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Diffraction images of bright incoherent disks in the presence of linear image motion

The paper discusses the intensity distribution in the images of uniform bright incoherent disks in the presence linear image motion. Results are plotted for the disks of various sizes for different amounts of image motion. It is shown that the presence of motion produces a decrease in intensity in the centre, broadens the image and shifts the position of peak intensity from the centre. Hence, an accurate knowledge of the image intensity distribution is desirable for the persons engaged in the work concerning aerial reconnaissance.

1. Introduction

It is now well known that the performance of modern cameras, such as those used in aerial photography, is limited by various types of image motion [1–7]. Although image motion compensation (IMC) techniques have been used, there is always some residual image motion present. This residual image motion may be linear, periodic, random or quadratic etc. Thus image movements are a characteristics feature of aerial photography. Degrading effects of image motion due to vibration etc. are now well know and have been adequately described elsewhere [7–9]. The influence of linear image motion has been considered with respect to its effect on imagery of periodic, bar and truncated objects [7–10] and image restoration [11–15].

Investigations concerning the diffraction images of isolated objects, such as line, edge, disks and annulus [16–21], gained considerable importance in reconnaissance work in the past. The study of light intensity distribution in the image of an incoherent disk is useful in many experimental situations and is widely discussed in our earlier papers [22–24]. Moreover a disk-shaped target is a natural extension of a point source and is two dimensional in nature. The presence of motion may hinder the recognition and parameter estimation in case of such an object.

The purpose of the present paper is, therefore, to study the effect of linear image motion on the intensity distribution in the diffraction images of uniform bright incoherent disks.

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2. Theory

In what follows, approach of Barakat and Houston [16, 18] suitably modified for a nonrotationally system will be used.

The image spectrum $I(\omega, \Theta)$ is related to the object spectrum by

$$I(\omega, \Theta) = T(\omega, \Theta)\cdot O(\omega) \tag{1}$$

where ω is the spatial frequency variable, Θ the corresponding azimuth in Fourier transform space and $T(\omega, \Theta)$ is the transfer function of the optical system under consideration.

The image intensity distribution $i(v, \psi)$, which is the inverse Fourier transform of $I(\omega, \Theta)$, is given by

$$i(v, \psi) = \frac{1}{2\pi}\int_0^2\int_0^{2\pi} T(\omega, \Theta)\, O(\omega) \times \times \exp[iv\omega\cos(\Theta-\psi)]\omega d\omega\, d\Theta. \tag{2}$$

Here v and ψ are the polar coordinates in the image plane; ψ defines the measurement direction, and v is a dimensionless distance parameter related to the parameter of the optical system by

$$v = \frac{(\pi D \sin\alpha)}{\lambda}$$

where D is the diameter of the aperture, $\varphi\lambda$ — the wavelength of light, and α — the semifield angle.

When the image move with linear motion the displacement is given by $\Delta r = vt$. The effective component or r along the x-axis is

$$x = r\cos\psi,$$

so

$$\Delta x = vt\cos\psi. \tag{3}$$

Equation (2) now becomes

$$i(v, \psi, t) = \frac{1}{2\pi} \int_0^2 \int_0^{2\pi} T(\omega, \Theta)\, O(\omega) \times$$

$$\times \exp\left[\left\{ i\omega \frac{\pi D}{\lambda f}(r - vt)\right\} \cos(\Theta - \psi)\right] \omega\, d\omega\, d\Theta. \quad (4)$$

The time average of intensity is

$$i(v, \psi) = \frac{1}{2\pi} \int_0^2 \int_0^2 T(\omega, \Theta) O(\omega) \exp[iv\omega \cos(\Theta - \psi)] \times$$

$$\times \left[\frac{1}{t_e} \int_0^{t_e} \exp\left[-i\omega \frac{\pi vt}{\lambda F} \cos(\Theta - \psi)\right] dt\right] \omega\, d\omega\, d\Theta \quad (5)$$

which gives

$$i(v, \psi) = \frac{1}{2\pi} \int_0^2 \int_0^{2\pi} T(\omega, \Theta) O(\omega) \cos\left[\left(v\omega - \frac{\pi A \omega}{2}\right)\right] \times$$

$$\times \cos(\Theta - \psi) \frac{\sin\left[\frac{\pi A \omega}{2} \cos(\Theta - \psi)\right]}{\pi A \omega \cos(\Theta - \psi)} \omega\, d\omega\, d\Theta. \quad (6)$$

Here $A = (2vt_e)/\lambda F$ is motion parameter, and t_e — the total exposure time.

Maximum dagration takes place along the direction of motion (i.e. $\psi = 0$), and the intensity is given as

$$i(v) = \frac{1}{2\pi} \int_0^2 \int_0^{2\pi} T(\omega, \Theta) O(\omega) \times$$

$$\times \cos\left[\left(v\omega - \frac{\pi A \omega}{2}\right) \cos\Theta\right] \times$$

$$\times \frac{\sin\left(\frac{\pi A \omega \cos\Theta}{2}\right)}{\pi A \omega \cos\Theta} \omega\, d\omega\, d\Theta. \quad (7)$$

For any other direction, the intensity can be evaluated by changing A to $A \cos\psi$ in equation (6).

