*Optica Applicata, Vol. XLI, No. 1, 2011*

# Acousto-optic method for liquids refractometry

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Various methods of liquids refractive index measurements were previously developed by others. They differ however in the measurement accuracy, the used light wavelength, the measurement range and the sensitivity to the temperature and pressure. In this work, we present and discuss an acousto-optic technique for measuring the index of refraction of transparent liquid materials. In the proposed technique, a diffraction pattern produced by an acousto-optic interaction is imaged by a liquid lens placed between an optical flat glass and a convergent glass lens. The diffraction pattern consists of two symmetrical dots that are digitized by a CCD camera. The focal shift, which is induced by the liquid sample, produces changes in the position of the diffracted orders. The spatial frequency measurement of the diffractive pattern leads to determine the sample refractive index. The current method presents the advantage to have an adjustable measurement range and can be easily interpreted geometrically.

Keywords: refractive index, acousto-optic diffraction, ultrasonic grating.

## 1. Introduction

Liquid refractometry is a method that uses optical techniques for measuring liquids refractive index. This optical parameter plays a vital role in many scientific domains. It can be measured by evaluating the light beam minimum deviation angle that passes through the liquid contained in a triangular or cylindrical cell [1–4], or by measuring the critical angle of total reflection at the boundary between the liquid sample and a reference prism (Abbe refractometer). The latter method depends however on the accurate knowledge of the prism refractive index [3–5]. NEMOTO [3] determined the refractive index by measuring the displacement of a laser beam which is obliquely incident on a rectangular cell filled with the sample liquid. The reached precision is $10^{-4}$. The method proposed by DE ANGELIS and co-workers [6] uses the imaging of the fringe pattern by a convergent lens and its projection through the liquid sample. The focal shift measurement using the spatial frequency of the fringe pattern led to obtaining a liquid refractive index, with an accuracy of $10^{-4}$. Both techniques do not require the cell parameters.

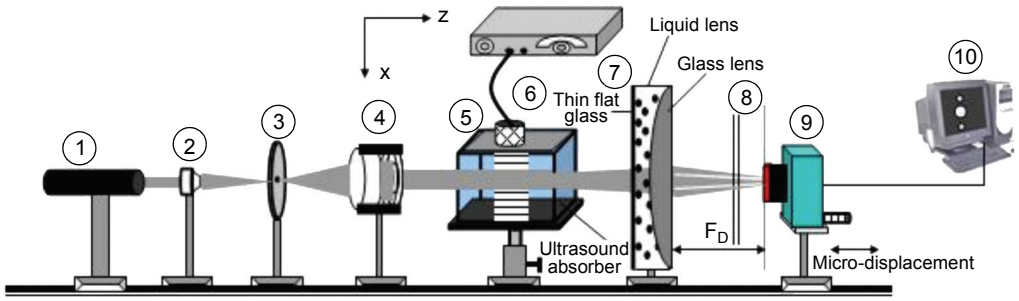


Fig. 1. Experimental set-up for measuring liquids refractive index; 1 – He-Ne laser source (output power 30 mW at $\lambda = 632.8$ nm), 2 – microscope objective (focal length = 8 mm), 3 – spatial filter (diameter = 20 μm) used as a beam cleaner, 4 – photographic objective with variable aperture diaphragm, used as a beam former, 5 – acousto-optic cell, 6 – frequency generator, 7 – convergent doublet, 8 – light attenuator used to prevent camera saturation, 9 – CCD camera (resolution 644×484 pixel with pixel size: 8.4×8.4 μm), 10 – computer.

In recent years, many optical miniature refractometers based on optical wave guides have been developed for different research fields. Their basic idea to determine the refractive index is the measurement of the outgoing intensity light modulation [7–10].

