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Guided modes in slab waveguides with both double-negative and single-negative materials

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A new dielectric slab waveguide with double-negative (DNG) film layer and single-negative (SNG) cover layer is proposed. The dispersion relations of an asymmetric left-handed materials (LHM) slab waveguide are investigated by using normalized parameters. An analytical method is then proposed to calculate the dispersion curves. TE-polarized guided modes differ dramatically from TM-polarized guided modes. For TE-polarized oscillating guided modes, fundamental modes exist all the time, but in a narrow range of low frequency values only. First order mode behaves as other higher order modes and exists up to infinite high frequency. Higher order modes have double degeneracy. For TE-polarized surface guided modes, the existence and the type of the mode solutions with respect to different parameters are classified systematically and discussed in detail. TE_0 surface mode exists only in one case. The dispersion curve of TE_1 surface mode continues with that of oscillating TE_0 mode. It seems that the two different kinds of modes compensate each other to form one whole mode. For TM-polarized wave, both oscillating and surface guided modes act as the waveguide with DNG (LHM) film sandwiched in between two normal dielectrics.

Keywords: guided modes, dispersion characteristic, double-negative left-handed materials (DNG LHM), single-negative (SNG), slab waveguide.

1. Introduction

In recent years, negative index materials (NIMs) have been a hot research issue because many physical properties are different from those of the conventional materials. The double-negative (DNG) materials with simultaneously negative permittivity ε and μ, sometimes are also known as left-handed materials (LHM) since the electric field E, the magnetic field H, and the wave vector k form a left-handed relation. Its unusual electromagnetic (EM) properties were first proposed by Veselago 40 years ago [1]. The SNG materials (one of the two parameters ε or μ is negative) include the epsilon-negative (ENG, $\varepsilon < 0$, $\mu > 0$) and the μ-negative (MNG, $\varepsilon > 0$, $\mu < 0$) media.

Nowadays, the study of the properties of electromagnetic waves guided by a DNG waveguide attracts much more attention. Waveguides with DNG film possess some peculiar properties, such as the absence of the fundamental mode [2], double

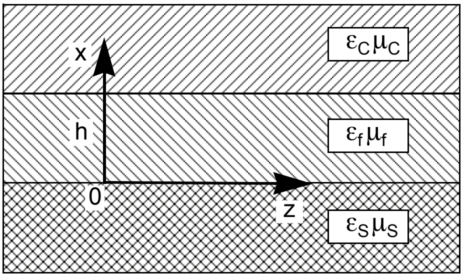

Fig. 1. Geometry of a slab waveguide with both DNG and SNG materials.

degeneracy of modes [3], slow propagation [4, 5], and sign-varying energy flux [6]. Compared with the studies of DNG waveguides, waveguide with both DNG and SNG materials have not been thoroughly studied.

In this paper, guided modes propagating along a dielectric slab waveguide with both DNG and SNG materials are investigated. By using normalized parameters [7], dispersion curves have been obtained, and salient properties of these guided modes are studied in terms of how these parameters of materials are chosen to be paired. Unlike with traditional slab waveguide and other LHM slab waveguide studied before, we find that guidance properties are strongly dependent on ε and μ of the substrate and cover layers. A number of unusual properties are proposed. Both TE and TM guided modes are discussed.

2. Setting up the model

Here, we investigate an asymmetric slab waveguide structure that is sketched in Fig. 1. A dielectric ($\varepsilon_0\varepsilon_f$, $\mu_0\mu_f$) slab occupying the region $0 < x < h$ is sandwiched between a substrate dielectric ($\varepsilon_0\varepsilon_s$, $\mu_0\mu_s$) occupying the region $-\infty < x < 0$ and a cover dielectric ($\varepsilon_0\varepsilon_c$, $\mu_0\mu_c$) occupying the region $h < x < \infty$. The substrate layer is a conventional dielectric material ($\varepsilon_f > 0$, $\mu_f > 0$), while the cover layer is made of an SNG material and the film layer is a DNG material (the dispersion characteristics are the same to the slab waveguide with DNG cover, SNG film and conventional substrate). Here, for SNG materials, we consider MNG ($\varepsilon_c > 0$, $\mu_c < 0$) materials only. Guided modes $|\varepsilon_f\mu_f| > |\varepsilon_s\mu_s| > |\varepsilon_c\mu_c|$ are also considered. Their structure is uniform in the y and z direction. Only the two-dimensional problem ($\partial/\partial y = 0$) is considered, all the fields have the harmonic time dependence $\exp(-i\omega t)$ (which can be ignored in calculation). The electromagnetic filed separates into TE and TM modes.

