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Partially coherent reconstruction of Fourier holograms. The contrast degrading function

On the basis of the generalized Schell's theorem it has been shown that the spatial frequency spectrum of an image obtained by partially coherent reconstruction of Fourier hologram is a product of the spatial frequency spectrum of an image obtained from the same hologram with coherent light and a function of spatial frequency dependent on partial coherence of illuminating light beam. An experimental measurement of this function is described in case of a flat, circular, uniformly radiating, quasimonochromatic and incoherent light source used for reconstruction of holograms, and the results obtained are presented.

1. Introduction

Holography is a technique of recording and reconstruction of images in which coherent light is usually employed. Therefore recording of holograms and reconstruction of images are usually described as coherent diffraction phenomena. It appears however, that the partially coherent light can be also applied to holography. Effects induced by partial coherence have been studied by several authors. For instance, the hologram recording process with help of partially coherent light was described by TSURUTA [1], BERTELOTTI et al. [2]. WEINGÄRTNER [3], ROSS [4], and LURIE [5] suggested an application of holograms recorded with partially coherent light to determining the degree of partial coherence in the light field used during recording. FUJIWARA and MURATA [6] have discussed the influence of the degree of coherence on a holographic image in case when both: hologram recording and image reconstruction were performed by means of a partially coherent light. Their analysis, however, was limited only to in-line Fresnel holography.

The present paper is devoted to the problem of reconstruction of Fourier holograms with the help of partially coherent light, the recording step being assumed to be completely coherent.

The following notation will be used hereafter:

A point in the space (x, y) is denoted by $\bar{P}$. Subscripts: $_0$, $_2$, $_4$ describe: the source plane $(\bar{P}_0)$, hologram plane $(\bar{P}_2)$, and observation plane $(\bar{P}_4)$, respectively (fig. 1).

The differential $d\bar{P}$ means an element of the surface:

$$d\bar{P} = dx \cdot dy, \tag{1}$$

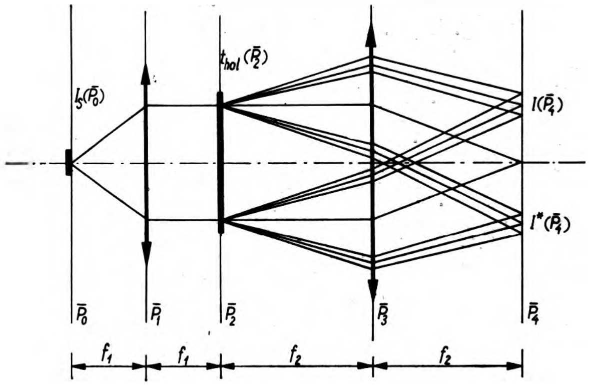

Fig. 1. Scheme of partially coherent reconstruction of Fourier holograms

and

$$\bar{P}_2 \cdot \bar{P}_4 = x_2 \cdot x_4 + y_2 \cdot y_4. \tag{2}$$

Spatial frequency is denoted by $\bar{q} = (q_x, q_y)$. Similarly to (1):

$$d\bar{q} = dq_x \cdot dq_y, \tag{3}$$

and

$$\bar{P} \cdot \bar{q} = x \cdot q_x + y \cdot q_y. \tag{4}$$

By $\otimes$ we denote convolution.

As it have been shown in [7] the mutual coherence function in the light field generated by flat, quasi-monochromatic, extended source in the far zone is quasi-stationary, i.e. it has the form (eq. (9) in [7]):

$$\Gamma(\bar{P}_2', \bar{P}_2') = \mathring{\Gamma}(\bar{P}_2' - \bar{P}_2'') \exp\left[\frac{ik}{2z}[\bar{P}_2'^2 - \bar{P}_2''^2)\right], \tag{5}$$

$\mathring{\Gamma}(\bar{P}_2' - \bar{P}_2'')$ being a spatially stationary part of the mutual coherence function in the hologram (fig. 1).

