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Coherent combination of laser radiations and fringe contrast ratios on far field patterns

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Fourier optics method has been used to study the far field of coherently combined laser beams. And an explicit expression of the far field has been derived for the case when emitters are positioned on apexes of multiple regular polygons with a common center. The statistical influence of the relative phases between constituent waves has been investigated and an expression for the fringes contrast ratio has been deduced with the aid of Ergodic hypothesis.

Keywords: beam combination, Fourier-optics, dark fringes, contrast ratio.

1. Introduction

In recent years, laser sources with high output power and good beam quality are required in various application areas, such as the laser weapon, laser radar, and the next generation laser communication in free-space, *etc*. While the output power of a single laser is limited by the gain saturation, thermal effects, and so on, coherent combination of laser beams from multiple lasers has been regarded as a hopeful option. And different types of lasers have been successfully phase locked to generate coherently combined beams [1–12]. There are many kinds of methods to achieve coherent combination of laser beams, such as phase control [7], mutual injections of radiations [2], *etc*. Meanwhile, previous studies have also been carried out to characterize the coherently combined lasers beams. Based on Collins integral formula, characteristics of the combined Hermite–Gaussian light beams have been studied [8]. In reference [9], a semi-empirical model is presented and the numerical results have been reported.

In order to coherently combine the laser beams, it is important that the relative phases of emitters should be fixed. Any fluctuation of the relative phases may induce a decrease of the fringe contrast ratios of the far field pattern. Several researchers have studied the relative phases of emitters so far [10–12]. The relation between injection current and the relative phase between the light emitted from each defect has been discussed in detail [10]. AUGST *et al.* [11] have presented detailed phase noise spectra generated in Yb-doped fiber amplifiers in coherent combination process. In ref. [12], the effect of the stochastic phase errors has been discussed.

In this work, by using Fourier optics [13], the far field of coherently combined laser beams has been analytically expressed in terms of the observation directions and an explicit expression has been derived for the case when the emitters are positioned on apexes of multiple regular polygons with a common center. With the aid of this expression, it has been shown that the dark fringes are far from being circles for the circular array when the number of emitters is not large enough. Meanwhile, the statistical influences of the parameter fluctuations on the far field pattern have been considered. For example, the fluctuations of the relative phases between constituent waves have been discussed in detail and an expression for the fringe contrast ratio has been deduced with the aid of Ergodic hypothesis [14].

2. Far field expression

Consider the case when N laser beams, with identical polarization and oscillation wavelength, are combined. Assume that the output field of the n-th laser is

$$e_n = A_n u_n(\mathbf{r} - \mathbf{r_n}) \exp\left[ik_0(z - z_n) + i\phi_n\right] \tag{1}$$

where A_n, ϕ_n and k_0 are the amplitude, phase and propagation constant of the field, respectively, $\mathbf{r_n}$ is the transversal coordinate of the beam center and z_n is longitudinal position of the output mirror (or facet), $u_n(\mathbf{r})$ describes the transversal distribution of the field. From Fourier optics, the far field of the combined fields is

$$\tilde{E}_T(\mathbf{f}) = \sum_n A_n \tilde{U}_n(\mathbf{f}) \exp(i\mathbf{f} \bullet \mathbf{r_n}) \exp\left[ik_0(z - z_n) + i\phi_n\right] \tag{2}$$

where $\mathbf{f}$ represents the spatial frequency, $\bullet$ stands for the dot product, and

$$\tilde{U}_n(\mathbf{f}) = \int U_n(\mathbf{r}) \exp(i\mathbf{f} \bullet \mathbf{r}) \mathrm{d}\mathbf{r} \tag{3}$$

and the far field intensity of the combined beams is

$$\tilde{I}_T = \left\langle \sum_{n,\, n'} \left\{ A_n A_{n'} \tilde{U}_n(\mathbf{f}) \tilde{U}_{n'}^*(\mathbf{f}) \exp\left[i\mathbf{f} \bullet (\mathbf{r_n} - \mathbf{r_{n'}}) + ik_0(z_{n'} - z_n) + i(\phi_n - \phi_{n'})\right] \right\} \right\rangle \tag{4}$$

where $*$ indicates complex conjugation, and $\langle\,\rangle$ denotes time average.

