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Degrees of cross-polarization of a partially coherent electromagnetic beam in uniaxial crystals

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Propagation of degrees of cross-polarization of a partially coherent electromagnetic beam in uniaxial crystals orthogonal to the optical axis is investigated. Based on the paraxial theory of beams, analytical propagation formulas for degrees of cross-polarization are derived and analyses are illustrated by numerical examples. It is found that values of the degree of cross-polarization in uniaxial crystals fluctuate more rapidly with larger deviation of correlation length parameters ($\delta_{ij} - \delta_{ii}$), but show slight variation with larger ratio of refractive index (n_e/n_0) or incident beam wavelength. Results also show that sources with sufficient correlation length parameters generate invariable degree of cross-polarization in uniaxial crystals.

Keywords: cross-polarization, uniaxial crystal, correlation length, refractive index.

1. Introduction

The concept of cross-polarization of stochastic electromagnetic beam-like fields was first introduced by ELLIS and DOGARIU [1] where the parameter called the mutual degree of cross-polarization (MDCP) was defined in a heuristic manner. The MDCP was defined not to exceed unity and was as the same as that for the spectral degree of coherence [2]. Then ANGELSKY *et al*. [3] applied the MDCP to study biological tissue coherent images. The analysis of the intensity fluctuations [4] of stochastic electromagnetic beams was carried out by WOLF *et al*. The quantity introduced in these papers was called the spectral degree of cross-polarization (SDCP) [5], it agrees in concept with the MDCP but has some differences with the later one. The SDCP reduces to the general spectral degree of polarization for coinciding spatial arguments. YU XIN *et al*. studied correlations of polarization in the far field and the SDCP of source beams in free space [6]. Very recently JIXIONG PU and KROTKOVA investigated behaviors of SDCP propagating in turbulent atmosphere [7]. On the other hand, propagation of various beams in uniaxial crystals have comprehensive applications in optical communications in recent years [8, 9]. To our best knowledge, behaviors of the SDCP propagating in anisotropic medium have not been investigated until now. So in this paper, propagation properties of the SDCP of a partially coherent electromagnetic beam in anisotropic uniaxial crystals are investigated. The obtained analytical results are illustrated by numerical examples, and some comparisons are made with previous published results.

2. Analytical formulas for the SDCP

The analytical expression for the degree of cross-polarization has been defined and discussed in previous research. It was introduced by means of the expression [4, 5, 7]:

$$P(r_1, r_2, z, \omega) = \sqrt{1 - \frac{4\,\mathrm{Det}\,W(r_1, r_2, z, \omega)}{\left[\mathrm{Tr}\,W(r_1, r_2, z, \omega)\right]^2}} \tag{1}$$

where Det and Tr stand for the determinant and the trace; W is the 2×2 cross-spectral density matrix of the electric field, which is given by the formula [2]:

$$W(r_1, r_2, z, \omega) = \langle E_i^*(r_1, z, \omega)\, E_j(r_1, z, \omega)\rangle, \qquad i, j = x, y \tag{2}$$

E_i, E_j are two mutual orthogonal components of electric field. The SDCP defined in Eq. (1) denotes the correlation of the general degree of polarization at two spatial positions (r_1, z) and (r_2, z).

Now let us consider the propagation law of a beam passing through uniaxial crystals orthogonal to the optical axis. We assume that optical axis of crystals coincides with the x-axis, and the beam propagates along the z-axis. With the framework of paraxial propagation theory [10], the projective electric components of the beam in uniaxial crystals are expressed as:

$$E_x(r_\perp, z) = \exp(ik_0 n_e z)\int d^2K_\perp \exp(iK_\perp \cdot r_\perp)\exp\left(-i\,\frac{n_e^2 k_x^2 + n_0^2 k_y^2}{2k_0 n_e k_0^2}\,z\right)\tilde{E}_x(K_\perp) = $$
$$= \exp(ik_0 n_e z)\, A_x(r_\perp, z) \tag{3a}$$

$$E_y(r_\perp, z) = \exp(ik_0 n_0 z)\int d^2K_\perp \exp(iK_\perp \cdot r_\perp)\exp\left(-i\,\frac{k_x^2 + k_y^2}{2k_0 n_0}\,z\right)\tilde{E}_y(K_\perp) = $$
$$= \exp(ik_0 n_0 z)\, A_y(r_\perp, z) \tag{3b}$$