The generalized Schell's theorem, as formulated in [7] (eq. (36)), states that the intensity distribution in a diffraction pattern, in partially coherent, paraxial

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diffraction, on a given transparency, is proportional to the convolution of the intensity distribution in a diffraction pattern on the same test, due to a point source and to a Fourier transform of the spatially stationary part of the mutual coherence function in the test plane. According to the adopted notation and fig. 1:

$$I_{p\,\mathrm{coh}}(\bar{P}_4) = I_{\mathrm{coh}}(\bar{P}_4) \otimes \iint \mathring{\Gamma}(\bar{P}_2)\exp\left(-2\pi i\bar{P}_2\frac{\bar{P}_4}{\lambda z}\right)d\bar{P}_2. \quad (6)$$

2. The contrast degrading function

Let us consider a Fourier-type hologram of a trans-illuminated object taken in a typical arrangement shown in fig. 2 [9]. The hologram is then reconstructed with partially coherent light in a setup shown schematically in fig. 3. During reconstruction the hologram is illuminated by a collimated, quasimonochromatic light beam originating from an extended, flat, incoherent source. The intensity distribution across its surface is $I_s(\bar{P}_0)$. This means that the light beam used for reconstruction is partially coherent.

Let us consider now an intensity distribution in anyone of two conjugate images $I(\bar{P}_4)$ or $I^*(\bar{P}_4)$. As the reconstruction of Fourier hologram is essentially a far field diffraction process, the generalized Schell's theorem can be applied to its description. Let $I_0(\bar{P}_4)$ denote an intensity distribution in the image obtained from the same hologram, but reconstructed with completely coherent light. By virtue of generalized Schell's theorem (6) $I(\bar{P}_4)$ can be expressed as [8]:

$$I(\bar{P}_4) = I_0(\bar{P}_4) \otimes H(\bar{P}_4), \quad (7)$$

where

$$H(\bar{P}_4) = \iint \mathring{\Gamma}(\bar{P}_2)\exp\left(-2\pi i\,\bar{P}_2\frac{\bar{P}_4}{\lambda f_2}\right)d\bar{P}. \quad (8)$$

The meaning of this function, as well as the equation (7), can be easily understood if we assume that all the functions in (7) are Fourier transforms:

$$\begin{aligned} I(\bar{P}_4) &= \iint i(\bar{q})\exp(-2\pi i\,\bar{P}_4\bar{q})d\bar{q}, \\ I_0(\bar{P}_4) &= \iint i_0(\bar{q})\exp(-2\pi i\bar{P}_4\bar{q})d\bar{q}, \quad (9) \\ H(\bar{P}_4) &= \iint h(\bar{q})\exp(-2\pi i\bar{P}_4\bar{q})d\bar{q}. \end{aligned}$$

Then, from the convolution theorem we have:

$$i(\bar{q}) = i_0(\bar{q})\cdot h(\bar{q}) \quad (10)$$

or

$$\frac{i(\bar{q})}{i_0(\bar{q})} = h(\bar{q}) = \iint H(\bar{P}_4)\exp(2\pi i\bar{P}_4\bar{q})d\bar{P}_4. \quad (11)$$

We shall call $h(\bar{q})$ *the contrast degrading function* (CDF) in partially coherent reconstruction of Fourier holograms.

In order to evaluate this function let us compare equations (8) and (11). It gives:

$$\begin{aligned} h(\bar{q}) &= \iint H(\bar{P})\exp(2\pi i\bar{P}\bar{q})d\bar{P} \\ &= \iint \iint \mathring{\Gamma}(\bar{S})\exp\left(-2\pi i\bar{S}\frac{\bar{P}}{\lambda f_2}\right)d\bar{S}\times \\ &\quad \times\exp(2\pi i\bar{P}\bar{q})d\bar{P} \\ &= \iint \mathring{\Gamma}(\bar{S})\iint \exp\left[-2\pi i\bar{P}\left(\frac{\bar{S}}{\lambda f_2}-\bar{q}\right)\right]d\bar{P}d\bar{S} \\ &= \lambda f_2\iint \mathring{\Gamma}(\bar{S})\delta(\bar{S}-\lambda f_2\bar{q})d\bar{S} \\ &= \lambda f_2\mathring{\Gamma}(\lambda f_2\bar{q}). \end{aligned} \quad (12)$$

The last equation means that the contrast degrading function is proportional to the spatially stationary part of the mutual coherence function in the hologram plane.