$T(\omega, \Theta)$ for a diffraction limited circular aperture is independent of Θ, and given by the well known expression

$$T(\omega) = \frac{2}{\pi} \left[\cos^{-1}\left(\frac{\omega}{2}\right) - \frac{\omega}{2}\left(1 - \frac{\omega^2}{4}\right)^{1/2}\right]. \quad (8)$$

The object spectrum for a disk object is given [18] by

$$O(\omega) = \frac{v_0 J_1(v_0 \omega)}{\omega}, \quad (9)$$

where v_0 is radius of the disk object in normalized diffraction units.

2.1. Time average intensity due to linear motion of an Airy pattern

The intensity distribution in the diffraction image of a point source is given as

$$I = I_0 \left[\frac{2J_1(v)}{v}\right]^2. \quad (10)$$

Now if we consider the Airy pattern to be moving with a linear velocity v, intensity distribution at any instant of time t is given as

$$i(v, t) = I_0 \left[\frac{2J_1\left(\frac{\pi r}{\lambda F} - \frac{\pi vt}{\lambda F}\right)}{\frac{\pi r}{\lambda F} - \frac{\pi vt}{\lambda F}}\right]^2 = I_0 \left[\frac{2J_1\left(v - \frac{\pi A t}{t_e}\right)}{\left(v - \frac{\pi A t}{t_e}\right)}\right]^2. \quad (11)$$

The time average of intensity for an exposure time t_e can then be obtained as

$$i(v) = \frac{1}{t_e} \int_0^{t_e} i(v, t) dt =$$

$$\frac{1}{t_e} \int_0^{t_e} \left[\frac{2J_1\left(v - \frac{\pi A k}{t_e}\right)}{v - \frac{\pi A k}{t_e}}\right]^2 dt. \quad (12)$$

3. Results and discussion

Integral in equation (7) was numerically evaluated by 32-point Gauss quadrature method on electronic computer. Results for some typical cases shown in figs 1–4. For comparison, images in the absence of image motion ($A = 0.0$) are shown by dotted curves in all the figures.

Four values of the disk radius $v = 2.0$, 4.0, 6.0, and 8.0 were taken for the amount of vibration parameter $A = 0.0$, 0.50, 1.0, 1.5, and 2.0. We have also calculated the intensity distribution using equation (12) and found that the results obtained agree with those obtained from relation (7) for $v_0 = 0.5$, i.e. for coherent illumination.

It is seen from the figures that an increase in motion parameter A leads to a decrease of intensity in the centre, broadening of the image, and shifting of the maximum intensity point away from the centre

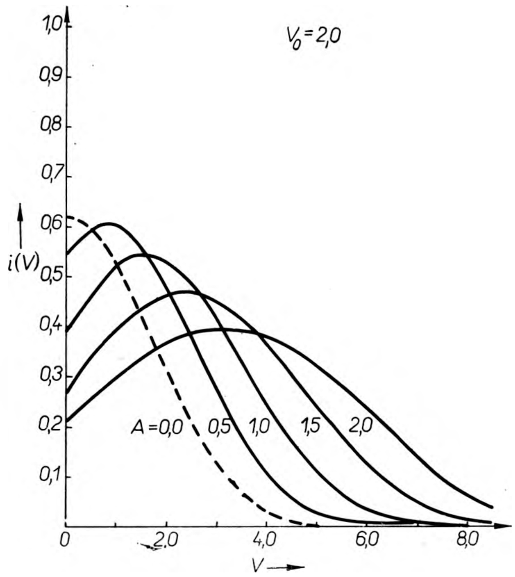

Fig. 1. Images of disks in the presence of linear image motion for $A = 0.5$ and $\nu_0 = 2.0$, 4.0, 6.0, and 8.0. Dotted curve represents the ideal case, i.e. $A = 0$

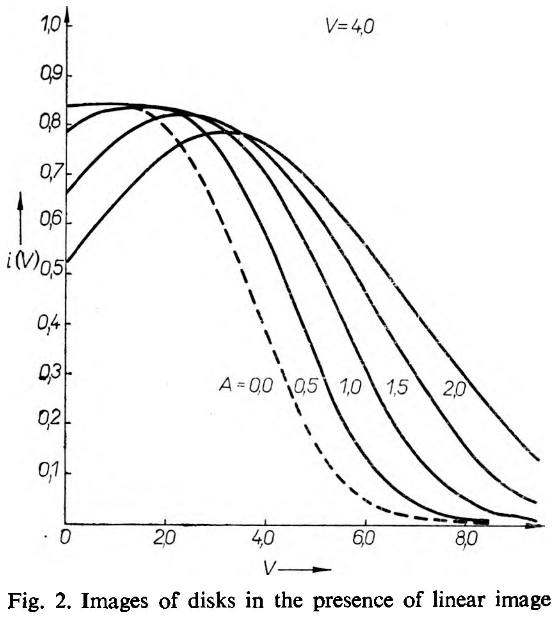

Fig. 2. Images of disks in the presence of linear image motion for $A = 1.0$ and $\nu_0 = 2.0$, 4.0, 6.0, and 8.0. Dotted curve represents the ideal case, i.e. $A = 0$