Generally, all the output mirrors (or facets) of the lasers are in the same plane, *e.g.*, $z_n = 0$, and all the laser beams have identical transversal field distribution, say $u(\mathbf{r} - \mathbf{r_n})$. If all the field amplitudes are the same, one has

$$\tilde{I}_T = A^2 \left|\tilde{U}(\mathbf{f})\right|^2 \sum_{n,\, n'} \exp[i\mathbf{f} \bullet (\mathbf{r_n} - \mathbf{r_{n'}})] \langle \exp\left[i(\phi_n - \phi_{n'})\right]\rangle \tag{5}$$

In principle, all parameters of an individual beam may fluctuate around their ideal values. However, in practice, efforts should always be made to minimizing these fluctuations. In many cases, the phase fluctuation is the most frequently encountered one. And, in this work, the discussions are focused on this fluctuation, as suggested by Eq. (5). Even though, we would like to point out that the studies made in this work can be extended to discuss other fluctuations with minor modifications. Thus, finding the time average of Eq. (5) becomes finding

$$\eta = \langle \exp(i\Delta\phi)\rangle \tag{6}$$

where, for simplicity, $(\phi_n - \phi_{n'})$ has been denoted by $\Delta\phi$ with its subscript (nn') omitted.

When one investigates the light intensity at the direction making a small angle θ with respect to the Z-direction, it can be shown that [13]

$$f \cong k_0 \theta = \frac{2\pi\theta}{\lambda} \tag{7}$$

where λ is the wavelength of the radiation. Inserting Eq. (7) into Eq. (5), the angular distribution of the far field of the combined beams can be found. From Eq. (5), it can be realized that the quantity $|\tilde{U}|^2$ acts as a profile of the far field pattern and the summation term dictates the fine structures of the pattern. If the lasers are scattered within a range considerably larger than the size of an individual beam, a considerable number of fringes can be observed on the far field pattern of the coherently combined beams.

In fact, Eq. (5) can also be cast as

$$\tilde{I}_T(\mathbf{f}) = \left|\tilde{U}(\mathbf{f})\right|^2 A^2 N + \left|\tilde{U}(\mathbf{f})\right|^2 A^2 \sum_{n \neq n'} \exp\left[i\mathbf{f} \bullet (\mathbf{r_n} - \mathbf{r_{n'}})\right] \langle \exp\left[i(\phi_n - \phi_{n'})\right]\rangle \tag{8}$$

On closer inspection on Eq. (8), it can be realized that the first term on the right side (RHS) represents the far field intensity of incoherently combined beams, and the second term indicates a redistribution of the light energy of the combined beams in the spatial directions, leading to bright and dark fringes on the far field pattern.

According to the law of energy conservation, it can be anticipated that the integration of the second term over **f** is equal to zero, since the integration of the first term represents the total energy of N constituent beams.

For the ideal case when N beams are coherently combined, the quantity $\Delta\phi$ has fixed values. For the so-called in-phase mode, $\Delta\phi$ is equal to zero and the quantity η is equal to unity. For the out-off-phase mode, $\Delta\phi_{nn'}$ may be equal to $(n - n')\pi$ and $\eta_{nn'}$ is equal to $(-1)^{n-n'}$. If there is no coherence between individual fields, the quantity $\Delta\phi$ may randomly pick up any values and η becomes zero. Inspecting on Eqs. (2) and (5), it can be seen that for the in-phase mode, the light intensity at the normal direction takes its maximum because **f** is equal to zero at this direction, and for the out-off-phase mode, the light intensity is equal to zero at Z-direction if N is an even number.

3. Calculating far field patterns

In order to calculate the far field pattern of the coherent combined waves, we consider the light intensity at the direction indicated by the ray $0P$ shown in Fig. 1. The angle between $0P$ and Z-axis is θ, and that between the projection of $0P$ inside the $z = 0$ plane and the X-axis is ψ. Inside the plane defined by $0P$ and Z-axis, one can find out that

$$\mathbf{f} \cong \frac{2\pi\theta}{\lambda}(\mathbf{i}\cos\psi + \mathbf{j}\sin\psi) \tag{9}$$

where we have assumed that θ is a very small angle, and **i** and **j** represent the unit vectors along X- and Y-directions, respectively. If we express $\mathbf{r_n}$ in the polar coordinate, *i.e.*,

$$\mathbf{r_n} = r_n(\mathbf{i}\cos\psi_n + \mathbf{j}\sin\psi_n) \tag{10}$$

it can be derived from (2), for the in-phase mode, that

$$\tilde{E}_T^{\mathrm{ip}}(\theta) = \tilde{U}(\theta)A\sum_n \exp\left[i\frac{2\pi\theta r_n}{\lambda}\cos(\psi - \psi_n)\right] \tag{11}$$

