Optica Applicata, Vol. XLI, No. 3, 2011

Application of imaging visibility to measurement of correlation coefficient of scattering potential

HONG LIU[1, 2], WEISHENG WANG[1*], JINGAO LIU[1], WEIDONG LIU[3]

[1]School of Information Science & Technology, East China Normal University, Shanghai, 200241, China

[2]Department of Physics and Electronic Engineering, Yangtze Normal University, Chongqing, 408100, China

[3]Module, Hisense R&D Center, Qingdao, 266071, China

[*]Corresponding author: wangweisheng@hotmail.com

It is shown that the imaging visibility of intensity correlated scattered field may be utilized to determine the normalized correlation coefficient of the scattering potential (CCSP) of the quasi-homogeneous (QH) media illuminated by a scalar plane wave. The relationship between the imaging visibility and the CCSP is constructed by analytical forms. As long as the visibility of the intensity correlated scattered field is known, the scaled width of the CCSP can be expressed by solutions of the inverse scattering problem.

Keywords: scattering theory, scattering measurements.

1. Introduction

The issue of scattering by random media was firstly introduced by WOLF *et al.* about two decades ago. They demonstrated that the spectrum of light may change when it scatters from an object [1–3]. Since then, many achievements related to the scattering theory have been reported, most of which focused on investigations of characteristics of light when it scatters from random media under various conditions [4–7]. Recently, the problem of inverse scattering has attracted much interest because of its potential applications in laser probing, remote sensing and detecting structural information about a scatterer [8]. FISCHER and WOLF derived the law for the inverse scattering problem under the condition that the light is scattered from quasi-homogeneous (QH) media [9]. GBUR and WOLF determined the density correlation functions of many-particle system with a high degree of symmetry, by measuring changes of the spectrum of polychromatic scattered wave [10]. Very recently, ZHAO *et al.* proposed a theoretical method for solving the inverse scattering problem, namely that of determining the correlation functions of homogeneous random media illuminated by a polychromatic plane wave with Gaussian spectral density, based on the knowledge of its spectrum in the scattered field [11]. Subsequently, LAHIRI *et al.* determined correlation functions of scattering potentials of stochastic media from scattering experiments [12]. WANG and ZHAO further extended the work for the determination of a pair-structure factor of scattering potential of a collection of particles [13].

In this paper, we present a novel method which might be viewed as an extension of the previous problem of inverse scattering [9–13], namely that of determining the normalized correlation coefficient of the scattering potential of QH media, as long as the imaging visibility of the intensity correlated scattered field is known.

2. Theory analysis

Assuming that a polychromatic scalar plane wave whose propagating direction is specified by a unit vector $\hat{\mathbf{S}}_\mathbf{0}$ is incident on QH media occupying a finite domain D (see Fig. 1). The polychromatic plane wave is represented in the analytical form

$$\mathbf{U}^{(i)}(\mathbf{r}, \omega) = a(\omega)\exp(ik\hat{\mathbf{S}}_\mathbf{0}\cdot\mathbf{r}) \tag{1}$$

with $k = \omega/c$ being the wave number, ω denoting the angular frequency, c representing the speed of light in vacuum. In Equation (1), $a(\omega)$ is a complex random variable which may depend on the frequency. The cross-spectral density function of the incident wave at points specified by position vectors $\mathbf{r_1}$ and $\mathbf{r_2}$ can be expressed by

$$\mathbf{W}^{(i)}(\mathbf{r_1}, \mathbf{r_2}, \omega) = \langle \mathbf{U}^{(i)*}(\mathbf{r_1}, \omega)\,\mathbf{U}^{(i)}(\mathbf{r_2}, \omega)\rangle =$$
$$= S^{(i)}(\omega)\exp\left[ik\hat{\mathbf{S}}_\mathbf{0}\cdot(\mathbf{r_2}-\mathbf{r_1})\right] \tag{2}$$

with

$$S^{(i)}(\omega) = \langle a^*(\omega)\,a(\omega)\rangle \tag{3}$$

is the spectral density of the incident polychromatic plane wave. The bracket denotes taking the ensemble average over the incident field and the asterisk represents the complex conjugation. Recalling the formula for the far-zone spectral density of light induced by the plane wave scattered from QH media [9, 14]

$$S^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \frac{S^{(i)}(\omega)\tilde{S}_F(0, \omega)}{r^2}\,\tilde{\eta}_F\left[k(\hat{\mathbf{s}}_\mathbf{0}-\hat{\mathbf{s}}), \omega\right] \tag{4}$$

where $\hat{\mathbf{s}}$ denotes the unit vectors along the direction of the scattered light, r is the distance from the original region of the scatterer (see Fig. 1). In Equation (4), $\tilde{S}_F$

