DNG isotropic media. In his study, general formulations for the wave interaction with stratified media were given, and the field solutions of guided waves in stratified media were obtained. The reflected and transmitted powers due to the interaction of electromagnetic waves with a double -negative slab were analyzed by SABAH, ÖGÜCÜ and UCKUN in 2006 [7, 8]. The transfer matrix method was used in the analysis to find the closed form formulations. Also, the effects of the structure parameters, incidence angle and the frequency on the reflected and the transmitted powers were presented numerically.

In this study, the analysis of the electromagnetic waves interacting with a double negative slab (DNS) which is located between two semi-infinite dielectric media is presented. We consider a monochromatic plane electromagnetic wave that is incident to the DNS. The electric and magnetic fields are determined in each region using the Maxwell's equations. Then, Snell's law is applied and boundary conditions are imposed at each interface to obtain the reflection and transmission coefficients. Finally, the numerical results are illustrated to show the effects of the structure parameters, the slab thickness, the incidence angle, and the frequency on the reflection and transmission coefficients.

2. Reflection and transmission coefficients – theoretical analysis

DNG medium which has negative permittivity and permeability has not been found in nature and is constructed artificially. Negative permittivity and permeability can be realized by an array of rods and split-rings over a certain frequency band, respectively [3]. The frequency-dependent permittivity and permeability can be described by the Lorentz medium model as [9]:

$$\mu(\omega) = \mu_0\left[1 - \frac{\omega_{mp}^2 - \omega_{mo}^2}{\omega^2 - \omega_{mo}^2 + j\gamma\omega}\right] \tag{1}$$

$$\varepsilon(\omega) = \varepsilon_0\left[1 - \frac{\omega_{ep}^2 - \omega_{eo}^2}{\omega^2 - \omega_{eo}^2 + j\gamma\omega}\right] \tag{2}$$

where ω_{mo} is the magnetic resonance frequency, ω_{mp} is the magnetic plasma frequency, ω_{eo} is the electronic resonance frequency, ω_{ep} is the electronic plasma frequency, γ is the dissipation factor, ω is the angular frequency, and $j = \sqrt{-1}$. Both the refractive index and the wave number are also negative for the DNG and they can be written as:

$$n = -\sqrt{|\varepsilon_r| \cdot |\mu_r|} \tag{3}$$

$$k = \omega\sqrt{\varepsilon\mu} = k_0 n \tag{4}$$

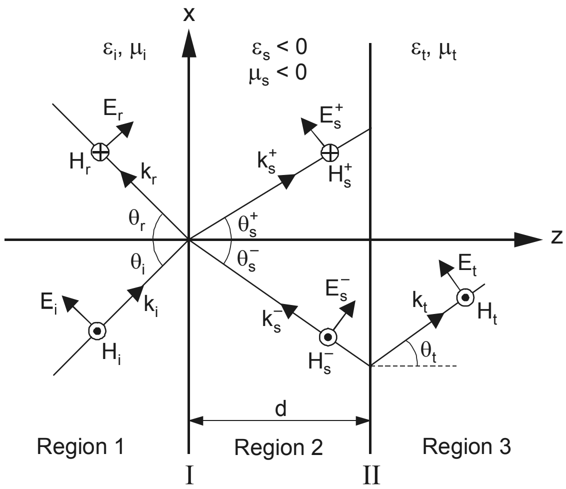

Fig. 1. The configuration of the DNS between two dielectric media.

where ε_r is the relative permittivity, μ_r is the relative permeability, and k_0 is the free space wave number.

In the analysis, we consider a transverse magnetic (TM) wave impinging on DNS from a semi-infinite dielectric medium as shown in Fig. 1. Note that $\exp(j\omega t)$ time dependence is assumed and suppressed. To obtain the general formulation for the reflection and transmission coefficients, it is necessary to examine the interfaces shown in Fig. 1 in detail.