3. Normalized dispersion relation of TE modes in slab waveguides with both DNG and SNG materials

For oscillating guided TE modes, the electric field is polarized along the y-axis and can be expressed as

$$E_y = E_c \exp\left[-\alpha_c(x-h)\right]e^{i\beta z}, \qquad \alpha_c^2 = \beta^2 - k^2\varepsilon_c\mu_c, \qquad x > h \tag{1}$$

$$E_y = E_f \cos\left(k_f x - \varphi_s\right) e^{i\beta z}, \quad k_f^2 = k^2 \varepsilon_f \mu_f - \beta^2, \quad 0 < x < h \tag{2}$$

$$E_y = E_s \exp(\alpha_s x) e^{i\beta z}, \quad \alpha_s^2 = \beta^2 - k^2 \varepsilon_s \mu_s, \quad x < 0 \tag{3}$$

where β is the propagation constant, $k = \omega\sqrt{\varepsilon_0 \mu_0}$, E_c, E_f, E_s are field amplitudes in the cover, film, and substrate, respectively; φ_s is a phase shift.

The number of independent parameters can be reduced by introducting normalized frequency and normalized propagation constant [8, 9]

$$V = kh\left(n_f^2 - n_s^2\right)^{1/2} \tag{4}$$

$$b = \frac{\beta^2 - k^2 n_s^2}{k^2 n_f^2 - k^2 n_s^2} = \frac{N^2 - n_s^2}{n_f^2 - n_s^2} \tag{5}$$

and the asymmetry measure

$$a = \frac{n_s^2 - n_c^2}{n_f^2 - n_s^2} \tag{6}$$

where $N = \beta / k$ is the effective index; $n_v = \sqrt{\mu_v \varepsilon_v}$, $v = f, s, c$ represent the film, substrate and the cover, respectively. Applying the boundary conditions such that E_y and $H_z = (i/\omega\mu)\frac{\partial E_y}{\partial x}$ are continuous at $x = 0$ and $x = h$, the dispersion relation in the normalized form [8] is obtained,

$$V = \frac{m\pi + \tan^{-1}\frac{\mu_f}{\mu_s}\sqrt{b/(1-b)} + \tan^{-1}\frac{\mu_f}{\mu_c}\sqrt{(a+b)/(1-b)}}{\sqrt{1-b}}, \quad 0 < b < 1 \tag{7}$$

For surface guided TE modes, the transverse wave number becomes purely imaginary $k_f = i\alpha_f$, where $\alpha_f^2 = \beta^2 - k^2 \mu_f \varepsilon_f$. Following the same procedure, we obtain the dispersion relation [9]

$$V = -\frac{\tanh^{-1}\frac{\mu_f}{\mu_s}\sqrt{b/(b-1)} + \tanh^{-1}\frac{\mu_f}{\mu_c}\sqrt{(a+b)/(b-1)}}{\sqrt{b-1}}, \quad b > 1 \tag{8}$$

for surface TE_0 mode and

$$V = -\frac{\coth^{-1}\dfrac{\mu_f}{\mu_s}\sqrt{b/(b-1)} + \coth^{-1}\dfrac{\mu_f}{\mu_c}\sqrt{(a+b)/(b-1)}}{\sqrt{b-1}}, \quad b > 1 \qquad (9)$$

for surface TE_1 mode.

Solving Eqs. (7), (8) and (9), we can directly get the dispersion curves, which are very important to design slab waveguides and have a practical use in optical engineering.

4. Dispersion characteristics of oscillating guided TE modes

In this case, $(\mu_f/\mu_s) < 0$, $(\mu_f/\mu_c) > 0$ the second term of Eq. (7) is negative while the third term is positive. Given 3 groups of assumptions ($\mu_f/\mu_s = -0.7$, $\mu_f/\mu_c = 1.9$), ($\mu_f/\mu_s = -1$, $\mu_f/\mu_c = 1$) and ($\mu_f/\mu_s = -1.9$, $\mu_f/\mu_c = 0.7$), we plot b as a function of V for the first four modes in Fig. 2, with solid and dot lines representing $a = 1$, 10, respectively.