If the light source used for reconstruction of hologram is flat, quasimonochromatic, incoherent, and the intensity distribution on its surface is $I_s(\bar{P}_0)$ then, according to the Van Cittert-Zernike theorem [10], the mutual coherence function in the hologram plane is (cf. fig. 1):

$$\mathring{\Gamma}(\bar{P}_2'-\bar{P}_2'') = \iint I_s(\bar{P}_0)\exp\left(-2\pi i\bar{P}_0\frac{\bar{P}_2'-\bar{P}_2''}{\lambda f_1}\right)d\bar{P}_0. \quad (13)$$

Inserting (13) into (12) we have:

$$h(\bar{q}) \approx \hat{I}_s\left(\bar{q}\frac{f_2}{f_1}\right). \quad (14)$$

Finally it may be concluded that the contrast degrading function of an image reconstructed from a Fourier hologram with an incoherent extended light source is determined by the light intensity distribution on the surface of the source.

3. An interpretation of the contrast degrading function

Let us consider the hologram of such object that the intensity distribution in a coherently reconstructed image has the form:

$$I_0(x_4) = 1 + A_0\cos\left(2\pi\frac{x_4}{d}\right). \quad (15)$$

Such an image contains only one different from zero spatial frequency $q_x = 1/d$; thus

$$i_0(q_x) = \delta(q_x)+\frac{1}{2}A_0\left[\delta\left(q_x-\frac{1}{d}\right)+\delta\left(q_x+\frac{1}{d}\right)\right]. \quad (16)$$

According to (10) in partial coherently reconstructed image we have:

$$i(q_x) = i_0(q_x)h(q_x) = h(q_x)\delta(q_x)+\frac{1}{2}A_0\cdot h(q_x)\left[\delta\left(q_x-\frac{1}{d}\right)+\right. \left.+\delta\left(q_x+\frac{1}{d}\right)\right]. \quad (17)$$

Hence the intensity distribution in that image is:

$$I(x_4) = \int i(q_x)\exp(-2\pi i q_x x_4)\,dq_x = h(0)+\frac{1}{2}A_0\left[h\left(-\frac{1}{d}\right)\exp\left(2\pi i\frac{x_4}{d}\right)+\right. \left.+h\left(\frac{1}{d}\right)\exp\left(-2\pi i\frac{x_4}{d}\right)\right]. \quad (18)$$

Note, that I_s being an intensity distribution is real, so $h(q)$ is an even function [11]. Then:

$$I(x_4)= h(0)+A_0 h\left(\frac{1}{d}\right)\cos\left(2\pi\frac{x_4}{d}\right). \quad (19)$$

Moreover, if the intensity distribution on the source is an even function (i.e. the source is symmetrical) then the contrast degrading function is real [11].

The equation (19) may now be interpreted as follows:

If the image reconstructed from a Fourier hologram with coherent light forms a system of sinusoidal fringes of spatial frequency q_x and contrast A_0, then the image reconstructed from the same hologram with partially coherent light forms also the system of sinusoidal fringes of the same direction and frequency when $h(q_x) \neq 0$, or gives uniformly illuminated light spot when $h(q_x) = 0$.

The contrast in partial coherently reconstructed image is diminished by an amount equal to the normalized value of the contrast degrading function for given frequency $h(q_x)/h(0)$, but as long as the light source is symmetrical there is no phase shift with respect to the coherently reconstructed image.

