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Properties of the angular gap in one-dimensional periodic structures containing left-handed metamaterials

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Wave propagation is investigated in one-dimensional periodic structures containing layers of left-handed metamaterials. For a special choice of parameters, an angular gap appears due to total internal reflection in a certain frequency region. This gap has been found insensitive to the polarization of light and the length scale of the periodic structure for linear wave propagation. We have studied phenomenon of bistability related to this gap for nonlinear wave propagation. As far as the sign of the Kerr nonlinearity is concerned, the said gap shows the same character as that related to the Bragg gap. It is further shown that the bistable character related to the gap is sensitive to the polarization of the incident radiation and length scale of the periodic structure.

Keywords: left-handed metamaterials, photonic gaps, optical bistability.

1. Introduction

Recently the experimental realization of left-handed metamaterials [1–7] has opened up a new research area. Metamaterials are such artificial composite structures in which the dimensions of the periodically repeated elements are much smaller than the wavelength of the incident radiation so that the structure appears to be a homogenous medium for the working wavelength. Left-handed metamaterial refers to a medium in which the electric permittivity and the magnetic permeability are simultaneously negative over a certain frequency range. The inclusion of left-handed metamaterials in photonic band gap (PBG) structures has led to the emergence of new mechanisms to produce photonic gaps [8–15]. In a one dimensional PBG structure containing alternate left-handed and regular material (also called right-handed) layers, the average refractive index of the structure becomes zero over a certain frequency range. Such a frequency range has been termed as a zero-n gap [8, 13, 14]. Another gap results due to the total internal reflection at an angular incidence in PBG structures containing alternate left-handed and right-handed layers [16–18]. Such a gap has been termed as the angular gap or the total internal reflection (TIR) gap and its properties are being investigated. Recently the appearance of TIR gap in a periodic structure containing both left-handed layers was shown theoretically [19]. The characteristics of the TIR gap are found to be quite distinct as compared to those of the Bragg gap in the conventional PBG materials. Here we investigate the phenomenon of bistability for nonlinear wave propagation associated with the TIR gap in a periodic structure containing both left-handed layers.

2. Theoretical model

The periodic structures considered here consist of layers A and B as shown schematically in Fig. 1. Both A and B layers are taken to be left-handed layers. Left-handed metamaterials are experimentally realized in the form of composite structures consisting of split ring resonators and periodic array of metal wires [2–7]. Frequency dispersions of electric permittivity and magnetic permeability are generally represented by the Drude model and the Lorentz model [2, 3].

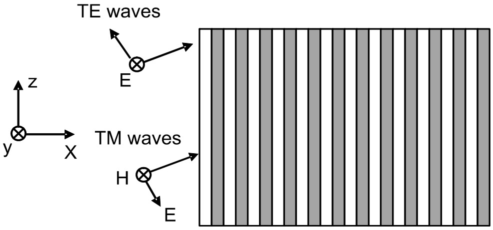

Fig. 1. Schematic diagram of the structure under consideration, white and gray layers represent layers A and B, respectively.

Left-handed materials are also realized in the form of special microstrips [20–22]. The frequency dispersions of electric permittivity and magnetic permeability of these microstrips are of the form as represented in Eqs. (1) below [19, 23]. Here we have taken $\varepsilon_A(\omega) = \mu_B(\omega)$ and $\varepsilon_B(\omega) = \mu_A(\omega)$ for simplicity. However it must be noted that it is not a necessary condition and the TIR gap and the resulting properties can be studied for any other choice of left-handed parameters;

$$\begin{aligned}
\varepsilon_A(\omega) &= 0.5 + \frac{25}{0.9^2 - \omega^2} + \frac{100}{11.5^2 - \omega^2} \\
\mu_A(\omega) &= 0.5 + \frac{9}{0.902^2 - \omega^2} \\
\varepsilon_B(\omega) &= 0.5 + \frac{9}{0.902^2 - \omega^2} \\
\mu_B(\omega) &= 0.5 + \frac{25}{0.9^2 - \omega^2} + \frac{100}{11.5^2 - \omega^2}
\end{aligned} \tag{1}$$