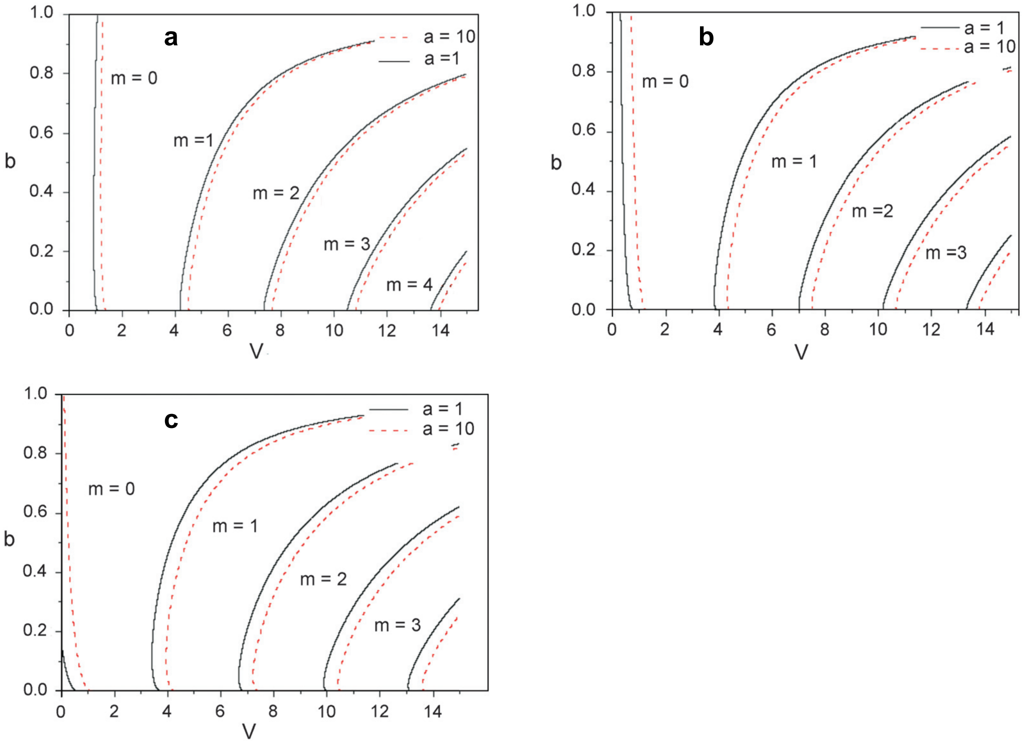

Fig. 2. Normalized dispersion curves of TE modes for a slab waveguide with SNG cover and DNG film. $\mu_f/\mu_s = -0.7$, $\mu_f/\mu_c = 1.9$ (**a**), $\mu_f/\mu_s = -1$, $\mu_f/\mu_c = 1$ (**b**), $\mu_f/\mu_s = -1.9$, $\mu_f/\mu_c = 0.7$ (**c**).

For $b = 0$, Eq. (7) yields the normalized cutoff frequency V_c

$$V_c = m\pi + \tan^{-1}\left(\left| \frac{\mu_f}{\mu_c} \right| \sqrt{a} \right), \qquad m = 1, 2, ... \tag{10}$$

In Figure 2, we see that the shape of dispersion curves of $m > 0$ modes is similar to that of $m > 1$ modes for a slab waveguide with a DNG (LHM) film [9]. Double degeneracy of the modes occurs near cutoff (one V value corresponding to two b values). When V is larger than cutoff, b increases monotonically with V and approaches unity. But in the case of modes different from $m > 1$ ones for a slab waveguide with a DNG (LHM) film [9], b decreases with a [10].