where

$$\tilde{E}_\perp(K_\perp) = \frac{1}{2\pi}\int \mathrm{d}^2 r_\perp \exp(-i K_\perp \cdot r_\perp) E_\perp(r_\perp, 0) \tag{4}$$

k_0 is the wave number in vacuum, n_0 and n_e are ordinary and extraordinary refractive indexes, $K_\perp = k_x \hat{e}_x + k_y \hat{e}_y$, $r_\perp = x\hat{e}_x + y\hat{e}_y$; A_x and A_y are the slowing varying amplitudes of electric field.

Substituting Eq. (4) into Eqs. (3) and inverting the integral order [10], after some tedious but straightforward vector operations and arrangements [11, 12], the electric components in uniaxial crystals are given by:

$$E_x(x, y, z) = \frac{k_0 n_0}{2\pi i z}\exp(ik_0 n_e z)\int \mathrm{d}x'\mathrm{d}y' \exp\left\{-\frac{k_0}{2izn_e}\left[n_0^2(x-x')^2 + n_e^2(y-y')^2\right]\right\}\times$$
$$\times E_x(x', y', 0) \tag{5}$$

$$E_y(x, y, z) = \frac{k_0 n_0}{2\pi i z}\exp(ik_0 n_0 z)\int \mathrm{d}x'\mathrm{d}y' \exp\left\{-\frac{k_0 n_0}{2iz}\left[(x-x')^2 + (y-y')^2\right]\right\}\times$$
$$\times E_y(x', y', 0) \tag{6}$$

From Eqs. (5) and (6), we can see that if we know the cross-spectral density matrix of the source, we can formulate the corresponding cross-spectral density matrix at distance z from the source in uniaxial crystals.

As a numerical example, we choose the electromagnetic Gaussian–Schell model (EGSM) [7] beam with uniform polarization to illustrate the propagation properties of the SDCP in anisotropic uniaxial crystals. The 2×2 cross-spectral density matrix of the EGSM beam in the source plane is expressed as [2]:

$$W_{ij}^0(x_1, y_1; x_2, y_2, 0) = A_i A_j B_{ij} \exp\left[-\frac{x_1^2 + y_1^2 + x_2^2 + y_2^2}{4\sigma^2} - \frac{(x_1 - x_2)^2 + (y_1 - y_2)^2}{2\delta_{ij}^2}\right] \tag{7}$$

here σ is the spot beam width, δ_{ij} is the source correlation length parameter, $B_{ij} = B_{ji}^* = |B|\exp(-i\varphi_{ij})$. If we assume that components of electric amplitudes are equal $A_x = A_y$. If we multiply Eqs. (5) and (6), together with Eqs. (2) and (7), and use the integral formula[13]:

$$\int_0^\infty \exp(-q^2 x^2)\mathrm{d}x = \frac{\sqrt{\pi}}{2q}$$

the 2×2 cross-spectral density matrix at distance z in uniaxial crystals is derived as:

$$W_{ij}(x_1, y_1; x_2, y_2, z) = \frac{k_0^2 n_0^2 |B|^2}{4z^2 \sqrt{c_{ij} d_{ij} e_{ij} f_{ij}}} \exp\left[\left(a + \frac{a^2}{c_{ij}} + \frac{a^2}{4c_{ij}^2 d_{ij} \delta_{ij}^4}\right)x_1^2 + \right.$$
$$+ \left(b + \frac{b^2}{e_{ij}} + \frac{b^2}{4e_{ij}^2 f_{ij} \delta_{ij}^4}\right)y_1^2 + \left(\frac{a}{d_{ij}} - a\right)x_2^2 +$$
$$\left. + \left(\frac{b^2}{f_{ij}} - b\right)y_2^2 - \frac{a^2}{c_{ij} d_{ij} \delta_{ij}^2} x_1 x_2 - \frac{b^2}{e_{ij} f_{ij} \delta_{ij}^2} y_1 y_2 \right] \tag{8}$$