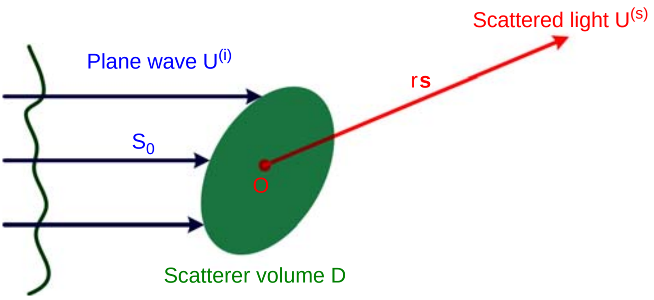

Fig. 1. Illustration of the notation for the scattering theory.

and η_F represent the three-dimensional spatial Fourier transformations of S_F and η_F, respectively,

$$\tilde{S}_F(k\hat{\mathbf{s}}, \omega) = \int S_F(\mathbf{R}_\mathbf{s}^+, \omega)\exp(ik\mathbf{R}_\mathbf{s}^+ \cdot \hat{\mathbf{s}})\,\mathrm{d}^3\mathbf{R}_\mathbf{s}^+ \tag{5a}$$

$$\tilde{\eta}_F(k\hat{\mathbf{s}}, \omega) = \int \eta_F(\mathbf{R}_\mathbf{s}^-, \omega)\exp(ik\mathbf{R}_\mathbf{s}^- \cdot \hat{\mathbf{s}})\,\mathrm{d}^3\mathbf{R}_\mathbf{s}^- \tag{5b}$$

with S_F and η_F being the strength and the normalized correlation coefficient of the scattering potential of QH media, respectively. Both of them constitute the correlation function of the scattering potential as follows [9, 14]

$$C_F(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sqrt{S_F(\mathbf{r}_1, \omega)S_F(\mathbf{r}_2, \omega)}\;\eta_F(\mathbf{r}_2 - \mathbf{r}_1, \omega) \tag{6}$$

Reference [11] has demonstrated the feasibility of determining the correlation function of the scattering potential of random media, by the measurement of correlation-induced spectral changes of scattered light. Comparably, in the present paper, the method for measuring the normalized correlation coefficient of scattering potential of QH media is shown, as long as the imaging visibility of the scattered light is determined in experiments.

Accordingly, the far-zone intensity distribution of light induced by a plane wave scattered from QH media is of the following form

$$I^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \frac{\tilde{S}_F(0, \omega)}{r^2}\,\tilde{\eta}_F\left[k(\hat{\mathbf{s}}_\mathbf{0} - \hat{\mathbf{s}}), \omega\right] \tag{7}$$

Recalling the formula for the far-zone spectrum of a plane wave scattered from random media [1, 11]

$$S^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \frac{V}{r^2}\,\tilde{C}_F\left[k(\hat{\mathbf{s}} - \hat{\mathbf{s}}_\mathbf{0}), \omega\right]S^{(i)}(\omega) \tag{8}$$

where V denotes the volume of the scatterer D (see Fig. 1), and

$$\tilde{C}_F(\hat{\mathbf{K}}, \omega) = \int C_F(\mathbf{r}, \omega)\exp(-i\,\hat{\mathbf{K}}\cdot\mathbf{r})\mathrm{d}^3\mathbf{r} \tag{9}$$

is the three-dimensional Fourier transformation of the correlation function of the scattering potential. One can see from Eq. (8) that

$$S^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \frac{V}{r^2}\,\tilde{C}_F(0, \omega)\,S^{(i)}(\omega) \tag{10}$$