The dispersion characteristic of the fundamental mode is different for different μ_s and μ_c. If $|\mu_f/\mu_s|$ increases (or $|\mu_f/\mu_c|$ decreases), the dispersion curve of fundamental mode moves to the left. When $|\mu_f/\mu_s| \le |\mu_f/\mu_c|$, the absolute value of the third term is larger than that of the second term in Eq. (7), V is positive. Fundamental mode exists for all the b values ($0 < b < 1$), but in a narrow range of V values. When $|\mu_f/\mu_s| = |\mu_f/\mu_c| = 1$, $0.785 > V > 0.293$ for $a = 1$ and $1.2645 > V > 0.698$ for $a = 10$ (Fig. 2**b**). If $|\mu_f/\mu_s| > |\mu_f/\mu_c|$, the fundamental mode exists when $\tan^{-1}(\sqrt{a}\,|\mu_f/\mu_c|) > V > 0$, the corresponding b value being in the range $0 < b < a|\mu_f/\mu_c|^2/(|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2)$. The dispersion curve exists in a narrow range of b values. For example, when $0.60244 > V > 0$ the corresponding b value is $0 < b < 0.15704$ for $a = 1$ and when $1.138 > V > 0$ corresponding b value is $0 < b < 1$ for $a = 10$, see Fig. 2**c**. Double degeneracy also exists in the fundamental mode, see, for example Fig. 2**a**, V decreases from cutoff ($V = 1.08321$) to the minimum value ($V = 0.92309$), and then increases to a large value ($V = 1.05639$) for $a = 1$, one V value corresponding to two b values in the range of $0.92309 < V < 1.05639$.

5. Dispersion characteristics of surface guided TE modes

$|\mu_f/\mu_s|\sqrt{b/(b-1)} > |\mu_f/\mu_c|\sqrt{(a+b)/(b-1)}$ is needed to ensure positive V in Eq. (8), and then solving the inequality we deduce

$$b > \frac{a\,|\mu_f/\mu_c|^2}{|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2}, \qquad \left| \frac{\mu_f}{\mu_s} \right| > \left| \frac{\mu_f}{\mu_c} \right| \tag{11}$$

$$b \in \Phi, \qquad \left| \frac{\mu_f}{\mu_s} \right| \le \left| \frac{\mu_f}{\mu_c} \right| \tag{12}$$

while $|\mu_f/\mu_s|\sqrt{b/(b-1)} < |\mu_f/\mu_c|\sqrt{(a+b)/(b-1)}$ is needed to ensure positive V in Eq. (9), then solving the inequality we obtain

$$b < \frac{a\,|\mu_f/\mu_c|^2}{|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2}, \qquad \left|\frac{\mu_f}{\mu_s}\right| > \left|\frac{\mu_f}{\mu_c}\right| \tag{13}$$

$$b > 1, \qquad \left|\frac{\mu_f}{\mu_s}\right| = \left|\frac{\mu_f}{\mu_c}\right| \tag{14}$$

$$b > \frac{a\,|\mu_f/\mu_c|^2}{|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2}, \qquad \left|\frac{\mu_f}{\mu_s}\right| < \left|\frac{\mu_f}{\mu_c}\right| \tag{15}$$

The dispersion characteristics can be classified in three sub-cases.

5.1. $|\mu_f/\mu_s| > |\mu_f/\mu_c|$

5.1.1. $|\mu_f/\mu_s| < 1$

In Figures 3 and 4, the corresponding dispersion curves of oscillating guided modes are also shown for comparison. A horizontal line $b = 1$ divides the figure into two parts: $b > 1$ part is the surface modes; while $0 < b < 1$ part is the oscillating modes.

In this case $|\mu_f/\mu_c| < 1$, too. Both TE_0 and TE_1 surface modes can exist. From Eq. (8) and inequality (11), the value range of b for TE_0 mode is

$$b > \max\left\{ \frac{1}{1 - |\mu_f/\mu_s|^2}, \frac{1 + a|\mu_f/\mu_c|^2}{1 - |\mu_f/\mu_c|^2}, \frac{a|\mu_f/\mu_c|^2}{|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2} \right\}$$

From Eq. (9) and inequality (13), the value range of b for TE_1 mode is

$$1 < b < \min\left\{ \frac{1}{1 - |\mu_f/\mu_s|^2}, \frac{1 + a|\mu_f/\mu_c|^2}{1 - |\mu_f/\mu_c|^2}, \frac{a|\mu_f/\mu_c|^2}{|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2} \right\}$$