where

$$a = -\frac{k_0 n_0^2}{2izn_e} \tag{9a}$$

$$b = -\frac{k_0 n_e}{2iz} \tag{9b}$$

$$c_{ij} = \frac{1}{4\sigma^2} + \frac{1}{2\delta_{ij}^2} - a \tag{9c}$$

$$d_{ij} = \frac{1}{4\sigma^2} + \frac{1}{2\delta_{ij}^2} + a - \frac{1}{4c_{ij}\delta_{ij}^4} \tag{9d}$$

$$e_{ij} = \frac{1}{4\sigma^2} + \frac{1}{2\delta_{ij}^2} - b_{ij} \tag{9e}$$

$$f_{ij} = \frac{1}{4\sigma^2} + \frac{1}{2\delta_{ij}^2} + b - \frac{1}{4e_{ij}\delta_{ij}^4} \tag{9f}$$

With Equations (1), (8) and (9), we can formulate the SDCP at arbitrary distance in uniaxial crystals.

3. Numerical examples

The calculation parameters used in this paper are $n_0 = 2.5$, $\sigma = 100$ μm, $|B| = 0.9$, $z_R = 4\pi n_0 \sigma^2/\lambda$ (λ is the incident beam wavelength, z_R is the Rayleigh length of the ordinary beam [8, 11]). In order to simplify calculations, we assume the EGSM model source is isotropic [2], so condition $\delta_{ii} = \delta_{ij}$ is fulfilled.

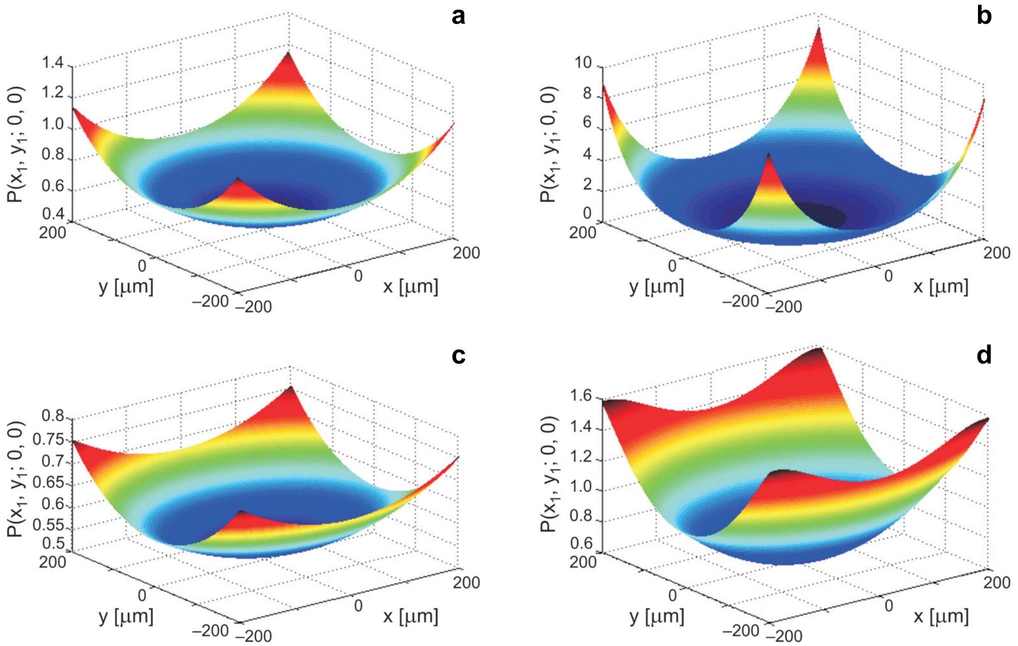

Fig. 1. Profiles of the degree of cross-polarization $P(x_1, y_1;\ 0,\ 0)$ of the EGSM beam in uniaxial crystals, with different propagation distances and correlation length parameters; $z = z_R$, $\delta_{xx} = 90$ μm, $\delta_{xy} = 100$ μm (**a**), $z = z_R$, $\delta_{xx} = 80$ μm, $\delta_{xy} = 120$ μm (**b**), $z = 3z_R$, $\delta_{xx} = 90$ μm, $\delta_{xy} = 100$ μm (**c**), $z = 3z_R$, $\delta_{xx} = 80$ μm, $\delta_{xy} = 120$ μm (**d**).