If the out-off-phase mode satisfies

$$\phi_n - \phi_{n'} = (n - n')\pi \tag{12a}$$

it can be obtained that

$$\tilde{E}_T^{\mathrm{op}}(\theta) \propto \tilde{U}(\theta)A\sum_n (-1)^{n-1}\exp\left[i\frac{2\pi\theta r_n}{\lambda}\cos(\psi - \psi_n)\right] \tag{12b}$$

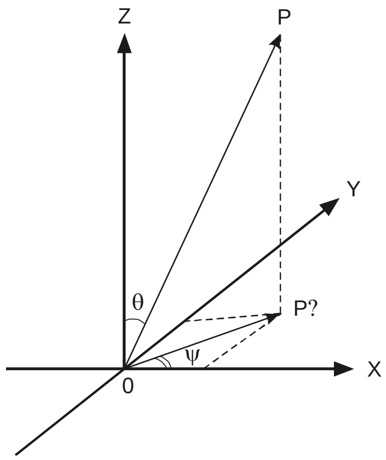

Fig. 1. Schematic showing the observation direction indicated by the ray *0P* and its defining parameters.

It may happen that all the output facets (or mirrors) of the lasers are positioned on the apexes of a regular polygon. Under those circumstances, r_n is equal to the radius r of the circumscribed circle of the polygon, and

$$\psi_n = \frac{2(n-1)\pi}{N} + \psi_1 \tag{13}$$

Inserting Eq. (13) into Eq. (11) and Eq. (12b), the explicit analytical expression of the far field of the coherently combined beams can be derived.

In Figure 2, variations of the light intensity of the in-phase mode, decided by the summation term (denoted by K) of Eq. (11), with the divergent angle θ inside the X-Z (*i.e.*, $\psi = 0$ and π) plane (**a**) and Y-Z (*i.e.*, $\psi = \pi/2$ and $3\pi/2$) plane (**b**), have been plotted for $N = 6$. In the calculations, $r/\lambda = 1000$ and $\psi_1 = 0$. The analytical expressions can be deduced from (11), which are

$$K^2(\psi = 0) = 4\left[\cos\left(\frac{2\pi r\theta}{\lambda}\right) + 2\cos\left(\frac{\pi r\theta}{\lambda}\right)\right]^2 \tag{14a}$$

$$K^2(\psi = \pi/2) = 4\left[1 + 2\cos\left(\frac{\sqrt{3}\,\pi r\theta}{\lambda}\right)\right]^2 \tag{14b}$$

Figure 2 gives only the fine structure of the intensity distribution. If the specific form of $\tilde{U}(\mathbf{f})$ is known, the number of the dark fringes, which can be observed, may be reduced considerably.

In Figure 3, dark fringes for $N = 4$ on the far field pattern of the in-phase mode have been shown. The data used are the same as those given after Fig. 2. In this diagram, the vector, from the frame origin O to the point under consideration, gives

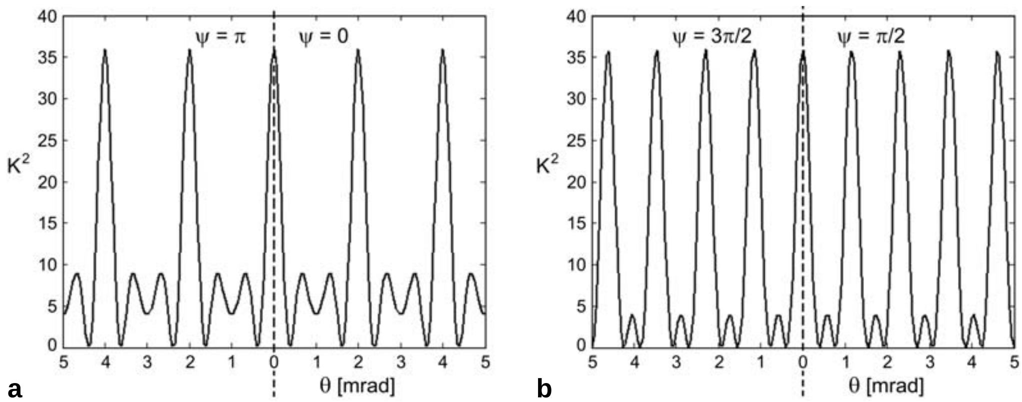

Fig. 2. Fine structure of the far field inside $\psi = 0$ (or π) (**a**) and $\psi = \pi/2$ (or $3\pi/2$) (**b**) planes for a circular array with 6 emitters.