For example, for $|\mu_f/\mu_s| = -0.9$, $|\mu_f/\mu_c| = 0.7$, the surface TE_0 and TE_1 mode dispersion curves are plotted in Fig. 3**c**. For surface TE_0 mode, $5.263 < b < +\infty$ for $a = 1$ and $15.3125 < b < +\infty$ for $a = 10$. For surface TE_1 mode, $1 < b < 1.531$ for $a = 1$ and $1 < b < 5.263$ for $a = 10$. For $a = 1$ the surface TE_0 mode dispersion curve decreases monotonically with V and approaches a constant ($b = 5.263$). For $a = 10$ the surface TE_0 mode dispersion curve exists only in a narrow range of low frequency values. The TE_1 surface mode dispersion curve connects with that of oscillating TE_0 mode, the whole curve behaves as other higher order oscillating modes: double degeneracy of the modes occurs near the cutoff. When V is larger than cutoff, increases monotonically with V and approaches a constant ($b = 5.263$).

5.1.2. $|\mu_f/\mu_s| \geq 1$

In this case, $|\mu_f/\mu_s|\sqrt{b/(b-1)} < -1$ is needed in Eq. (8), TE$_0$ surface mode does not exist. Only TE$_1$ surface modes can exist. Whatever the value of μ_f/μ_c is, inequality (13) is needed to ensure positive V in Eq. (9). Considering the surface mode $b > 1$, TE$_1$ surface mode can exist when

$$a > \frac{|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2}{|\mu_f/\mu_c|^2}$$

For example, when $\mu_f/\mu_s = -2.5$, $\mu_f/\mu_c = 1.3$, the surface TE$_1$ mode dispersion curves are plotted in Fig. 3**a**. In this case,

$$\frac{|\mu_f/\mu_s|^2 - |\mu_f/\mu_c|^2}{|\mu_f/\mu_c|^2} = 2.1065$$

and no TE$_1$ surface mode exists for $a = 1$. For $a = 10$,the TE$_1$ surface mode exists, and with increasing V, the b value decreases monotonically from 3.706 to 0. For