Figure 1 shows the SDCP distribution generated by the EGSM model source in transversal planes of uniaxial crystals orthogonal to optical axis. From Figure 1 we find that as the propagation distance increases, values of the SDCP also become larger than previous ones. Source correlation length parameters also influence the SDCP a lot. We can notice that as the deviation of the source correlation length parameters ($\Delta = \delta_{ij} - \delta_{ii}$) become larger, the SDCP shows more fluctuating features. These obtained results are very different from previous published papers [7], in which the SDCP is hardly affected by the propagation distance or correlation length parameters in isotropic turbulent atmosphere. Figure 2 further show correlations of the SDCP in two cross-sections with different correlation length parameters of the source. Associating Fig. 2 with Fig. 1, we can conclude that with larger deviation of correlation length parameters ($\Delta = \delta_{ij} - \delta_{ii}$), the SDCP shows more fluctuating features, and the values of SDCP are much more easily affected by autocorrelation length parameter δ_{xx} than cross-correlation length δ_{xy}. On the other hand, if the incident light is the exact fundamental Gaussian beam, namely the condition $(\delta_{ii}, \delta_{ij} \to \infty)$ is fulfilled, values of the SDCP in two cross-sections in Fig. 2 are invariable, it can be also analytically derived with Eqs. (9) and (10) by setting $\delta_{ii}, \delta_{ij} \to \infty$. This obtained result is somewhat different from previous research, in which the necessary and sufficient condition [7] for invariable of the SDCP is the equalization of

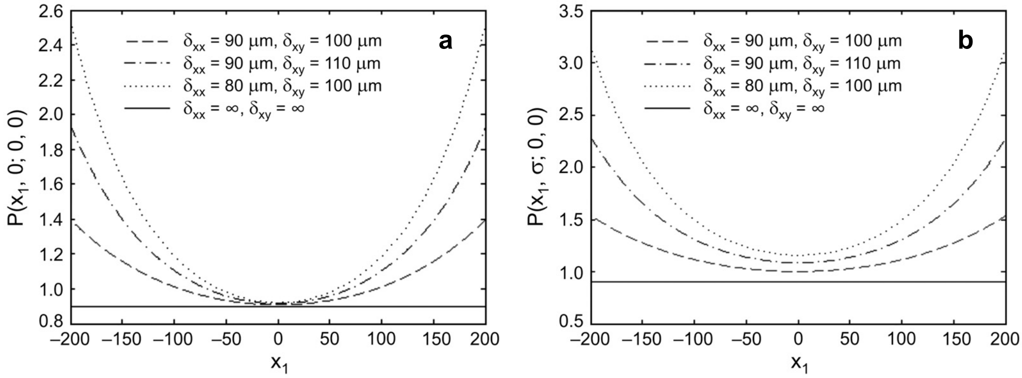

Fig. 2. Profiles of the degree of cross-polarization $P(x_1, \sigma; 0, 0)$ of the EGSM beam in the cross-section $y_1 = 0$ (**a**) and $y_1 = \sigma$ (**b**) in uniaxial crystals, with different correlation length parameters of the source at distance $z = z_R$.

the autocorrelation length parameters, whereas the invariable condition in uniaxial crystals is the sufficient large dimensions of correlation length parameters. This is the main result of this paper.

Figure 3 shows the variation of the SDCP in two cross-sections $y_1 = 0$ and $y_1 = \sigma$ at propagation distances $z = 3z_R$, with different ratio of refractive index n_e/n_0. One can conclude that values of the SDCP in cross-section are also influenced by the ratio of refractive index of uniaxial crystals. As the ratio of refractive index is larger, the corresponding values of the SDCP are larger in paraxial region, whereas inversely in abaxial region. Distributions of the SDCP in uniaxial crystals of a large ratio of refractive index show flat features.