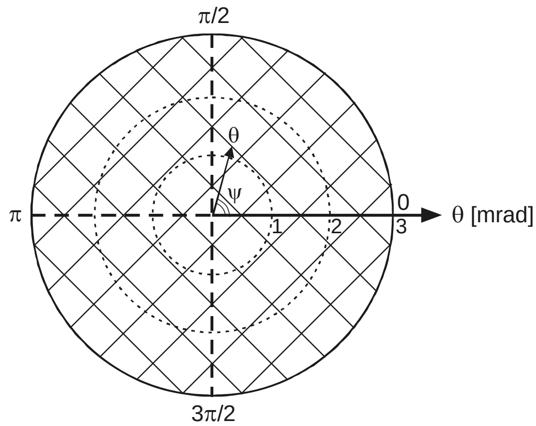

Fig. 3. Distribution of dark fringes on the far field for a circular array with 4 emitters.

the number set (θ, ψ) in the polar coordinate, *i.e.*, the modular and angle of the vector represent θ and ψ, respectively. The points on the dark fringes, defining the number set (θ, ψ), satisfy $K(\theta, \psi) = 0$. While the expression of K, derived from (11), reads

$$K = 2\cos\left(\frac{2\pi r\theta\cos\psi}{\lambda}\right) + 2\cos\cos\left(\frac{2\pi r\theta\sin\psi}{\lambda}\right) \tag{15}$$

And the points on the dark fringe satisfy

$$\theta_m^{(0)} = \frac{(2m+1)\lambda}{2r(\cos\psi \pm \sin\psi)} \qquad (m = 0, 1, 2...) \tag{16}$$

From Figure 3, it can be realized that the dark fringes are far from being circles. Rather, they look like “squares” although the “sides” are not the exact straight lines. It is not difficult to understand these results. Since the four light sources are located at $\psi = 0$, $\pi/2$, π, and $3\pi/2$, respectively, there is no reason that the light intensities at the four directions should be different from each other.

It should be pointed out that the quantity K, *i.e.* the summation term of Eq. (11), only represents the fine structures of the far field pattern. A comprehensive pattern requires the knowledge of $\tilde{U}(\mathbf{f})$ or $u(\mathbf{r})$, which serves as an envelope covering the fine structure. For most lasers, the functions of $U(\mathbf{r})$ are known. What we would like to address is that if the light sources are distributed within an area considerably larger than that of the individual beam, there may be large amounts of peaks, which appear when $|K^2|$ takes its maxima, on the far field pattern. To make sure that a considerable amount of coherently combined energy may be directed to the central peak of the far field pattern, the ratio of the beam size to the quantity r should be kept as large as possible. In many cases, the sizes of the output mirrors of the lasers are larger than the beam sizes. This, to certain extent, frustrates the attempts of reducing the quantity r, if the entire output mirror should be positioned on one circle.

Before ending this section, we want to point out that Eq. (11) can be extended to the case where the light sources are positioned on several circles with different radii. Assume that these sources are positioned on the apexes of M regular polygons with a common center, and the m-th polygon, whose radius of the circumscribed circle is r_m, has N_m apexes, it can be deduced that the far field of the in-phase mode reads

$$\tilde{E}_T^{\mathrm{ip}}(\psi, \theta) \propto \sum_{n=1}^{N} \sum_{m_n=0}^{M_n-1} \exp\left\{ i \frac{2\pi\theta r_m}{\lambda} \cos\left[\psi - \frac{2(m_n-1)\pi}{M_n} - \psi_{on} \right] \right\} \tag{17}$$

where ψ_{on} is the polar angle of the zeroth emitter on the n-th polygon.

Using Eq. (17), the far fields of many practical arrays can be specified. For example, we can calculate the far field for the configuration, in ref. [12], where 19 emitters are on two hexagons and one at the center of these hexagons.