Substituting Eq. (10) into Eq. (7), the far-zone intensity distribution of scattered field yields

$$I^{(\infty)}(r\hat{\mathbf{s}}, \omega) = \frac{S^{(\infty)}(r\hat{\mathbf{s}}, \omega)}{VS^{(i)}(\omega)}\,\tilde{\eta}_F\Big[k(\hat{\mathbf{s}}_\mathbf{0} - \hat{\mathbf{s}}), \omega\Big] \tag{11}$$

Equation (11) indicates that the far-zone intensity distribution of light scattered from QH media is proportional to the Fourier transformation of the normalized CCSP $\tilde{\eta}_F$ and the spectrum $S^{(\infty)}(r\hat{\mathbf{s}}_\mathbf{0}, \omega)$ (specified at the scattering angle $\theta = 0$) is anti-proportional to the scatterer volume V and the original spectrum $S^{(i)}(\omega)$.

3. The imaging visibility of intensity correlated scattered field

To analyze the imaging quality of the intensity correlated field induced by plane wave scattered from QH media, the arbitrary N-th-order intensity correlation function of light is introduced [15]

$$\Gamma^{(N)}(r_1, \ldots, r_N) = \langle I(r_1)\ldots I(r_N)\rangle = \langle E^*(r_1)\ldots E^*(r_N)E(r_N)\ldots E(r_1)\rangle \tag{12}$$

where $I(r_1)$ $(i = 1, \ldots, N)$ is the instantaneous intensity distribution specified at points r_i. Therefore, the imaging visibility of the N-th-order intensity correlated field can be defined as

$$\overline{V}_s^{(N)} = \frac{\Gamma_{\max}^{(N)} - \Gamma_{\min}^{(N)}}{\Gamma_{\max}^{(N)} + \Gamma_{\min}^{(N)}} \tag{13}$$

where $\Gamma_{\max}^{(N)}$ and $\Gamma_{\min}^{(N)}$ are the maximum and minimum values of $\Gamma^{(N)}$, respectively. Furthermore, Eq. (12) can be expanded in terms of the normalized first-order field correlation functions by utilizing the moment theorem for a Gaussian random process [15, 16]

$$\begin{aligned}\Gamma^{(N)} &= \sum_{N!}\langle E^*(r_1)E(r_{\underline{1}})\rangle\ldots\langle E^*(r_N, r_{\underline{N}})\rangle = \\ &= \prod_{l=1}^{N}\langle I(r_l)\rangle\sum_{N!}\mu(r_1, r_{\underline{1}})\ldots\mu(r_N, r_{\underline{N}})\end{aligned} \tag{14}$$

where $\Sigma_{N!}$ denotes taking summations over $N!$ permutations of underlined indices ($\underline{1}$, ..., $\underline{N}$). $\mu(r_i, r_j)$ is the normalized first-order field correlation function

$$\mu(r_i, r_j) = \frac{\langle E^*(r_i) E(r_j) \rangle}{\sqrt{\langle I(r_i) \rangle \langle I(r_j) \rangle}}, \qquad (i, j = 1, \dots, N) \tag{15}$$

In general, $|\mu(r_i, r_j)| \le 1$ is always satisfied according to the Schwarz inequality [17]. Therefore, the minimum value $\Gamma_{\min}^{(N)}$ can be reached when $N = 1$, namely,

$$\Gamma_{\min}^{(N)} = \langle I(r\hat{\mathbf{s}}_{\mathbf{1}}, \omega) \rangle = \frac{S^{(\infty)}(r\hat{\mathbf{s}}_{\mathbf{0}}, \omega)}{VS^{(i)}(\omega)} \tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}_{\mathbf{1}}), \omega \right] \tag{16}$$