High accurate refractive index measurements are obtained by interferometric instruments such as Fabry–Perot, Mach–Zehnder, Michelson, Jamin and Rayleigh [11]. Those instruments measure in principle the changes in the interferometric fringe pattern caused by the changes in the optical path of one beam of the interferometric instrument. The precision reaches $5\times10^{-6}$ [11, 12]. In fact interferometric refractometers are sensitive to a number of disturbances such as temperature variations, and vibrations; these can give rise to signals which are greater than those due to the refractive index changes of interest, as explained in Ref. [6].

In the proposed method, the liquid sample is placed between an optical flat glass and a plano-convex glass lens. The obtained optical doublet is positioned behind an acousto-optic cell filled by distilled water, in which a progressive acoustic wave is generated by a piezoelectric transducer. The laser beam interaction with this acoustic wave leads to observe the diffraction pattern on the focal plane of the convergent doublet. For a fixed acoustic frequency of the transducer, the position of the focal plane depends on the refractive index of the liquid sample. When the refractive index is higher than that of air, the focal plane is shifted far away from the cell. This focal shift can be calculated from the spatial frequency of the diffracted orders. The proposed method has an adjustable measurement range and presents an estimated measurement error of $\pm7\times10^{-3}$. Details of the set-up are shown in Fig. 1.

## 2. Principle of the method

In a parallelepiped acousto-optic (AO) cell made of transparent glass and filled with distilled water, an ultrasonic wave is generated by a piezoelectric circular transducer

(20 mm in diameter) and driven by a variable frequency generator. To ensure progressive wave propagation, an absorbing material is used in the cell bottom. Vibrations of the piezoelectric transducer caused variations of the refractive index of the medium inside the AO cell, so the distribution of the refractive index is expressed by the following equation [13]:

$$N(x, t) = N_0 + \Delta_N \sin\left[2\pi f_a\left(t - \frac{x}{V_a}\right)\right] \tag{1}$$

where $t$ denotes the time, $x$ is the direction of the acoustic wave propagation, $N_0$ is the refractive index of the undisturbed medium, $\Delta_N$ refers to the amplitude of the refractive index variation, $f_a$ is the ultrasound frequency, $V_a = \lambda_a f_a$ is the ultrasound velocity and $\lambda_a$ is the acoustic wavelength in the cell medium. The second term in Eq. (1) is a result of the acoustic pressure due to the longitudinal vibration of the transducer.

An expanded and collimated He-Ne laser beam (5 mm in diameter) illuminates in $z$ direction a progressive ultrasonic wave inside the AO cell. We assume after Raman and Nagendra Nath that ultrasound can act as a pure phase grating [14, 15]. Having left the AO cell, the intensity of the diffracted light in the far field can be observed on the focal plane of the convergent doublet. This last consists of plano--concave liquid lens, inserted between a flat glass (1 mm in length) and plano-convex glass lens (of curvature radius $r = 40.95$ mm and refractive index $n_g = 1.5563$). The three optical elements are mounted on a cylindrical holder made of polypropylene and carefully manufactured, as shown in Fig. 2.

As shown in Fig. 3, the diffracted orders positions $x_{1D}$ and $x_{1R}$ are measured by recording the image of the diffracting pattern on the open active area of the CCD camera which is mounted on a holder with one dimensionally moving bench with a step of 10 μm, along $z$ direction. By filling the liquid inside the convergent doublet, the focal distance changes from $F_R$ to $F_D$ and therefore the diffraction order spacing changes as well from $x_{1R}$ to $x_{1D}$. For each position, the camera is readjusted until it is

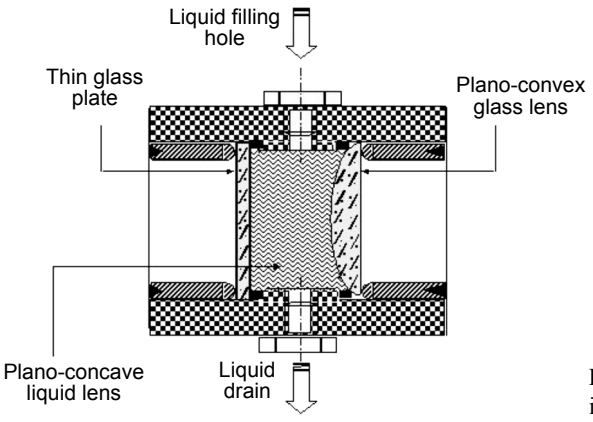


Fig. 2. Convergent doublet with liquid lens inside.