Here ω is the frequency measured in GHz. The widths of the two layers are taken to be same, *i.e.*, $d_A = d_B = 5$ mm. We have considered wave propagation through the structure using the transfer matrix approach, *i.e.*, the transfer matrix for the j-th structure can be written as:

$$m_j = \begin{bmatrix} \cos(k_j d_j) & -\frac{1}{q_j}\sin(k_j d_j) \\ q_j \sin(k_j d_j) & \cos(k_j d_j) \end{bmatrix} \tag{2}$$

where:

$$q_j = \frac{\sqrt{\varepsilon_j}}{\sqrt{\mu_j}}\sqrt{1 - \frac{\sin^2\theta}{\mu_j\,\varepsilon_j}} \quad \text{for the TE field} \tag{3a}$$

and

$$q_j = \frac{\sqrt{\mu_j}}{\sqrt{\varepsilon_j}}\sqrt{1 - \frac{\sin^2\theta}{\mu_j\,\varepsilon_j}} \quad \text{for the TM field} \tag{3b}$$

The tangential components of the electric and magnetic field at the incident side $x = 0$ and at the transmitted side $x = L$ are related by:

$$\begin{bmatrix} E_1 \\ H_1 \end{bmatrix}_{x=0} = M \begin{bmatrix} E_N \\ H_N \end{bmatrix}_{x=L} \tag{4}$$

where:

$$M = \prod_{j=1}^{N+1} m_j \tag{5}$$

while N is the total number of layers in the structure. The transmission coefficient T of the finite structure is calculated by applying the boundary conditions at the incident and the transmitted ends and is given by the following expression:

$$T = \frac{2q_0}{(q_0 M_{11} + q_0 M_{22}) - (q_0^2 M_{12} + M_{21})} \tag{6}$$

where $q_0 = \sqrt{1 - \dfrac{\sin^2\theta}{\mu_0\,\varepsilon_0}} = \cos\theta$, as there is air, *i.e.*, $\varepsilon_0 = \mu_0 = 1$ (in cgs units) on the incident and the transmitted sides of the structure; M_{lm} are the elements of the matrix M.

3. Results and discussion

The transmission coefficient T versus frequency W curve for linear wave propagation is shown in Fig. 2. The solid line in Fig. 2 corresponds to a normally incident wave. The long-dashed line is plotted for an oblique incidence at an angle of $\theta = 30°$. An angular gap appears in the frequency region 3.5–6.2 GHz due to total internal reflection. In this frequency region, total internal reflection takes place as light is travelling from a denser (air) to a rarer (periodic structure of left-handed metamaterials). Also the condition $\varepsilon(\omega)\mu(\omega) - \sin^2\theta < 0$ is satisfied in this frequency range. Such a TIR (total internal reflection) gap is not possible in a conventional PBG structure as light always enters from a medium of lower refractive index to a medium of higher refractive index. The width of the gap strongly depends on the angle of incidence as shown by the short-dashed line which is plotted for an angle of $\theta = 45°$; the greater the angle of incidence, the greater is the width of the TIR gap.

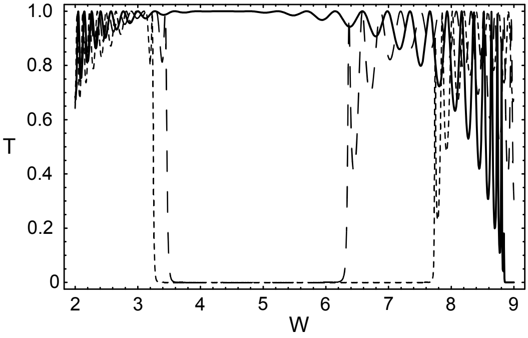

Fig. 2. The figure shows the appearance of TIR gap for the choice of parameters given in Eq. (1). For normal incidence, there is no gap as shown by the solid line. When the angle of incidence is 30°, a gap appears for 3.5 to 6.2 GHz frequency region as shown by the long-dashed line. The width of the gap increases as the angle of incidence increases to 45° as shown by the short-dashed line.