4. Fringe contrast ratio

Now, we consider the quantity η defined by Eq. (6). For a practical phase-locked laser system, it is likely that the constituent laser fields may experience certain fluctuations, which, in turn make $\Delta\phi$ randomly fluctuate around the ideal value $\Delta\phi^{(0)}$. If the probability density $p(\Delta\phi)$ that the quantity $\Delta\phi$ takes the value of $\Delta\phi$ can be described by a Gaussian function, *i.e.*

$$p(\Delta\phi) = \frac{1}{\sqrt{\pi}\,\delta_\phi} \exp\left[-\frac{\left(\Delta\phi - \Delta\phi^{(0)}\right)^2}{\delta_\phi^2} \right] \tag{18}$$

while δ_ϕ is the standard deviation of $\Delta\phi$, the quantity η can be evaluated. When the sampling time of the detector is much longer than the time scale at which the fluctuations take place, or the average of Eq. (6) is made within a period during which the constituent fields have experienced a large number of fluctuations, one may employ the Ergodic hypothesis and use the assemble average to replace the time average. Thus, one can deduce that

$$\eta = \exp\left(i\Delta\phi^{(0)}\right)\exp\left(-\frac{\delta_\phi^2}{4}\right) \tag{19}$$

For the in-phase mode, $\Delta\phi^{(0)}$ is equal to zero. Under the circumstances, Eq. (8) can be put down as

$$\tilde{I}_T = (1-\eta)\,\tilde{I}_{\mathrm{inc}}(\mathbf{f}) + \eta\,\tilde{I}_{\mathrm{coh}}(\mathbf{f}) \tag{20}$$

where the subscripts "inc" and "coh" are used to indicate the incoherent and coherent light intensities, respectively, and

$$\tilde{I}_{\mathrm{inc}}(\mathbf{f}) = NA^2\left|\tilde{U}(\mathbf{f})\right|^2 \tag{21a}$$

$$\tilde{I}_{\mathrm{coh}}(\mathbf{f}) = A^2\left|\tilde{U}(\mathbf{f})\right|^2\left|\sum_n \exp(i\mathbf{f}\bullet\mathbf{r_n})\right|^2 \tag{21b}$$

Since the light intensity is zero at the directions of the dark fringes, from the above equations it can be derived that the fringe contrast ratio Γ of the far field pattern is

$$\Gamma = \frac{\eta N}{2(1-\eta)+\eta N} \tag{22}$$

In deriving Eq. (22), we have assumed that the angular spacing between the bright fringe and its nearest dark fringe is considerably smaller than the divergent angle (FWHM) of $|\tilde{U}|^2$, thus the roll-off of $|\tilde{U}|^2$ can be neglected within this angular spacing. Equation (22) tells us that the quantity η or δ_ϕ can be obtained by measuring the fringe contrast on the far field pattern.

In Figure 4, dependences of δ_ϕ (**a**) and η (**b**) on the contrast ratio Γ have been plotted for $N = 4$, 8 and 19, respectively. From this diagram, it can be seen that the standard deviation δ_ϕ of $\Delta\phi$ reduces, while, the η increases with the contrast ratio Γ. The smaller the emitter number N is, the tighter the phase fluctuation should be controlled for realizing the same contrast ratio Γ. In practice, it is conceivable that it is difficult to force a large number of field phases to fall into a very small range. Fortunately, the phase control is not so tight when the emitter number becomes large. In laboratory, a contrast ratio of ≥ 0.8 has been observed from an external cavity phase-locked diode array with 19 broad-stripe emitters [15]. However, if one wants to

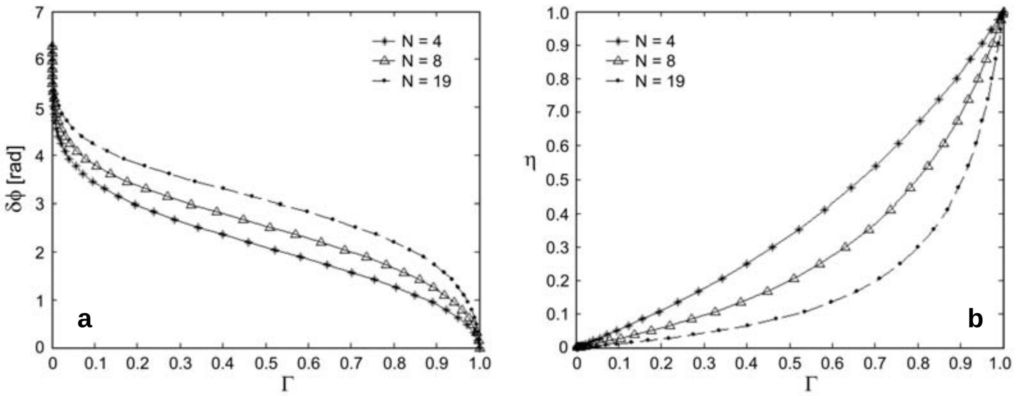

Fig. 4. Variation of the standard deviation $\delta\phi$ of $\Delta\phi$ (**a**) and η (**b**) with contrast ratios Γ for $N = 4, 8, 19$, respectively.

realize a contrast ratio approaching unity, the phase fluctuations should be strictly controlled. In other words, for a completely coherent combination of laser beams, it may become more difficult if the emitter number is larger.