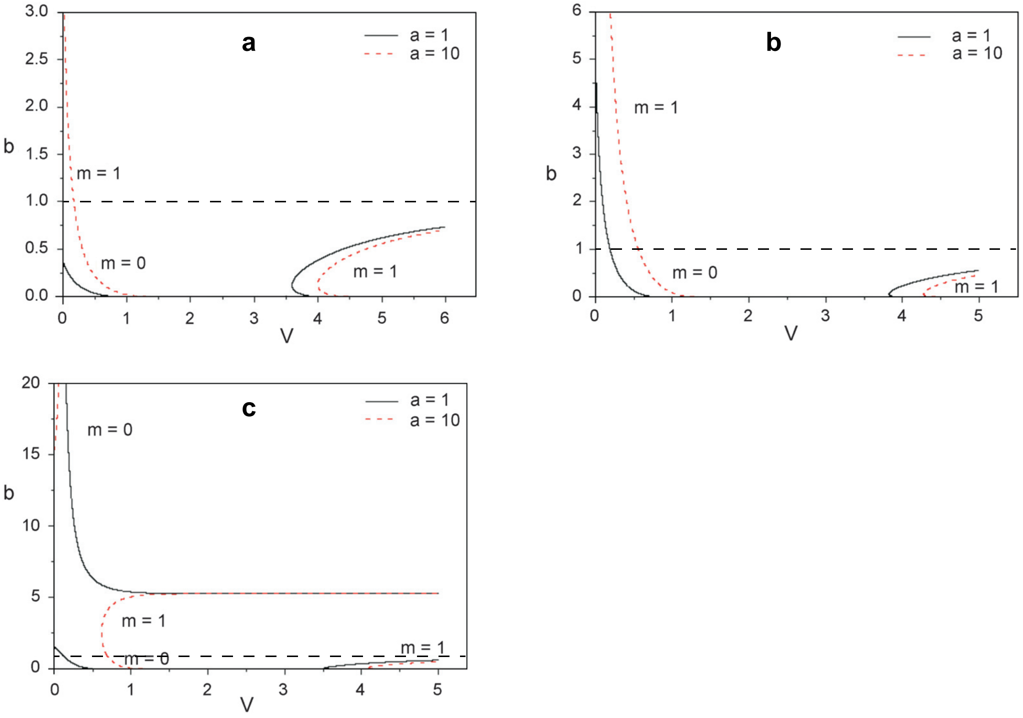

Fig. 3. Normalized dispersion curves of surface TE modes ($b > 1$ part) for a slab waveguide with SNG cover and DNG film, when $|\mu_f/\mu_s| > |\mu_f/\mu_c|$; $\mu_f/\mu_s = -2.5$, $\mu_f/\mu_c = 1.3$ (**a**), $\mu_f/\mu_s = -1.2$, $\mu_f/\mu_c = 1.1$ (**b**), $\mu_f/\mu_s = -0.9$, $\mu_f/\mu_c = 0.7$ (**c**).

$\mu_f/\mu_s = -1.2$, $\mu_f/\mu_c = 1.1$, the surface TE$_1$ mode dispersion curves are plotted in Fig. 3**b**. Both $a = 1$ and $a = 10$ TE$_1$ surface modes can exist. From inequality (13), we know the values of b: $1 < b < 5.2606$ and $1 < b < 52.606$ corresponding to $a = 1$ and $a = 10$, respectively. In Figures 3**a** and 3**b**, we can see that the surface TE$_1$ mode dispersion curve smoothly connects with that of oscillating TE$_0$ mode, but the whole curve monotonically decreases with V and only exists within small frequency.

5.2. $|\mu_f/\mu_s| = |\mu_f/\mu_c|$

The absolute value of the first term of Eq. (8) is smaller than the second one, V is negative. TE$_0$ surface mode does not exist. Only TE$_1$ surface mode can exist.

When $|\mu_f/\mu_s| = |\mu_f/\mu_c| \geq 1$, TE$_1$ surface mode always exists, and TE$_1$ surface mode dispersion curve connects with that of oscillating TE$_0$ mode. For example, when $\mu_f/\mu_s = -1.9$, $\mu_f/\mu_c = 1.9$, surface TE$_1$ mode is plotted in Fig. 4**a**. Both $a = 1$ and $a = 10$ surface TE$_1$ modes exist, the surface TE$_1$ mode dispersion curve connects with that of oscillating TE$_0$ mode, but the whole curve monotonically decreases with V and only exists in a narrow range of low frequency values.

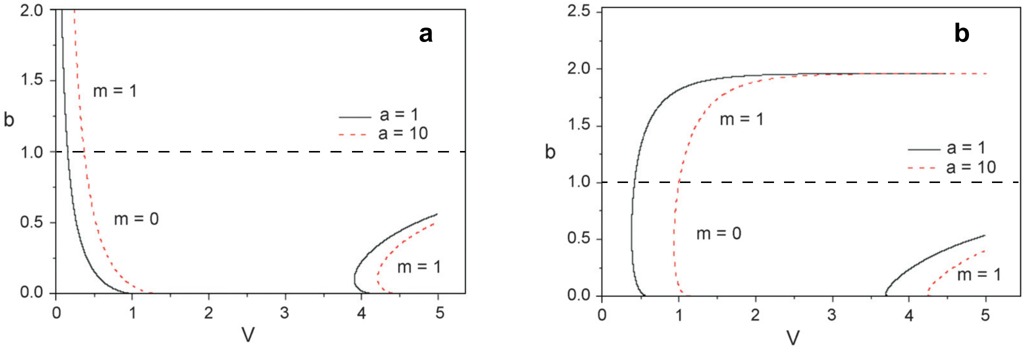

Fig. 4. Normalized dispersion curves of surface TE modes ($b > 1$ part) for a slab waveguide with SNG cover and DNG film when $|\mu_f/\mu_s| = |\mu_f/\mu_c|$; $\mu_f/\mu_s = -1.9$, $\mu_f/\mu_c = 1.9$ (**a**), $\mu_f/\mu_s = -0.7$, $\mu_f/\mu_c = 0.7$ (**b**).