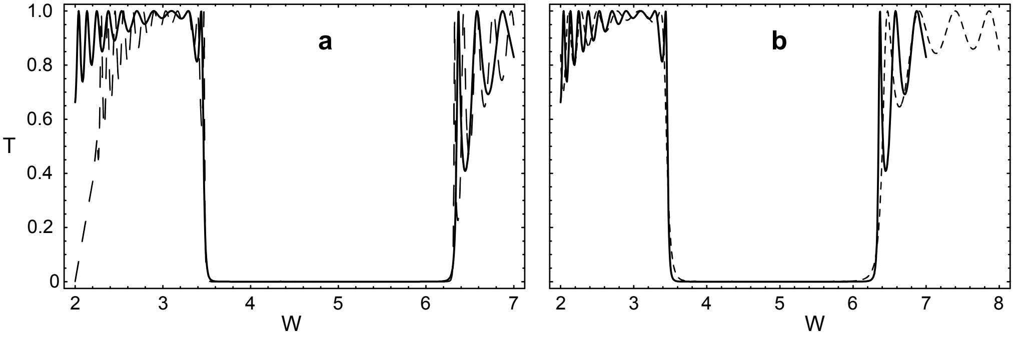

Fig. 3. Effect of polarization, the solid line shows TE mode, whereas the dashed line shows TM mode (**a**). Effect of changing the length scale of the periodic structure, the solid line shows the structure with $d_A = d_B = 5$ mm, whereas the dashed line corresponds to the structure with $d_A = d_B = 10$ mm (**b**).

Figure 3**a** shows the effect of polarization of the incident radiation. The solid line corresponds to transverse electric (TE) polarization and the dashed line corresponds to transverse magnetic (TM) polarization. The two plots seem to overlap each other showing that the gap remains invariant under the effect of polarization. Figure 3**b** shows the effect of change of widths of the layers of the periodic structure. The solid line corresponds to a structure with $d_A = d_B = 5$ mm, whereas the dashed line corresponds to a structure with $d_A = d_B = 10$ mm, the two lines overlap each other indicating the scale invariance of the gap for linear wave propagation

Now we investigate the bistability behavior for the nonlinear wave propagation near the edges of this gap. Bistability behavior is very important from the application point of view because it is used as switching mechanism in optical devices [20, 21]. It is important to investigate the dependence of threshold of the bistable behavior on the characteristics of the incident radiation such as its polarization and the properties of the periodic structure such as the widths of the layers involved. As the left-handed metamaterials have recently been realized in the microwave region, we have considered here GHz frequencies and widths of the order of millimeter; however with the experimental realization of these composite structures in the optical range; the same results can be investigated for the higher frequency ranges. For nonlinear wave propagation, the relative permittivity of the layer A of the structure is taken to have Kerr nonlinearity, *i.e.*,

$$\varepsilon_{\mathrm{NL}}(\omega) = \varepsilon_{\mathrm{A}}(\omega) + \alpha|E|^2 \tag{7}$$

Here α is the nonlinear Kerr coefficient. Kerr nonlinearity is a kind of weak nonlinearity which gives rise to the different nonlinear phenomena such as self phase modulation, self focusing and modulation instability under different situations [24]. All materials including left-handed materials exhibit Kerr nonlinearity [24, 25]. In our computational work, the Kerr coefficient and the incident intensity are normalized with respect to the characteristic intensity so that $\alpha = \pm 1$, where the plus sign corresponds to a self-focusing nonlinearity and the negative sign corresponds to a defocusing nonlinearity [25].