Similarly, the maximum value $\Gamma_{\max}^{(N)}$ can be reached when all the terms $|\mu(r_i, r_j)| \equiv 1$, namely,

$$\Gamma_{\max}^{(N)} = \prod_{l=1}^{N} \langle I(r_1) \rangle \dots \langle I(r_N) \rangle \times N! =$$

$$= \left[\frac{S^{(\infty)}(r\hat{\mathbf{s}}_{\mathbf{0}}, \omega)}{VS^{(i)}(\omega)} \right]^N \prod_{l=1}^{N} \tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}_{\mathbf{1}}), \omega \right] \dots \tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}_{\mathbf{N}}), \omega \right] \times N! \tag{17}$$

Due to the fact that $\tilde{\eta}_F$ is a slow function of its internal arguments, hence the following approximation can be made [14]

$$\tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}_{\mathbf{1}}), \omega \right] \approx \tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}_{\mathbf{2}}) \right] \approx \dots \approx \tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}_{\mathbf{N}}) \right] \tag{18}$$

Substituting Eq. (18) into Eq. (17), yields

$$\Gamma_{\max}^{(N)} = \left\{ \frac{S^{(\infty)}(r\hat{\mathbf{s}}_{\mathbf{0}}, \omega)}{VS^{(i)}(\omega)} \tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}), \omega \right] \right\}^N \times N! \tag{19}$$

Substituting Eqs. (16) and (19) into Eq. (13), the imaging visibility of the N-th-order intensity correlated scattered field yields

$$\overline{V}_s^{(N)}(r\hat{\mathbf{s}}, \omega) = 1 - \frac{2}{\left\{ \dfrac{S^{(\infty)}(r\hat{\mathbf{s}}_{\mathbf{0}}, \omega)}{VS^{(i)}(\omega)} \tilde{\eta}_F \left[k(\hat{\mathbf{s}}_{\mathbf{0}} - \hat{\mathbf{s}}), \omega \right] \right\}^{N-1} \times N! + 1} \tag{20}$$

Equation (20) indicates that, for the case where a scalar plane wave scatters from QH media, the imaging visibility of the N-th-order intensity correlated scattered field depends on the intensity correlated order N and the Fourier transformation of the normalized CCSP $\tilde{\eta}_F$. This is because, in experiments related to scattering, both the scatterer volume V and the spectrum of incident wave $S^{(i)}(\omega)$ are known. Besides, $S^{(\infty)}(r\hat{\mathbf{s}}_\mathbf{0}, \omega)$ can be obtained by the measurement of the far-zone spectrum of light around the region where the scattering angle $\theta = 0$. Therefore, one can define the following factor

$$\overline{C}_0(\omega) = \frac{S^{(\infty)}(r\hat{\mathbf{s}}_\mathbf{0}, \omega)}{V S^{(i)}(\omega)} \tag{21}$$

It can be observed from Eq. (20) that, when $N = 1$ is substituted, $\overline{V}_s^{(1)} \equiv 0$. This result indicates a well-known fact that the visibility of the first-order intensity correlated scattered field is equal to zero, and this is because interference fringes of scattered field cannot be formed. Therefore, in general, the imaging visibility must be measured in the second-order or even higher-order correlated scattered field, such as classical ghost images and optical coherence tomography by using scattered light [18, 19]. Let us now consider the inverse scattering problem when the intensity correlated order $N \geq 2$. It follows from Eqs. (20) and (21) that

$$\tilde{\eta}_F\left[k(\hat{\mathbf{s}}_\mathbf{0} - \hat{\mathbf{s}}), \omega\right] = \frac{1}{\overline{C}_0(\omega)}\left\{\frac{1 + \overline{V}_s^{(N)}(r\hat{\mathbf{s}}, \omega)}{N!\left[1 - \overline{V}_s^{(N)}(r\hat{\mathbf{s}}, \omega)\right]}\right\}^{\frac{1}{N-1}}, \quad N \geq 2 \tag{22}$$