When $|\mu_f/\mu_s| = |\mu_f/\mu_c| < 1$, from Eq. (9), the range of values of b is

$$1 < b < \frac{1}{1 - |\mu_f/\mu_s|^2}$$

For example, when $\mu_f/\mu_s = -0.7$, $\mu_f/\mu_c = 0.7$, the TE$_1$ surface mode is plotted in Fig. 4**b**. Both $a = 1$ and $a = 10$ surface TE$_1$ modes exist, the surface TE$_1$ mode dispersion curve connects with that of oscillating TE$_0$ mode, the whole curve increases monotonically with V and approaches unity ($b = 1.9608$).

5.3. $|\mu_f/\mu_s| < |\mu_f/\mu_c|$

In inequalities (12) and (15), we see that TE$_0$ surface mode does not exist. But TE$_1$ surface mode exists no matter what the value of *a* is.

5.3.1. $|\mu_f/\mu_s| \geq 1$

In this case, $|\mu_f/\mu_c| > 1$ the range of values of b is $1 < b < +\infty$ for TE$_1$ surface mode. For example, when $\mu_f/\mu_s = -1.5$, $\mu_f/\mu_c = 2.5$, the surface TE$_1$ mode curves are similar to those of Fig. 4**a**. Both $a = 1$ and $a = 10$ surface TE$_1$ modes exist, the surface TE$_1$ mode dispersion curve connects with that of oscillating TE$_0$ mode, the whole curve monotonically decreases with V and only exists in a narrow range of low frequency values.

5.3.2. $|\mu_f/\mu_s| < 1$

Inequality (15) can be tenable all the time. $1 < b < (1 - |\mu_f/\mu_s|^2)$ is obtained in Eq. (9) when $|\mu_f/\mu_c| \geq 1$ and $1 < b < \min\{1/(1 - |\mu_f/\mu_s|^2), (1 + a|\mu_f/\mu_c|^2)/(1 - |\mu_f/\mu_c|^2)\}$ is obtained in Eq. (9) when $|\mu_f/\mu_c| < 1$. The surface TE$_1$ mode curves are similar to those of Fig. 4**b**. Both $a = 1$ and $a = 10$ surface TE$_1$ modes exist, the surface TE$_1$ mode dispersion curve connects with that of oscillating TE$_0$ mode, the whole curve increases monotonically with V and approaches unity.

6. TM mode dispersion characteristics

For TM modes, the magnetic field is polarized along the y-axis and can be expressed as Eqs. (1), (2) and (3). After the same algebraic manipulation, we get the same dispersion equations: Eq. (7) for oscillating TM modes and Eqs. (8) and (9) for surface TM modes, but replacing the ratios of μ_f/μ_s, μ_f/μ_c by $\varepsilon_f/\varepsilon_s$, $\varepsilon_f/\varepsilon_c$, respectively.

In this case, $\varepsilon_f < 0$, $\varepsilon_s > 0$ and $\varepsilon_c > 0$. Thus, all the dispersion characteristics for TM modes in this case are similar to a slab waveguide with LHM film layer [11].

7. Conclusions

By introducing normalized waveguide parameters to the double-negative (DNG) and single-negative (SNG) materials of slab waveguide, the dispersion equations in normalized form are derived. The dispersion curves are calculated analytically by solving the dispersion equation in a reverse way. The existence and novel properties of the guided modes are investigated in detail in the asymmetric slab waveguide with both DNG film and SNG cover layer. A number of unusual properties were proposed. We find that guidance properties are strongly dependent on ε and μ of the substrate and cover layers. For TE-polarized oscillating guided modes, fundamental modes exist all the time, but only in a narrow range of low frequency values. First order mode behaves as other higher order modes and exists up to an infinite high frequency. Higher order modes have double degeneracy. For TE-polarized surface guided modes, the existence and the type of the mode solutions with respect to different parameters are classified systematically and discussed in detail. TE_0 surface mode exists only when $0 < |\mu_f/\mu_c| < |\mu_f/\mu_s| < -1$. The dispersion curve of TE_1 surface mode continues with that of oscillating TE_0 mode. It seems that the two different kinds of modes compensate each other to form one whole mode. For TM-polarized wave, both oscillating and surface guided modes act as the waveguide with DNG (LHM) film surrounded by two normal dielectrics.

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