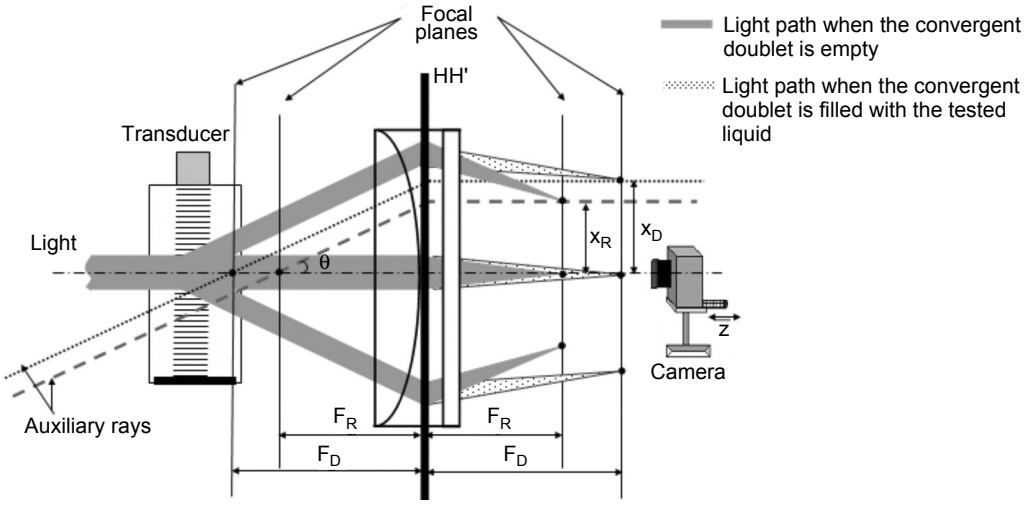


Fig. 3. Displacement of the CCD camera in $z$ direction to record the diffraction pattern.

located at the focal plane of the convergent doublet, where the smallest and focused laser spot can be obtained. To determine the exact position of the diffracted orders, the output of the camera is fed to the USB port of the computer for image reconstruction and processing via Image Pro Plus software (IPP), as it will be shown in the next section.

In order to obtain a relationship between the refractive index of the sample liquid and the spatial position of the diffracted order, we used the known grating equation [16]:

$$\lambda_a \sin\theta = \lambda \tag{2}$$

where $\theta$ refers to the first diffraction angle for the collimated incident light; $\theta$ can also be deduced by referencing to Fig. 3, when no sample is inserted, as follows:

$$\tan\theta = \frac{x_{1R}}{F_R} \tag{3}$$

As $\theta$ is less than 1° in the experiment conditions ($f_a$ = 10 MHz), thus we can get:

$$\sin\theta \approx \tan\theta \tag{4}$$

Using Eqs. (2) and (3) with the help of Eq. (4), we obtain finally:

$$F_R = \frac{x_{1R} V_a}{\lambda f_a} \tag{5a}$$

By analogy with Eq. (5a), when the liquid sample is inserted, the focal distance of the convergent doublet can be written as:

$$F_D = \frac{x_{1D} V_a}{\lambda f_a} \tag{5b}$$

Since the convergent doublet is a combination of two thin lenses in contact (the glass lens and the liquid lens), we obtain for the power of this combination [11]:

$$\frac{1}{F_D} = \frac{1}{F_L} + \frac{1}{F_R} \tag{6}$$

where $1/F_L$ refers to the vergence of a plano-concave liquid lens and in turn it can be written using the lens maker's formula as [11]:

$$\frac{1}{F_L} = \frac{n(\lambda) - 1}{r} \tag{7}$$

where $r$ denotes the curvature radius of the glass lens and $n(\lambda)$ refers to the refractive index of the sample liquid. By substituting Eqs. (5a), (5b) and (7) into Eq. (6), the desired formula of the sample refractive index can be written as follows:

$$n(\lambda) = r\left(\frac{1}{x_{1D}} - \frac{1}{x_{1R}}\right)\frac{\lambda f_a}{V_a} + 1 \tag{8}$$

## 3. Results and discussion

In this experiment, distilled water was used in the acousto-optic cell as an interaction medium. The acoustic velocity inside this cell medium, as a function of pressure $P$ in bars and temperature $T$ in °C, can be predicted within 0.05% from the experimentally determined formula [17]:

$$V_a(P, t) = 1402.7 + 488t - 482t^2 + 135t^3 + \left(15.9 + 2.8t + 2.4t^2\right)\frac{P}{100} \tag{9}$$

where $t = T/100$. To maintain the liquids temperature at 20±1 °C, the measurements were carried out in a laboratory equipped with an air conditioner and controlled by a digital thermometer (with temperature resolution of 0.1 °C). As a consequence, the predicted acoustic velocity inside the acousto-optic cell is 1482±3 $\mathrm{ms}^{-1}$.

The image of the diffractive pattern and its intensity line profile are presented in Fig. 4, for two different cases. First, the convergent doublet was empty (the glass lens

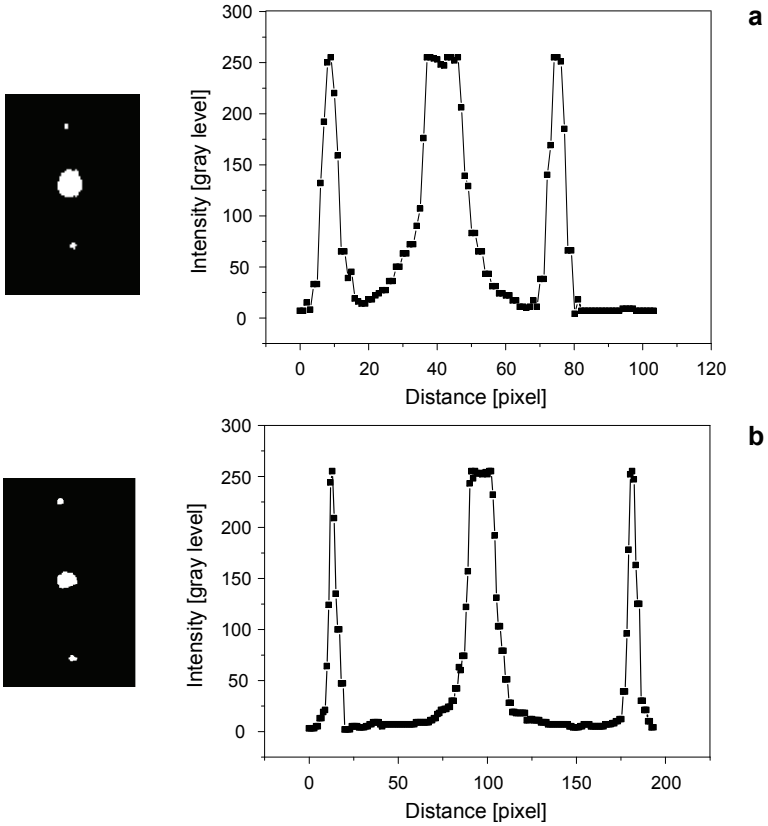


Fig. 4. Diffraction spectrum and its intensity line profile. A glass lens alone (**a**). Acetone liquid used as a test sample (**b**); $f_a$ = 10 MHz and $\lambda_a$ = 632.8 nm.

alone); secondly, the convergent doublet was filled with acetone taken as a sample liquid. In order to obtain the exact distance between the two diffractive spots, a cross--section of the image of the diffractive orders intensity surface was carried out using IPP software.