As shown in Fig. 1, the TM polarized incident magnetic field in Region 1 can be expressed as:

$$\mathbf{H_i} = \mathbf{a_y} H_i \exp\left[-j(k_{ix}x + k_{iz}z)\right] \tag{5}$$

where k_i is the wave number and H_i is the magnitude of the incident magnetic field which impinges onto the dielectric-DNS interface I at an angle of θ_i. Similarly the reflected magnetic field at this interface can be written as

$$\mathbf{H_r} = -\mathbf{a_y} H_r \exp\left[-j(k_{rx}x - k_{rz}z)\right] \tag{6}$$

where k_r is the wave number, H_r is the magnitude of the reflected magnetic field. The transmitted magnetic field in Region 3 can be expressed as

$$\mathbf{H_t} = \mathbf{a_y} H_t \exp\left[-j(k_{tx}x + k_{tz}(z - d))\right] \tag{7}$$

where H_t represent magnitude of the transmitted magnetic field and k_t is the wave number of Region 3.

The magnetic field in the DNS is represented as a sum of the two field components, one of them is propagating toward the interface I and the other toward the interface II. Hence in the DNS, the magnetic field can be written as

$$\mathbf{H_s} = -\mathbf{a_y} H_s^{+} \exp\left[-j(k_{sx}x + k_{sz}z)\right] + \mathbf{a_y} H_s^{-} \exp\left[-j(k_{sx}x - k_{sz}z)\right] \tag{8}$$

where the superscripts + and – signs refer to the forward and the backward waves, respectively, within the slab. Note that the wave number $k_s = |\mathbf{k_s^+}| = |\mathbf{k_s^-}|$, $\mathbf{k_s^+}$ being the propagation vector of forward wave and $\mathbf{k_s^-}$ the propagation vector of the backward wave as shown in Fig. 1. The electric fields in all regions can easily be obtained by using the Maxwell's equations.

By imposing the continuity of the tangential components of the electric and magnetic fields at the interfaces, both in phase and magnitude, the reflection and the transmission coefficients at each interface can be found. The continuity of these fields in phase requires

$$k_i \sin\theta_i = k_r \sin\theta_r = k_s \sin\theta_s^{+} = k_s \sin\theta_s^{-} = k_t \sin\theta_t \tag{9}$$

where θ_s^{+} and θ_s^{-} are the reflection angles at interface I and the transmission angle at interface II, respectively. Equation (9) is known as Snell's law which gives the relation between the incidence, reflection and transmission angles. From Equation (9), we obtain $k_i = k_r$, $\theta_i = \theta_r$, and $\theta_s = \theta_s^{+} = \theta_s^{-}$. After applying the boundary conditions at the interfaces with the use of Eq. (9), the relationships between the incident, reflected and transmitted fields can easily be determined. Then, we can find the reflection and transmission coefficients which are defined as the ratios of the amplitudes of the reflected and the transmitted electric fields to that of the incident field, respectively. So that, the formulations of the reflection and transmission coefficients for TM wave can be expressed in the explicit form as:

$$R^{\mathrm{TM}} = \frac{(-k_{iz}\varepsilon_s + k_{sz}\varepsilon_i)(k_{sz}\varepsilon_t + k_{tz}\varepsilon_s)e^{j\varphi} - (k_{iz}\varepsilon_s + k_{sz}\varepsilon_i)(k_{sz}\varepsilon_t - k_{tz}\varepsilon_s)e^{-j\varphi}}{(k_{iz}\varepsilon_s + k_{sz}\varepsilon_i)(k_{sz}\varepsilon_t + k_{tz}\varepsilon_s)e^{j\varphi} + (k_{iz}\varepsilon_s - k_{sz}\varepsilon_i)(k_{sz}\varepsilon_t - k_{tz}\varepsilon_s)e^{-j\varphi}} \tag{10}$$