5. Conclusions

Using Fourier optics method, an analytical expression of the far field of coherently combined laser beams is obtained, and an explicit expression has been derived for the case when emitters are positioned on apexes of multiple regular polygons with a common center. Based upon this expression, it has been shown that the dark fringes are far from being circles for the circular array when the emitter number is not large enough. The fluctuation of the relative phase between constituent waves has been discussed, and an expression for the fringes contrast ratio has been deduced with the aid of Ergodic hypothesis. From this expression, it can be realized that for the case when a larger number of beams should be coherently combined, the control on the relative phase between individual fields may become less tight if one wants to achieve a medium contrast ratio, say < 0.9. This is, no doubt, an encouragement for people who want to phase lock more laser beams.

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Reference

[1] LUO B., WANG C., DU J., MA C., GUO Y., YAO J., *Design and analysis of phase gratings for laser beams coherent combination*, Microelectronic Engineering **83**(4–9), 2006, pp. 1368–71.

[2] LU D., CHEN J., YANG H., CHEN H., LIN X., GAO S., *Theoretical analysis on phase-locking properties of a laser diode array facing an external cavity*, Optics and Laser Technology **38**(7), 2006, pp. 516–22.

[3] KOZLOV V.A., HERNANDEZ-CORDERO J., MORSE T.F., *All fiber coherent beam combining of fiber lasers*, Optics Letters **24**(24), 1999, pp. 1814–6.

[4] BOCHOVE E.J., CHEO P.K., KING G.G., *Self-organization in a multicore fiber laser array*, Optics Letters **28**(14), 2003, pp. 1200–2.

[5] MAKLER S.S., CAMPS I., *The coherence of the AlGaAs-GaAs phonon laser*, Physica B **316–317**, 2002, pp. 300–3.

[6] VENUS G.B., SEVIAN A., SMIRNOV V.I., GLEBOV L.B., *Stable coherent coupling of laser diodes by a volume bragg grating in PTR glass*, Proceedings of the SPIE **6104**, 2006, p. 61040S-1.

[7] APOLLONOV V.V., DERZHAVIN S., KISLOV V., KUZMINOV V., MASHKOVSKIY D., PROKHOROV A.M., *Phase-locking of the 2D structures*, Optics Express **4**(1), 1999, pp. 19–26.

[8] BAIDA LU, HONG MA, *Beam combination of a radial laser array: Hermite–Gaussian model*, Optics Communications **178**(4–6), 2000, pp. 395–403.

[9] HUO Y., CHEO P.K., KING G.G., *Fundamental mode operation of a 19-core phase-locked Yb-doped fiber amplifier*, Optics Express **12**(25), 2004, pp. 6230–9.

[10] LEHMAN A.C., RAFTERY J.J., JR., CARNEY P.S., CHOQUETTE K.D., *Coherence of photonic crystal vertical-cavity surface-emitting laser arrays*, IEEE Journal of Quantum Electronics **43**(1), 2007, pp. 25–30.

[11] AUGST S.J., FAN T.Y., SANCHEZ A., *Coherent beam combining and phase noise measurements of ytterbium fiber amplifiers*, Optics Letters **29**(5), 2004, pp. 474–6.

[12] LI Y., QIAN L., LU D., FAN D., WEN S., *Coherent and incoherent combining of fiber array with hexagonal ring distribution*, Optics and Laser Technology **39**(5), 2007, pp. 957–63.

[13] LAUTERBORN W., KURZ T., WIESENFELDT M., *Coherent Optics: Fundamentals and Applications*, Springer Verlag, New York 1993.

[14] REIF F., *Fundamentals of Statistical and Thermal Physics*, Vol. 5 – Statistical Physics, McGraw-Hill Book Company, 1965.

[15] SHI P., CHEN J., QIAN L., YAN D., LEI J., *Phase-locking observed among high-order lateral modes of a broad-stripe diode array inside an external cavity*, Optics and Laser Technology **39**(5), 2007, pp. 953–6.

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