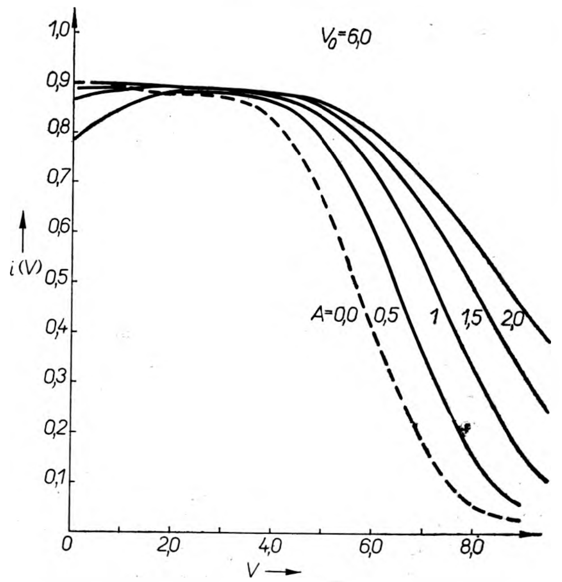

Fig. 3. Images of disks in the presence of linear image motion for $A = 1.5$ and $\nu_0 = 2.0$, 4.0, 6.0, and 8.0. Dotted curve represents the ideal case, i.e. $A = 0$

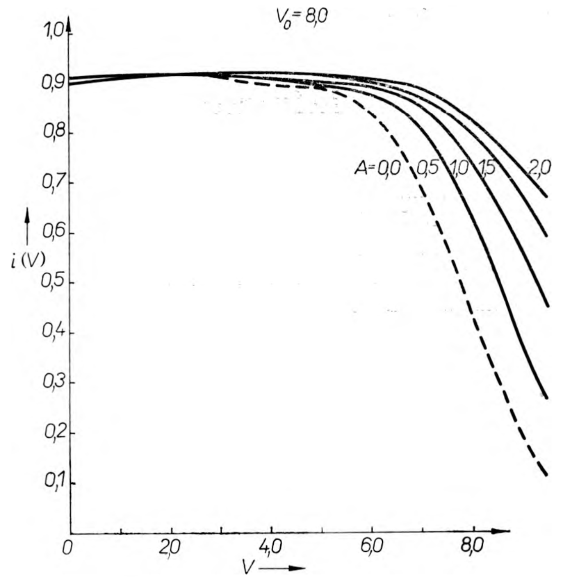

Fig. 4. Images of disks in the presence of linear image motion for $A = 2.0$ and $\nu_0 = 2.0$, 4.0, 6.0, and 8.0. Dotted curve represents the ideal case, i.e. $A = 0$

(i.e. $\nu = 0$). Shift of the maximum intensity point is linear, since the phase is a linear function of A. A disk under these conditions may be interpreted as an annulus object. This effect may thus modify the interpretation of microphotometry of small particles. Hence, it is evident that the influence of the linear motion is very pronounced and it considerably distorts the object.

The slop of the intensity distribution curves is the largest for the motion free case and decreases at the edges of the disks as the amount of linear motion increases. In fact, the intensity distribution tends to a Gaussian shape. For motion free case and when ν_0 is very large (such as $\nu_0 = 8.0$) the disk behaves as an edge object [24]. The irradiance at the geometrical radius in the image shows that for large sizes of the disk, the value of this irradiance approaches one half of that at the centre of the image. However, this does not hold good in the presence of image motion. As reported by earlier workers [17] the image of an incoherent bright disk on a dark background is complimentary to that of a disk on a light background. This holds good also in presence of linear image motion. The effect of linear motion is maximum in the motion direction $\psi = 0$, and mimimum in the direction $\psi = \pi/2$.

It is also interesting to note that expression (7) is analogous to the equation for the intensity distribution in the far field diffraction pattern in partially coherent light [25]. Hence results for intensity distribution are also valid for Fraunhofer diffraction in partially coherent light.

Дифракционные изображения некогерентных светлых дисков при наличии линейного смещения изображения

В работе обсуждается распределение интенсивности в изображении некогерентного равномерно светящего диска в случае линейного движения изображения. Выполнены графики результатов для дисков различных размеров и при различных смещениях подвижного изображения. Показано, что смещение снижает интенсивность изображения в середине, расширяет изображение и перемещает положение пика интенсивности из середины. Поэтому точное знание распределения интенсивности в изображении имеет значение для занимающихся авиаразведкой.

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