$$T^{\mathrm{TM}} = \frac{k_t}{k_i}\,\frac{4\,k_{iz}k_{sz}\varepsilon_i\varepsilon_s}{(k_{iz}\varepsilon_s + k_{sz}\varepsilon_i)(k_{sz}\varepsilon_t + k_{tz}\varepsilon_s)e^{j\varphi} + (k_{iz}\varepsilon_s - k_{sz}\varepsilon_i)(k_{sz}\varepsilon_t - k_{tz}\varepsilon_s)e^{-j\varphi}} \tag{11}$$

where $k_{pz} = k_p\cos(\theta_p)$, $(p = i, s, t)$ and $\varphi = k_{sz}d$. Note that ε_i, ε_s, and ε_t are the permittivities of the incident, DNS, and transmitted media.

The reflection and transmission coefficients for an incident transverse electric (TE) polarized wave can be found by using a similar approach as:

$$R^{\mathrm{TE}} = \frac{(k_{iz}\mu_s - k_{sz}\mu_i)(k_{sz}\mu_t + k_{tz}\mu_s)e^{j\varphi} + (k_{iz}\mu_s + k_{sz}\mu_i)(k_{sz}\mu_t - k_{tz}\mu_s)e^{-j\varphi}}{(k_{iz}\mu_s + k_{sz}\mu_i)(k_{sz}\mu_t + k_{tz}\mu_s)e^{j\varphi} + (k_{iz}\mu_s - k_{sz}\mu_i)(k_{sz}\mu_t - k_{tz}\mu_s)e^{-j\varphi}} \tag{12}$$

$$T^{\mathrm{TE}} = \frac{4\, k_{iz}\, k_{sz}\, \mu_s\, \mu_t}{(k_{iz}\mu_s + k_{sz}\mu_i)(k_{sz}\mu_t + k_{tz}\mu_s)e^{j\varphi} + (k_{iz}\mu_s - k_{sz}\mu_i)(k_{sz}\mu_t - k_{tz}\mu_s)e^{-j\varphi}} \tag{13}$$

where μ_i, μ_s, and μ_t are the permeabilities of the incident, DNS, and transmitted media.

For both polarizations the conservation of the power is obtained as follows [7, 8]:

$$\left|R^{\mathrm{TE,\,TM}}\right|^2 + \left|\frac{k_t \cos\theta_t}{k_i \cos\theta_i}\,\frac{\mu_i}{\mu_t}\right|\left|T^{\mathrm{TE,\,TM}}\right|^2 = 1 \tag{14}$$

Note that the incident power is normalized to unity in Eq. (14).

3. Numerical results

In this section, the effects of the structure parameters, the slab thickness, the incidence angle, and the frequency on the reflection and transmission coefficients are studied by some numerical examples for both TE and TM wave cases. To check the results of the analysis used in these computations, the conservation of power given in Eq. (14) is first checked and it is clear that it is satisfied for all examples. The power conversion results are not presented here, but interested readers are referred to [7] and [8]. A second method used to check the results is to derive a transmission line equivalent circuit model for the structure given in Fig. 1 [10] and find the reflection and transmission coefficients using this equivalent model. It is seen that both methods give the same numerical values for the reflection and transmission coefficients. Thus, the theoretical results are verified by means of two concepts, the conservation of power and the transmission line equivalent circuit model.

The reflection and transmission coefficients are calculated as a function of frequency, incidence angle, and structure parameters (such as permittivity and slab thickness). In the calculations, the operation frequency is assumed to be $f_o = 11$ GHz. The permeabilities of Region 1 and the Region 3 in Fig. 1 are selected to be equal to the permeability of the free space ($\mu_i = \mu_t = \mu_o$). The permittivity and the permeability of the DNS are calculated using Lorentz medium model equations given in Eqs. (1) and (2) with no loss case, *i.e.*, $\gamma = 0$.