The general case is described by the equation (10). This formula states that if there exist a Fourier component of any spatial frequency $\bar{q}$ in the intensity distribution in the image reconstructed from a Fourier hologram with coherent light, then the presence of the same component in the image reconstructed with partially coherent light depends on the contrast degrading function. Namely, if the function $h(\bar{q})$ is equal to zero, then the component of a corresponding spatial frequency in that image will be absent. If the contrast degrading function vanishes for all frequencies higher than q_0, then this frequency may be treated as „cut-off" frequency due to partial coherence of illuminating light. If $h(\bar{q})$ differs from zero then the contrast in the corresponding component is decreased only by $h(\bar{q})/h(0)$. Eventually, a phase shift in this component will occur if a light source used for partially coherent reconstruction is unsymmetrical.

Note however, that the contrast degrading function should not be confused with a "transfer function", because it describes the relation between two images reconstructed from the same hologram with help of two different sources (i.e. between a completely coherent (point) one, and an extended incoherent one) rather than the "object"—"image" relationship.

4. Measurement and experimental results

To verify experimentally the results derived above the contrast in the images of a test object reconstructed from the Fourier hologram with laser, light and with the light from an incoherent extended source had to be compared.

4.1. Test object

A sinusoidal grating of known spatial frequency would be the best object for testing. However, due to technological difficulties this test had to be replaced by Ronchi ruling. Several rectangular gratings of this kind served as test objects.

4.2. Registration of holograms

A setup for registration of test holograms is shown in fig. 2. He-Ne CW laser of LG-600 type working at $\lambda = 628$ nm has been used. 20^x microobjective and a photographic Sonnar objective of focal length $f_1 = 185$ mm formed a beam expander. Another 20^x microobjective and a pinhole of diameter about 100 μm were used to form a reference point source. The Fourier transforming objective had the focal length $f_2 = 500$ mm. Holograms have been registered on 10E75 Agfa-Gevaert plates.

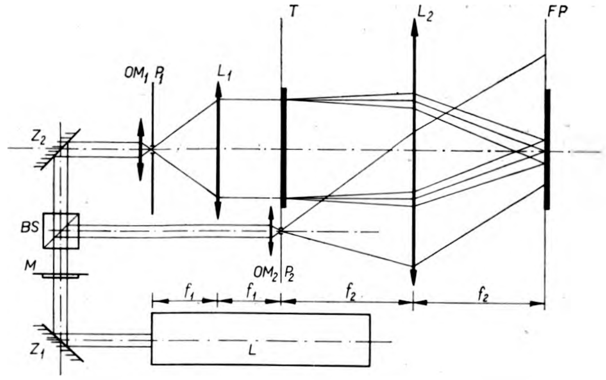

Fig. 2. Diagram of an arrangement for registration of Fourier holograms:

L — laser LG-600; Z_1, Z_2 — mirrors; BS — beamsplitter; M — shutter; OM_1, OM_2 — 20^x microobjectives; P_1, P_2 — pinholes; L_1 — collimating lens (f_1 = 185 mm); T — test-object; L_2 — Fourier transforming lens (f_2 = 500 mm) FP — photoplate

4.3. Reconstruction of holograms

A setup for partially coherent reconstruction of holograms is shown in fig. 3. A XBO-101 high pressure mercury lamp illuminated an exchangeable pinhole through two lenses and an interference filter for $\lambda = 546$ nm. The pinhole was placed in a back

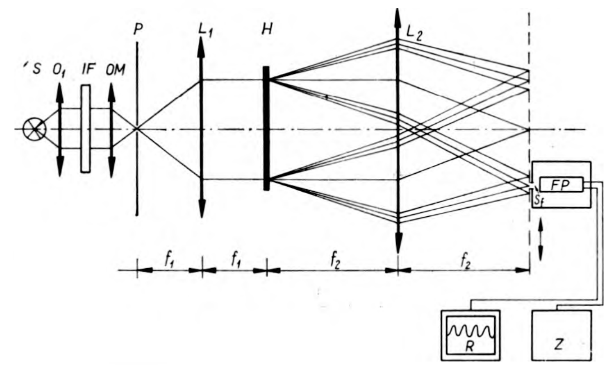

Fig. 3. Diagram of an arrangament for partially coherent reconstruction of Fourier holograms, and measurement of the intensity distribution in the image:

S — high pressure mercury lamp of XBO-101 type; O_1 — lens; IF — interference filter for λ = 546 nm; OM — 20^x microobjective; P — exchangeable pinhole; L_1 — collimating lens (f_1 = 185 mm); H — hologram; L_2 — Fourier transforming lens (f_2 = 500 mm); FP — photomultiplier; Sf — photomultiplier slit; Z — high voltage supply; R — X-Y recorder

focal plane of the collimating lens. If the arc of the mercury lamp is sharply imaged onto the pinhole, the latter can be approximately treated as an incoherent, flat, and quasimonochromatic light source of known shape and dimensions.

The collimating and Fourier transforming lenses have been identical with those used during registration. A photomultiplier with 50 μm slit placed in a back focal plane of transforming lens have been used for measuring the intensity distribution in the reconstructed images.

Contrast in reconstructed image has been calculated from the intensity distribution curve registrated by a plotter.

In case of a completely coherent reconstruction instead of the mercury lamp the laser has been used.

4.4. Foreseen shape of the contrast degrading function

The construction of an "incoherent secondary source" described above, justifies the assumption that the source is an incoherent, uniformly radiating circle of radius $2r$. For this case:

$$I_s(\bar{P}_0) = \mathrm{circ}\left(\frac{\bar{P}_0}{r}\right). \tag{20}$$

So the contrast degrading function should be:

$$h(\bar{q}) = \int\int \mathrm{circ}\left(\frac{\bar{P}_0}{r}\right)\exp\left(2\pi i(f_2/f_1)\bar{P}_0\bar{q}\right)d\bar{P}_0 = \frac{2J_1(2\pi\bar{q}r(f_2/f_1))}{2\pi\bar{q}r(f_2/f_1)}, \tag{21}$$

where J_1 — 1-st kind, 1-st order Bassel function.

Let us introduce a "relative spatial frequency"

$$\bar{\nu} = r\frac{f_2}{f_1}\bar{q}. \tag{22}$$

Then, the contrast dagrading function in reconstruction of Fourier holograms, with an extended incoherent source in form of uniformly radiating disc takes the form:

$$h(\bar{\nu}) = \frac{2J_1(2\pi\bar{\nu})}{2\pi\bar{\nu}}. \tag{23}$$

This function is plotted in fig. 4 and is denote there as "sinusoidal test".

Slightly different results should be expected if a rectangular test is used instead of the sinusoidal one. Such test (Ronchi ruling) is characterized by intensity distribution:

$$I_0(x) = \begin{cases} 1 \text{ for } |x-2Nd| < \dfrac{d}{2}, \\ 0 \text{ for } |x-(2N-1)d| < \dfrac{d}{2}, \end{cases} \quad (N = 0, \pm 1, \pm 2, \ldots). \tag{24}$$

According to (8), (14) and (20) we can write:

$$I(x) = I_0(x) \otimes \mathrm{circ}\left(\frac{\sqrt{x^2+y^2}}{r}\right). \tag{25}$$

Michelson visibility

$$\vartheta = \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} \tag{26}$$

of the reconstructed image (25) acts now as the contrast degrading function. Straightforward calculations show that in this case:

$$\vartheta(\nu) = \frac{2I_M - \pi}{\pi}, \quad (27)$$

I_M is defined in intervals:
for

$$\begin{aligned} &\nu \in [0, 1) \quad I_M = \pi, \\ &\nu \in [1, 3) \quad I_M = \pi - I_1(\nu), \\ &\nu \in [3, 5) \quad I_M = \pi - I_1(\nu) + I_2(\nu), \\ &\nu \in [5, 7) \quad I_M = \pi - I_1(\nu) + I_2(\nu) - I_3(\nu), \end{aligned} \quad (28)$$

where:

$$I_k(\nu) = \pi - 2\arccos\left(\frac{k}{\nu}\right) - \frac{2k}{\nu}\sqrt{1-\left(\frac{k}{\nu}\right)^2} \quad (k = 1, 2, \ldots). \quad (29)$$

The contrast degrading function evaluated in this way is plotted also in fig. 4, and is denoted as "rectangular test".