The obtained distance between the diffractive orders is first expressed in pixels and then converted into millimeters using the size of the camera pixel.

To verify the performance of the method, a series of tests have been made on three different liquids: distilled water, acetone and ethanol. The measurements of the refractive index are presented in Tab. 1.

The results presented in Tab. 1 agree well with the results obtained in Refs. [3] and [5]. However, our results deviated slightly from their results starting from the third decimal number.

To evaluate the uncertainty that affects the refractive index measurement, an error analysis was carried out by adopting the law of uncertainty propagation [18]

T a b l e 1. Refractive index measurements.

| Ultrasound frequency [Mhz] | Refractive index | | |
|---|---|---|---|
| | Distilled water | Acetone | Ethanol |
| 8 | 1.336 | 1.358 | 1.364 |
| 10 | 1.334 | 1.355 | 1.362 |
| 12 | 1.333 | 1.356 | 1.361 |
| Average refractive index (our work) | 1.334 | 1.356 | 1.362 |
| Refractive index from Ref. [3] | 1.3321 | – | 1.3603 |
| Refractive index from Ref. [5] | – | 1.3578 | 1.3604 |

T a b l e 2. A summary of calculations used to measure the refractive index uncertainty at liquid temperature $T = 20$ °C.

| Measurement parameters | Measurement uncertainties | Partial derivatives | Calculations to obtain $Un$ | |
|---|---|---|---|---|
| | $Ur = \pm 0.10$ mm | $\frac{\partial n}{\partial r} = \frac{n(\lambda)-1}{r} = 0.0081\ \text{mm}^{-1}$ | $\frac{\partial n}{\partial r} Ur = 0.00081$ | |
| $n_{\text{water}} = 1.334$<br>$\lambda_{\text{He-Ne}} = 632.8$ nm | $Uf_a = \pm 0.01$ MHz | $\frac{\partial n}{\partial f_a} = \frac{n(\lambda)-1}{f_a} = 0.334 \times 10^{-7}$ s | $\frac{\partial n}{\partial f_a} Uf_a = 0.00033$ | |
| $r = 40.95$ nm<br>$f_a = 10$ MHz<br>$x_{1D} = 656.6$ µm | $Ux_{1D} = \pm 3.4$ µm | $\frac{\partial n}{\partial x_{1D}} = -r\frac{\lambda f_a}{V_a x_{1D}^2} = -405.95\ \text{m}^{-1}$ | $\frac{\partial n}{\partial x_{1D}} Ux_{1D} = -0.00138$ | $Un = \pm 0.007$ |
| $x_{1R} = 290.9$ µm<br>$V_a = 1482$ m/s | $Ux_{1R} = \pm 3.4$ µm | $\frac{\partial n}{\partial x_{1R}} = r\frac{\lambda f_a}{V_a x_{1R}^2} = 2069.08\ \text{m}^{-1}$ | $\frac{\partial n}{\partial x_{1R}} Ux_{1R} = 0.00703$ | |
| | $UV_a = \pm 3$ m/s | $\frac{\partial n}{\partial V_a} = \frac{n(\lambda)-1}{V_a} = -0.00022\ \text{sm}^{-1}$ | $\frac{\partial n}{\partial V_a} UV_a = -0.00067$ | |

$$Un = \pm\left[\left(\frac{\partial n}{\partial r}Ur\right)^2 + \left(\frac{\partial n}{\partial f_a}Uf_a\right)^2 + \left(\frac{\partial n}{\partial V_a}UV_a\right)^2 + \left(\frac{\partial n}{\partial x_{1D}}Ux_{1D}\right)^2 + \left(\frac{\partial n}{\partial x_{1R}}Ux_{1R}\right)^2\right]^{\frac{1}{2}} \tag{10}$$