In the first example, it is assumed that the Region 1 and Region 3 are air and mica whose relative permittivities are 1.0 and 6.0, respectively. The permittivity and permeability of DNS were calculated as being equal to $\varepsilon_s = -2.8732\varepsilon_o$ and $\mu_s = -2.0143\mu_o$. In the calculations, the following parameters were used [9, 11]:

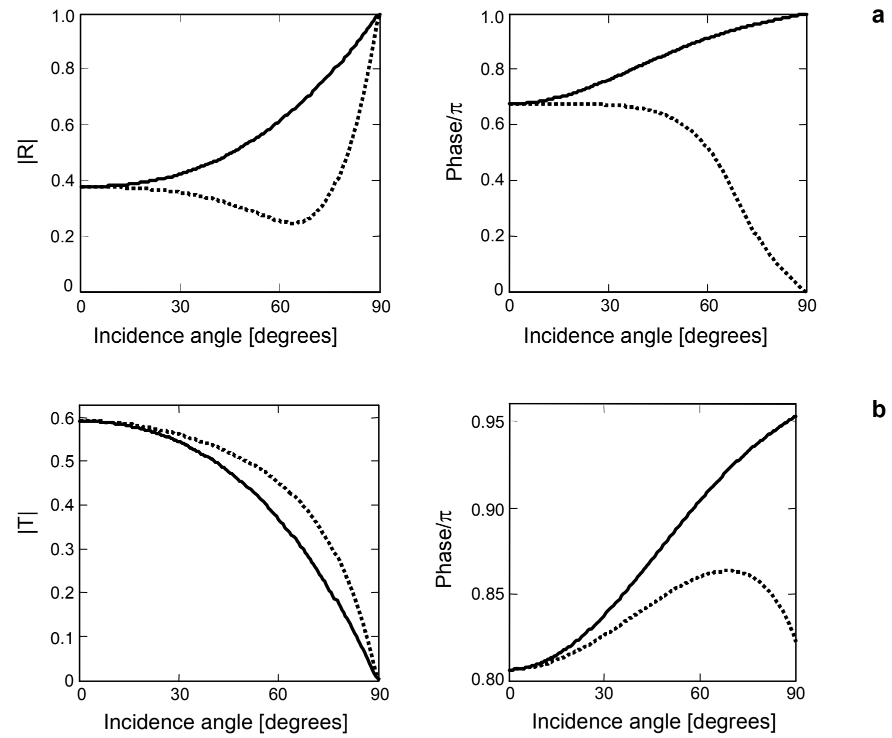

Fig. 2. Reflection and transmission coefficients for TE and TM cases versus incidence angle. The solid line corresponds to TE wave while the dotted line to TM wave.

$f_{mp} = 8.50$ GHz, $f_{mo} = 12.05$ GHz, $f_{ep} = 12.80$ GHz, and $f_{eo} = 10.30$ GHz where $f_i = \omega_i/2\pi$ ($i = mp, mo, ep, eo$). A quarter-wavelength slab is assumed. Figure 2 shows the reflection and the transmission coefficients for TE (solid line) and TM (dotted line) cases versus incidence angle. From Figure 2**a**, the magnitude of the reflection coefficient for TE wave increases with the increase of the angle of incidence. But the magnitude of the reflection coefficient for TM wave decreases up to $\theta_i = 64°$ and above this angle it increases. The phase of the reflection coefficient for TE wave increases when the incidence angle increases. But for the TM wave it is around 0.65 radians up to the incidence angle of $\theta_i = 43°$ and above this angle it decreases. From Figure 2**b**, the magnitude of the transmission coefficients for TE and TM waves decline as θ_i increases. The phase of the transmission coefficient for TE wave increases from 0.806 radians to 0.953 radians. For TM wave the phase rises up to $\theta_i = 69°$ and after this angle it decreases. It is clearly seen that the magnitude of the transmission coefficient for TM wave is dominant over a wide range of the incidence angle. But for TE wave, it is dominant for the incident angle less than 30°. Also, there is no transmission when the incidence angle is 90°.