By performing the inverse Fourier transformation of Eq. (22), the normalized CCSP of QH media yields

$$\eta_F(\mathbf{r}', \omega) = \int \frac{1}{\overline{C}_0(\omega)}\left\{\frac{1 + \overline{V}_s^{(N)}(r\hat{\mathbf{s}}, \omega)}{N!\left[1 - \overline{V}_s^{(N)}(r\hat{\mathbf{s}}, \omega)\right]}\right\}^{\frac{1}{N-1}} \exp(-i\hat{\mathbf{K}} \cdot \mathbf{r}')\mathrm{d}^3\hat{\mathbf{K}} \tag{23}$$

where $\hat{\mathbf{K}} = k(\hat{\mathbf{s}}_\mathbf{0} - \hat{\mathbf{s}})$ is the momentum transferred vector. In practical experiments such as classical ghost images or optical coherence tomography by using scattered light [18, 19], the intensity correlated order $N = 2$. In this case, Eq. (23) can be simplified to

$$\eta_F(\mathbf{r}', \omega) = \frac{1}{2}\int \frac{1}{\overline{C}_0(\omega)} \frac{1 + \overline{V}_s^{(2)}(r\hat{\mathbf{s}}, \omega)}{1 - \overline{V}_s^{(2)}(r\hat{\mathbf{s}}, \omega)} \exp(-i\hat{\mathbf{K}} \cdot \mathbf{r}')\mathrm{d}^3\hat{\mathbf{K}} \tag{24}$$

4. Discussion

Equations (23) and (24) are the main results of this paper, which may provide a novel approach for determining the normalized correlation coefficient of scattering potential (CCSP) of QH media, as long as the visibility $\overline{V}_s$ of the intensity correlated scattered field is measured by the interference fringes formed. Firstly, the factor $\overline{C}_0(\omega)$ can be known from experimental measurements according to Eq. (21). In this step, a spectrograph should be required to have knowledge of $S^{(i)}(\omega)$ and $S^{(\infty)}(r\hat{\mathbf{s}}_\mathbf{0}, \omega)$, respectively. Secondly, values of the visibility $\overline{V}_s$ of intensity correlated field can be determined by calculating the interference fringes of images obtained. In fact, there also exist many other algorithmic methods to obtain the imaging visibility $\overline{V}_s$. Finally, by performing the FFT algorithm [20] which integrals over the variable $\hat{\mathbf{K}}$, the normalized CCSP $\eta_F(\mathbf{r}', \omega)$ can be reconstructed from the experimental data. Let us now consider a more detailed numerical example. At first, it is assumed that the distribution of normalized CCSP is of the Gaussian profile, *i.e.*,

$$\eta_F(\mathbf{r}', \omega) = \frac{A}{(2\pi\delta_F^2)^{3/2}} \exp\left(-\frac{\mathbf{r}'^2}{2\delta_F^2}\right) \tag{25}$$

The factor $\overline{C}_0(\omega)$ can be approximated to the simple form

$$\overline{C}_0(\omega) = \overline{C}_0 \tag{26}$$

It is also assumed that the interference fringes of intensity correlated images are of the Airy disk [17]

$$\overline{V}_s^{(2)}(\theta, \omega) = \left[\frac{J_1(2k\theta)}{k\theta}\right]^2 \tag{27}$$

where J_1 denotes the first kind Bessel functions with the first order. Substituting Eqs. (25)–(27) into Eq. (24), the solution to the inverse scattering problem of determining the scaled width $k\delta_F$ of the CCSP yields

$$k\delta_F = \frac{1}{\sqrt{2}\sin\left(\frac{\theta}{2}\right)} \sqrt{-\ln\left[f(\theta, \omega)\right]} \tag{28}$$

with the kernel part

$$f(\theta, \omega) = \frac{1 + \left[\frac{J_1(2k\theta)}{k\theta}\right]^2}{2\overline{C}_0 k^4 \left\{1 - \left[\frac{J_1(2k\theta)}{k\theta}\right]^2\right\}} \tag{29}$$

Comparing Eqs. (28) and (29) with Eqs. (12) and (13) of Ref. [11], one can observe that our method for reconstructing $\eta_F(\mathbf{r}', \omega)$ is somewhat similar to that for $C_F(\mathbf{r}', \omega)$

determining in Ref. [11]. However, for above two methods, their kernel parts $f(\theta, \omega)$ appear to be totally different in analytical forms.

Theoretical results such as Eqs. (22)–(24) and Eqs. (28), (29) may find potential applications in optical coherence tomography or classical ghost images, the aim of which is to determine internal structures of unknown scatterer by utilizing the intensity correlated scattered images.