where $Un$ is the uncertainty of the refractive index measurement, $Ur$ is the uncertainty of the measurement of the lens curvature radius, $Uf_a$ refers to the uncertainty of the acoustic frequency, $UV_a$ stands for the uncertainty of the acoustic velocity, which depends on the temperature variation as shown in Eq. (9), $Ux_{1D}$ and $Ux_{1R}$ are respectively the uncertainties of the measurement of the diffractive orders spacing $x_{1D}$ and $x_{1R}$ which are evaluated by considering the measurement error related to the exact localization of the focal plane (estimated to ±2 μm) and the reading error related to the use of the camera pixels (estimated to ±2.8 μm).

Table 2 summarises the steps taken to measure $Un$ in the case of distilled water. It should be noted that the same uncertainty was found for ethanol and acetone. The final results of the propagation error show that $Un$ depends mainly on the diffractive orders spacing measurement and particularly $x_{1R}$. This fact makes it possible to approximate $Un$ as follows:

$$Un \approx \pm\frac{1}{x_{1R}^2}\,\frac{r\lambda_a f_a}{V_a}\,Ux_{1R} \tag{11}$$

$Un$ and the square value of $x_{1R}$ are related by a hyperbolic function, therefore the larger the value of $x_{1R}$ is, the weaker $Un$ becomes. To obtain a large value of $x_{1R}$, it is possible to use a glass lens with high focal length or using high acoustic frequency, which in turn gives a large value of $x_{1R}$, as shown in Eq. (5a). In addition, the lower is the pixel size of the camera; the lower is the measurement error.

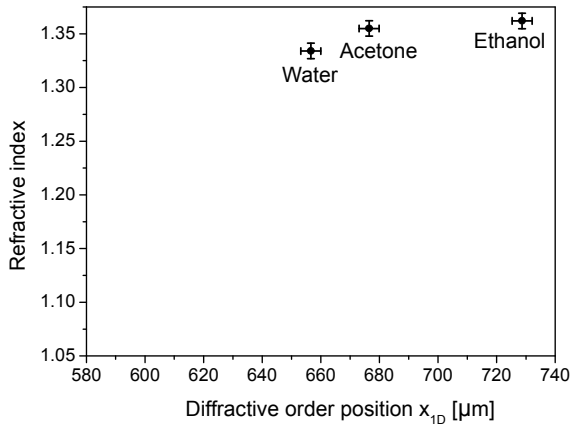


Fig. 5. Refractive index of the liquids used according to the diffractive order position $x_{1D}$.

The following figure summarizes the variation of the refractive index of the three liquids (water, acetone and ethanol) according to the diffractive order spatial position ($x_{1D}$) for 10 MHz of acoustic frequency.

Another case of practical interest is the refractive index measurement range. In the proposed method the measurement range depends: first on the sign of the optical doublet vergence, as long as this vergence is positive; which in more details means that, as long as the liquid lens refractive index is lower than that of the glass lens, the measurement is possible. In the contrary case, the optical doublet will become divergent and the measurement cannot be achieved, therefore we must in this case use a reference glass lens with higher refractive index. Secondly, when the refractive index of the sample is close to the refractive index of air, the measurement will become more difficult since $x_{1D} \approx x_{1R}$. To overcome this difficulty, we increase the acoustic frequency, which in turn increases the difference ($\Delta x$) between $x_{1D}$ and $x_{1R}$. This can be seen in the following equation, obtained by the combination of Eqs. (5a) and (5b):

$$\Delta x_2 = \frac{f_{a2}}{f_{a1}} \Delta x_1 \tag{12}$$

where, $\Delta x_1$ is the difference obtained using the acoustic frequency $f_{a1}$ and $\Delta x_2$ obtained using the acoustic frequency $f_{a2}$, with $f_{a2} > f_{a1}$. Equation (12) describes how the measurement range is magnified by a ratio ($f_{a2}/f_{a1}$), this can easily be performed by adjusting the rf generator applied to the piezoelectric transducer.