Figure 4 displays values of the SDCP with different incident beam wavelength in two cross-sections at propagating distance $z = 3z_R$. From Figs. 4**a** and 4**b**, it is found

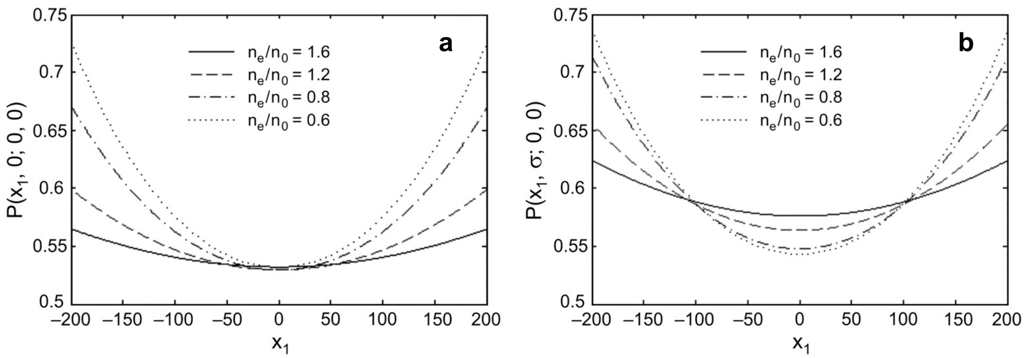

Fig. 3. Profiles of the degree of cross-polarization $P(x_1, \sigma; 0, 0)$ of the EGSM beam in the cross-section $y_1 = 0$ (**a**) and $y_1 = \sigma$ (**b**) in uniaxial crystals, with different ratio of refractive index n_e/n_0 at distance $z = 3z_R$. The values of the source parameters are: $\delta_{xx} = 90$ μm, $\delta_{xy} = 100$ μm.

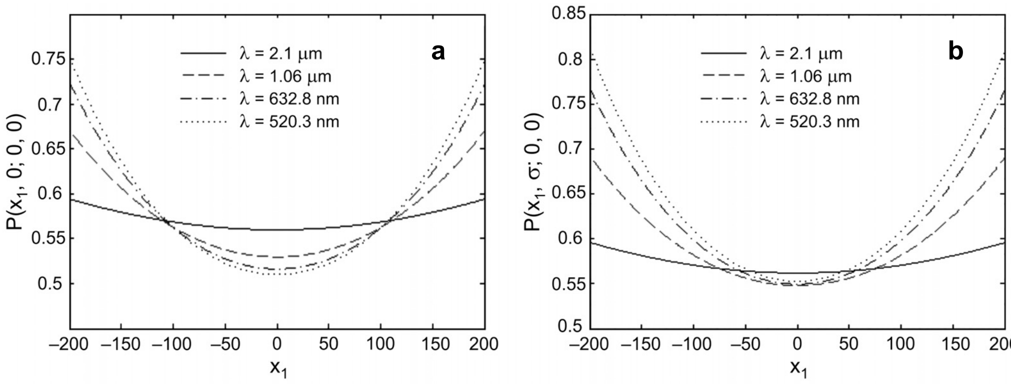

Fig. 4. Profiles of the degree of cross-polarization $P(x_1, \sigma; 0, 0)$ of the EGSM beam in the cross-section $y_1 = 0$ (**a**) and $y_1 = \sigma$ (**b**) in uniaxial crystals, with different incident beam wavelength at distance $z = 3z_R$. The values of the source parameters are the same as in Fig. 3.

that values of the SDCP in cross-sections show flat distribution while applying near--infrared wavelength beams as the source. Inversely, values of the SDCP in uniaxial crystals display large fluctuating features for short wavelength beams.

4. Conclusions

In conclusion, propagation of the degree of cross-polarization of a partially coherent electromagnetic beam in uniaxial crystals orthogonal to optical axis is investigated; analytical formula for the SDCP in uniaxial crystals is carried out based on the beam paraxial propagating theory. The EGSM beam is chosen as the numerical example. Results show that values of the SDCP is influenced by different correlation length parameters, the ratio of refractive index and incident wavelength. Results also show that degrees of cross-polarization of sources with sufficient large correlation length parameters are invariable in cross-sections of uniaxial crystals at arbitrary propagation distance. Though we have investigated some features of the SDCP in uniaxial crystals, further work needs to be carried out, such as studying conditions of propagating in other anisotropic medium or along the optical axis of uniaxial crystals, *etc*.

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