4.5. Experimental results

Three different holograms have been used for measurements. The spatial frequencies in the tests used were: $\frac{1}{648}\ \mu m^{-1}$, $\frac{1}{725}\ \mu m^{-1}$ and $\frac{1}{788}\ \mu m^{-1}$. Six different diameters of "secondary source" pinholes ($d_1 = 125\ \mu m$, $d_2 = 183\ \mu m$, $d_3 = 211\ \mu m$, $d_4 = 314\ \mu m$, $d_5 = 416\ \mu m$, $d_6 = 589\ \mu m$) allow to obtain almost 20 values of relative frequencies, varying from about 0.21 to 1.24. This corresponds to the most interesting part of the contrast degrading function curve.

Contrast in the coherent reconstruction fluctuates about the value 0.88, due to speckling and hologram inperfections. The values of contrast in images reconstructed with partially coherent light, related to the corresponding values of contrast in images reconstructed with the laser light for different spactial frequencies are plotted in fig. 4 together with the theoretical curves. As it is easily seen the consistency between theoretical shape of the contrast degrading function and the experimental data can be considered to be good.

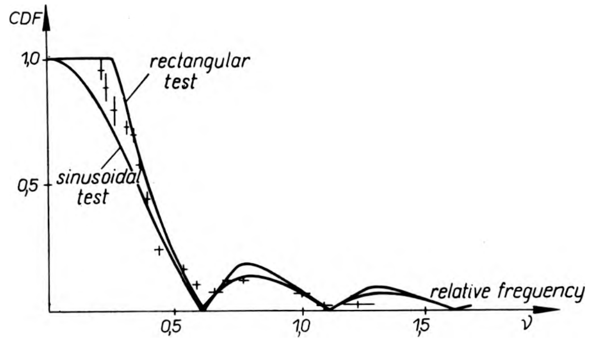

Fig. 4. Contrast degrading function versus relative frequency

5. Conclusions

Theoretical considerations, as well as the experimental results described above, justify the conclusion that it is possible to reconstruct Fourier holograms with partially coherent light, e.g. if the light source used for reconstruction is spatially extended incoherent and quasimonochromatic. This way of reconstruction leads to blurring of the image, that is to degradation of the contrast and possible lowering of the cut-off frequency in reconstructed image. Those effects are described quantitatively by the contrast degrading function which depends only on the light source parameters and the geometry of optical system. Thus the contrast degrading function contains information about permissible shape and dimensions of the light source, that corresponds to the desired cut-off frequency and contrast degradation in the image.

On the other had, it seems that in particular cases the partially coherent reconstruction may be even more advantageous than the coherent one. Laser light causes speckling, being sometimes very ardous in visual observation. The same effects can also result in some errors if measurements are carried out with help of small-size detectors. Sometimes it is advisable to avoid these inconveniencies even at the expense of reasonable loss of resolution and diminishing of the contrast.

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Частично когерентное восстановление Фурье голограмм функция понижения контраста

В работе применена обобщенная теорема Шелла для анализа влияния частичной когерентности света, употребленного для восстановления фурье-голограммы на восстановленное изображение. Показано, что спектр пространственных частот изображения, полученного от такой голограммы путем восстановления частично когерентным светом является произведением спектра пространственных

частот изображения, восстановленного по той же голограмме когерентным светом, и некоторой функции пространственных частот, зависимой от частичной когерентности восстанавливающего пучка. Эта функция, названная функцией понижения контраста, характеризует деградацию изображения, восстановленного по голограмме, вызванную частичной некогерентностью восстанавливающего пучка. Представлены результаты измерения функции понижения контраста в случае, когда для восстановления употреблялась модель плоского кругового однородно светящего квазимонохроматического источника некорентного света.

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