As a second example, the variations of the reflection and the transmission coefficients for TE and TM waves with the frequency are investigated. The structure

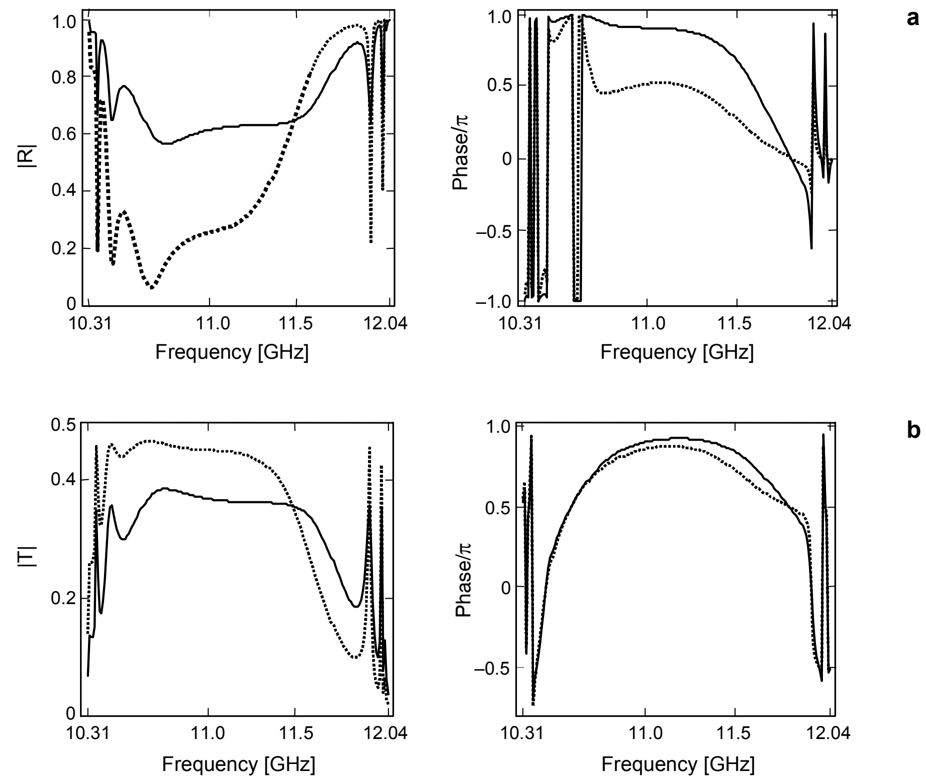

Fig. 3. Reflection and transmission coefficients for TE and TM waves as a function of frequency. The solid line corresponds to TE wave while the dotted line to TM wave.

parameters are the same with the previous example except for the permittivity and permeability of DNS. In this case, they are not constant and are expressed as a function of the frequency given in Eqs. (1) and (2). The angle of incidence is chosen to be 60°. The reflection and transmission coefficients are presented as a function of frequency in Fig. 3. It is observed that, the behaviors of the reflection and the transmission coefficients for TE and TM waves are very sensitive to the varying frequency due to the frequency-dependent permittivity and permeability. The structure has a slight pass band for the reflected wave both for TE and TM waves between the 11.5–12.04 GHz. The phases of reflection coefficient for TE and TM waves change form –1.0 radian to 1.0 radian, and the phases of transmission coefficient for both waves show similar behavior.

In the third example the effect of the slab thickness on the reflection and transmission coefficients is investigated. The incidence angle is assumed to be 70°. The permittivities and permeabilities are the same as those in the first example. The slab thickness d is allowed to change from $\lambda/5$ to λ. Figure 4 illustrates variations of the reflection and transmission coefficients for TE and TM waves as the slab thickness changes. From Figure 4**a**, $|R^{\mathrm{TE}}|$ changes between 0.21 and 0.74, while $|R^{\mathrm{TM}}|$