Initially we discuss bistability for the transverse electric (TE) mode. Figure 4**a** shows the bistable behavior near the lower edge inside the gap at frequency 3.6 GHz. The angle of incidence is taken to be 30° and the Kerr coefficient is taken to be negative for this case. Figure 4**b** shows bistability near the upper edge inside the gap at frequency 6.1 GHz for the same gap. However the Kerr coefficient is taken to be positive for this case. In this regard the behavior is quite similar to that of the Bragg gap. The phenomenon of bistability near the lower edge of Bragg gap takes place when the Kerr coefficient is negative and to observe bistable behavior near the upper edge of the Bragg gap, the Kerr coefficient is taken to be positive [26, 27]. This suggests that the nature of changes occurring for the wave propagation related to bistability is the same when both the layers in a periodic structure are either positive (Bragg gap)

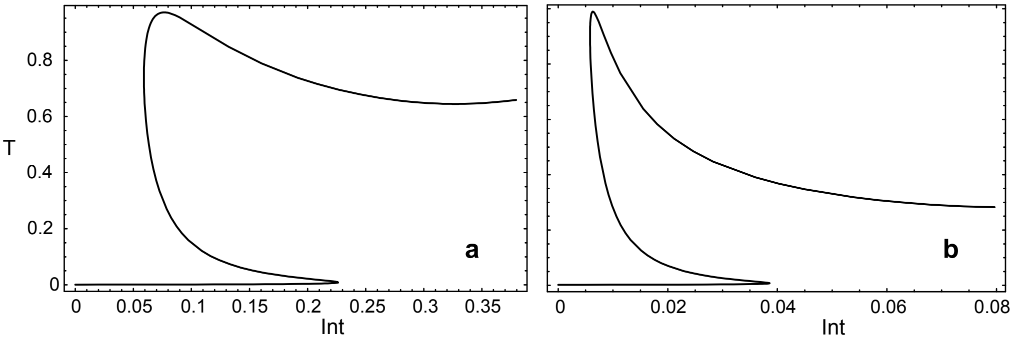

Fig. 4. The figure shows bistability for the TE mode. Part (**a**) shows the optical bistability near the lower edge of the TIR gap where α is taken to be –1 whereas part (**b**) shows the same phenomenon near the upper edge of the same gap where α is taken to be +1. Other parameters are same as in Fig. 1.

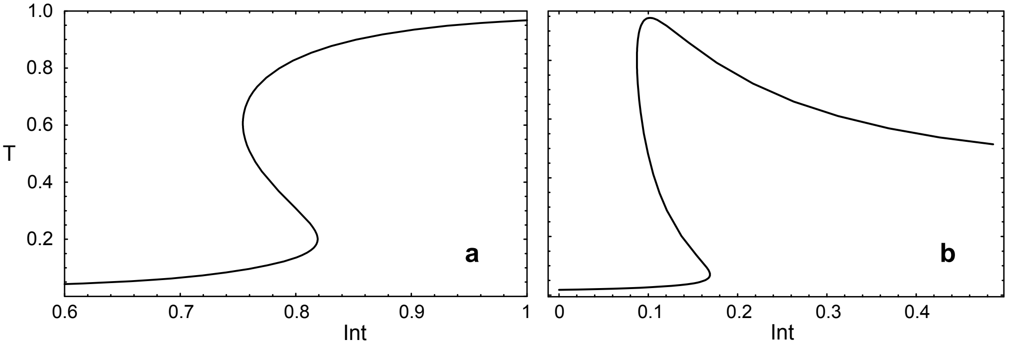

Fig. 5. The figure shows bistability for the TM mode. The part (**a**) shows the bistability near the lower edge of the TIR gap where α is taken to be –1 whereas the part (**b**) shows the same phenomenon near the upper edge of the same gap where α is taken to be +1. Other parameters are the same as in Fig. 1.

or negative (TIR gap considered here). However, when the same phenomenon is observed in zero-n gap where alternate layers in the periodic structure are left-handed and right-handed, the Kerr coefficients are taken to be of the opposite sign to those taken here, *i.e.*, it is taken positive near the lower edge of the gap, whereas it is taken negative near the upper edge of the gap [12, 14].