References

[1] WOLF E., FOLEY J.T., GORI F., *Frequency shifts of spectral lines produced by scattering from spatially random media*, Journal of the Optical Society of America A **6**(8), 1989, pp. 1142–1149.

[2] FOLEY J.T., WOLF E., *Frequency shifts of spectral lines generated by scattering from space-time fluctuations*, Physical Review A **40**(2), 1989, pp. 588–598.

[3] JAMES D.F.V., SAVEDOFF M.P., WOLF E., *Shifts of spectral lines caused by scattering from fluctuating random media*, Astrophysical Journal **359**, 1990, pp. 67–71.

[4] WANG T., ZHAO D., *Determination of correlation function of scattering potentials of a random medium from the scattered spectral density*, Physics Letters A **375**(3), 2011, pp. 780–783.

[5] LI J., XIN Y., CHEN Y.R., *et al.*, *Fourth-order correlation statistics of electromagnetic plane wave scattered by quasi-homogeneous medium*, Journal of the Optical Society of America A, (in press).

[6] SHIRAI T., ASAKURA T., *Multiple light scattering from spatially random media under the second--order Born approximation*, Optics Communications **123**(1–3), 1996, pp. 234–249.

[7] JIA LI, YANRU CHEN, SHIXUE XU, YONGQING WANG, MUCHUN ZHOU, QI ZHAO, YU XIN, FEINAN CHEN, *Condition for invariant spectral degree of coherence of an electromagnetic plane wave on scattering*, Optics Communications **284**(3), 2011, pp. 724–728.

[8] HOWARD J., *Laser probing of random weakly scattering media*, Journal of the Optical Society of America A **8**(12), 1991, pp. 1955–1963.

[9] FISCHER D.G., WOLF E., *Inverse problems with quasi-homogeneous random media*, Journal of the Optical Society of America A **11**(3), 1994, pp. 1128–1135.

[10] GBUR G., WOLF E., *Determination of density correlation functions from scattering of polychromatic light*, Optics Communications **168**(1–4), 1999, pp. 39–45.

[11] ZHAO D., KOROTKOVA O., WOLF E., *Application of correlation-induced spectral changes to inverse scattering*, Optics Letters **32**(24), 2007, pp. 3483–3485.

[12] LAHIRI M., WOLF E., FISCHER D.G., SHIRAI T., *Determination of correlation functions of scattering potentials of stochastic media from scattering experiments*, Physical Review Letters **102**(12), 2009, p. 123901.

[13] WANG T., ZHAO D., *Determination of pair-structure factor of scattering potential of a collection of particles*, Optics Letters **35**(3), 2010, pp. 318–320.

[14] VISSER T.D., FISCHER D.G., WOLF E., *Scattering of light from quasi-homogeneous sources by quasi-homogeneous media*, Journal of the Optical Society of America A **23**(7), 2006, pp. 1631–1638.

[15] MANDEL L., WOLF E., *Optical Coherence and Quantum Optics*, Cambridge University Press, New York, 1995.

[16] HONG-CHAO LIU, DONG-SHI GUAN, LU LI, SU-HENG ZHANG, JUN XIONG, *The impact of light polarization on imaging visibility of Nth-order intensity correlation with thermal light*, Optics Communications **283**(3), 2010, pp. 405–408.

[17] BORN M., WOLF E., *Principle of Optics*, Cambridge University Press, 1999.

[18] CROSBY S., CASTELLETTO S., ARULDOSS C., SCHOLTEN R.E., ROBERTS A., *Modelling of classical ghost images obtained using scattered light*, New Journal of Physics **9**, 2007, p. 285.

[19] Fercher A.F., Hitzenberger C.K., *Optical coherence tomography*, [In] *Progress in Optics*, [Ed.] Wolf E., Vol. 44, Chap. 4, Elsevier, 2002, pp. 215–302.

[20] Boruah B.R., Neil M.A.A., *Focal field computation of an arbitrary polarized beam using fast Fourier transforms*, Optics Communications **282**(24), 2009, pp. 4660–4667.

Received December 23, 2010
in revised form January 27, 2011