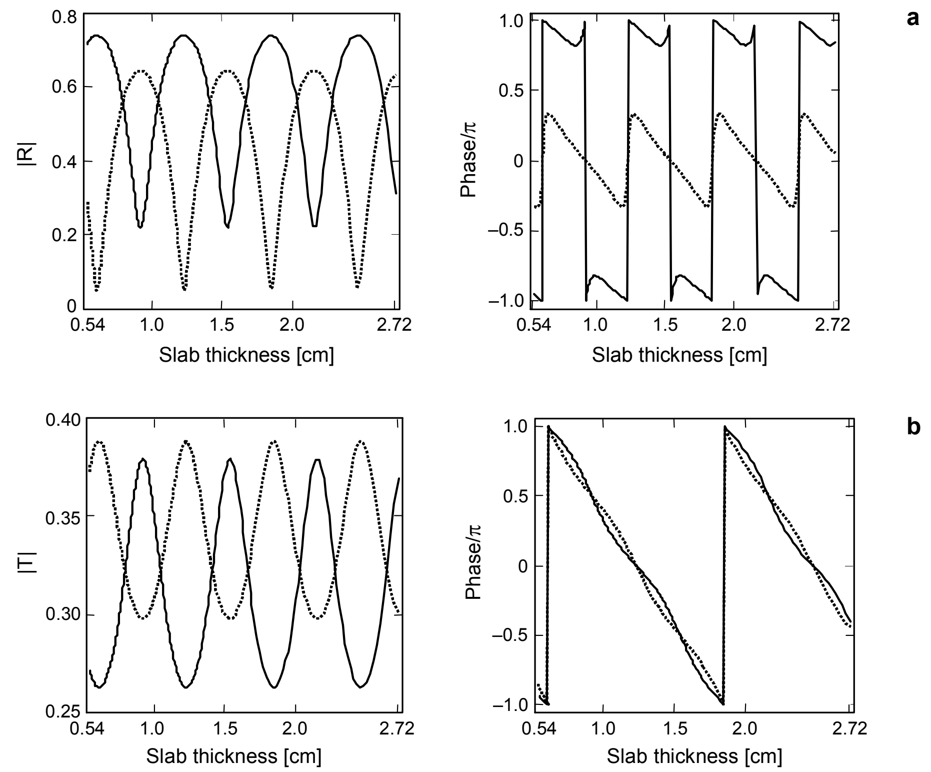

Fig. 4. Reflection and transmission coefficients for TE and TM waves versus slab thickness. The solid line corresponds to TE wave while the dotted line to TM wave.

between 0.045 and 0.65. Also the phase of R^{TE} varies from –1.0 radian to 1.0 radian, while the phase of R^{TM} ranges from –0.33 radians to 0.33 radians. From Figure 4**b**, $|T^{\mathrm{TE}}|$ changes between 0.26 and 0.38, but $|T^{\mathrm{TM}}|$ varies between 0.29 and 0.39. The phases of them change between –1.0 radian to 1.0 radian and show similar behavior. Figure 4 points out that the magnitudes of the reflection and transmission coefficients for TE and TM waves vary periodically as expected. Furthermore, the phases of them change monotonically with the frequency in the given range.

4. Conclusions

The reflection and transmission coefficients of the electromagnetic waves in a frequency dispersive DNS are analyzed in detail. The plane wave interaction with a DNS is examined and the reflection and the transmission coefficients for TE and TM waves are found analytically. Then, the effects of the frequency, the incidence angle, and the slab thickness are studied by the numerical results. It is shown that these parameters affect the reflection and transmission coefficients. The transmission coefficients both for TE and TM waves are dominant in some range of the incidence angle. The characteristics of the reflection and the transmission coefficients for both waves are very sensitive to the varying frequency due to the frequency-dependent permittivity and permeability. Also, there is a pass band for the reflected wave both for TE and TM waves in the frequency range of 11.5 to 12.04 GHz. The theoretical and numerical results obtained here can easily be extended for further studies such as lossy DNS, multilayer DNG slabs, *etc*.

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