As shown in Fig. 6, the use of the flat glass of thickness $d$ and refractive index $n_f$, as a wall in the convergent doublet gives rise to a focal shift $\delta$ along $z$ direction [6] due to $n_f$, $d$ and the refractive index $n$ inside the convergent doublet, however in

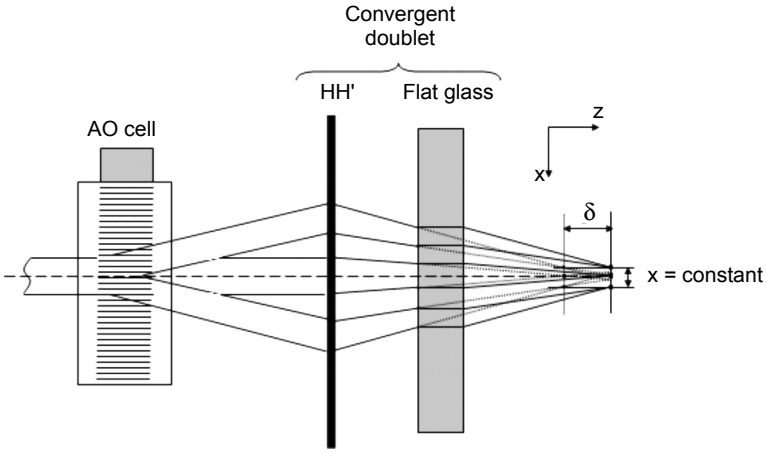


Fig. 6. The focal shift due to the flat glass in the convergent doublet.

*x* direction, the distance between the diffractive orders is unchanged and remains constant.

Figure 6 shows how it is more practical to use the diffraction pattern to determine the refractive index of the sample liquid regardless of the refractive indexes and thicknesses of the glass lens and the flat glass.

## 4. Conclusions

The technique we propose, addresses the field of liquid optics using a liquid phase grating for evaluating a liquid lens refractive index.

The liquid lens is placed between a thin flat glass and a convergent glass lens. The obtained convergent doublet, positioned behind an acousto-optic cell, enabled us to obtain the spatial Fourier transform of the light beam leaving the acousto-optic cell. Regardless of the glass lens and the flat glass refractive indexes and thicknesses, the sample liquid refractive index could be obtained by measuring the spatial frequency of the diffracted orders in two cases. First, when the convergent doublet is empty and secondly, when it is filled with the liquid.

The measurement is based on the difference between the two diffraction patterns recorded by means of a CCD camera. Once the image of the diffraction patterns is taken, an intensity line profile of the diffraction orders is drawn and this leads to the measurement of the diffracted orders spacing.

The accuracy in the presented measuring procedure depends mainly on the diffractive orders spacing measurements. This uncertainty can be reduced by the choice of a reference glass lens with high focal length, or by increasing the acoustic frequency, or using a CCD camera with high resolution. Although the obtained accuracy is not very high compared with interferometric methods, it is enough for many practical purposes. In addition, its simplicity and versatility make it very suitable for a variety of applications within this accuracy. We have tested this method by measuring the refractive index of acetone, ethanol and distilled water; the obtained results are in a good agreement with results of other workers in the field.

The measurement range can be adjusted by changing the acoustic frequency. Moreover, the fully computerized data acquisition made it possible to control temporal index variations.

*Acknowledgements* – The authors wish to thank the staff of the Institute of Experimental Physics of the University of Gdańsk, Poland, for receiving one member of their team as a guest scientist.

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*Received March 27, 2010*
*in revised form June 24, 2010*