Let us now consider bistability associated with the transverse magnetic (TM) mode of the same gap.

Figure 5**a** has been plotted for the same frequency (3.6 GHz) near the lower edge of the TIR gap for the TM mode. The Kerr coefficient is taken to be –1. If we compare Fig. 5**a** with Fig. 4**a**, it becomes obvious that the threshold of bistability for the TIR gap near the lower edge for the TE mode (Fig. 4**a**) and for the TM mode (Fig. 5**a**) are quite different. Let us compare Fig. 5**b** with Fig. 4**b** which have been plotted at 6.1 GHz frequency near the upper edge of the TIR gap for the TM and TE modes,

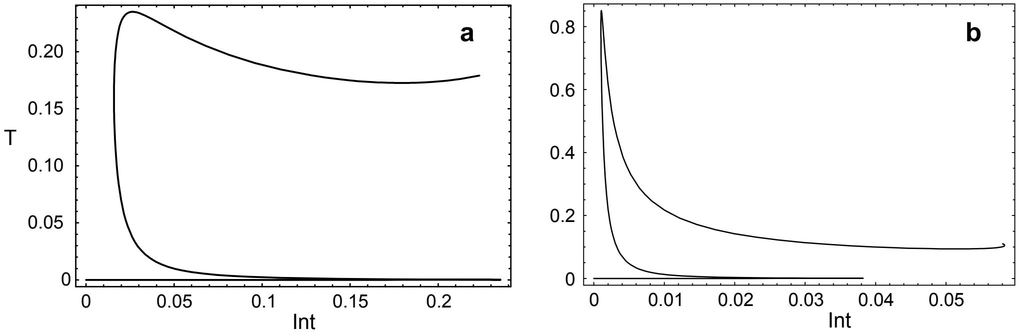

Fig. 6. The figure shows the optical bistability for the same parameters as in Fig. 4, except for the fact that the widths of the two layers are taken to be $d_A = d_B = 10$ mm.

respectively. The sign of the Kerr coefficient is taken to be positive in both cases. The switch up and the switch down fields, which are the basic characteristics of the bistability, are quite different in the two cases. It means that the TE and TM modes of the TIR gap are distinguishable for the nonlinear wave propagation, whereas it was shown previously (Fig. 3**a**) that the TE and TM modes are indistinguishable for the linear wave propagation.

It has been found (Fig. 3**b**) that the TIR or the angular gap is not affected when the length scale of the periodic structure is changed. Let us now investigate this effect for the bistable behavior of the structure.

Figure 6**a** shows the bistability curve at 3.6 GHz and Fig. 6**b** shows the same curve at 6.1 GHz for the TE mode. However the widths of the two layers are taken to be $d_A = d_B = 10$ mm. The comparison of Fig. 6**a** with Fig. 4**a** and Fig. 6**b** with Fig. 4**b** shows that the change in length scale has changed the switch-up and switch-down intensities for the bistable character of the gap. Whereas it was found previously that for the linear wave propagation, the change of length scale remains insensitive to the TIR gap (Fig. 3**b**).

4. Conclusions

The bistability character associated with TIR gap in a one-dimensional periodic structure containing left- handed metamaterials has been investigated. It is found that the behavior of the gap for the sign of the Kerr coefficient is the same as that related to the Bragg gap. It is further shown that the said gap is sensitive to the polarization of light and the length scale of the periodic structure for the phenomenon of bistability. As the left-handed metamaterials have recently been realized in the microwave region, we have considered here GHz frequencies and lengths of the order of millimeter; however with the experimental realization of these composite structures in the optical range, the same results can be investigated for the higher frequency ranges.

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