Optica Applicata VII/3, 1977

František Petrů *

A laser measurement system with approximately circularly polarized light**

The principle allowing to obtain two signals in quadrature for a laser interferometer with a single-frequency laser is explained in the introduction. The subject matter of this article includes the problem of light dividing in the laser measurement system for length measurement in more coordinates, further the design of the polarization optics of particular interferometers and the design of the detection unit for laser measurement system determined for measurements of lengths, flatness, dynamic processes etc.

1. Introduction

During several last years laser interferometry has been successfully applied to many branches of measuring techniques, especially in measurement of lengths and speeds in the machinery industry [1]. High precision, large measuring range, high productivity and possibility of automatization of the measuring process with record and data processing are the main advantages of this method. At present one observes an increasing importance of a laser measuring system, since it allows to measure not only lengths and speeds, but also many other geometrical quantities, e.g. angles, flatness, perpendicularity of displacement etc. [2]. The design of a laser measuring system involves solution of many problems in order to obtain the universality of some parts, easy variability and adaptability to specific way of measurement. It is also important that the measured values be in metric units and at immediate disposal after the measurement is accomplished. A simultaneous data processing is also desirable.

The design of the laser measuring system is based on the following main features:

a) only one single-frequency laser is employed as a source of radiation,

b) simple dividers and reflectors are used for the measurement in more coordinates, and for the direction change of the light beam,

c) two signals in quadrature are applied to determine the movement sense,

d) a unified detection unit is employed.

For these reasons the polarization of the entering light and the polarization of the leaving light must be the same for all interferometers involved. To assure a reliable functioning of the equipment it is desirable that each particular interferometer supply both signals in quadrature with high contrast.

For the time being the laser measuring system comprises the remote interferometer for linear measurements, the interferometer for flatness measurements and the interferometer for dynamic measurements. In the future other types will be also developed.

2. Problems involved with laser measurement system design

The attention is to be paid to the problems of polarization optics in order to get two signals in quadrature and obtain the variability and flexibility of the system.

2.1. Obtaining of two signals in quadrature with high contrast

At the laser measurement system, in which two signals in quadrature are employed to determine the movement sense of movable part of the interferometer it is possible to use the approximately circulary polarized light. For the sake of universality it is important that in this case the supplied light of particular interferometers differ as little as possible from the

* Institute of Scientific Instruments, Czechoslovak Academy of Sciences, Brno, Czechoslovakia.

** This paper has been presented at the Third Czechoslovak-Polish Optical Conference in Nove Město, Czechoslovakia, 27 September–1 October 1976.

precisely circularly polarized light. Account must be taken of the real properties of individual parts of interferometer, especially of dividing layers; it is advisable for the deviations from exactly circularly polarized light to be caused by production reproducibility of elements. In this case for any interferometer a sufficient contrast for both signals in quadrature can be obtained from the universal detection unit with fixed set polarization filters.

2.2. Dividing of light

In a multiple axis coordinate system the laser beam before entering interferometer should be divided into corresponding number of coordinates by means of the light dividers. The light dividers, e.g. for the reflected light, possess the properties expressed by matrix:

$$\begin{bmatrix} \sqrt{R_\perp}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & \sqrt{R_\parallel}\exp\left(-i\frac{\delta_2}{2}\right) \end{bmatrix}.$$

By dividing the circularly polarized light, e.g. of the type

$$\varepsilon_1 = E_1\begin{bmatrix} -i \\ 1 \end{bmatrix}$$

after reflection we get

$$\varepsilon_2 = E_1\begin{bmatrix} \sqrt{R_\perp}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & R_\parallel\exp\left(-i\frac{\delta_2}{2}\right) \end{bmatrix}\begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$= E_1\exp\left(-i\frac{\delta_2}{2}\right)\begin{bmatrix} \sqrt{R_\perp}\exp\left(-i\left(\frac{\delta}{2}-\delta_2\right)\right) \\ \sqrt{R_\parallel} \end{bmatrix}.$$

Note: Amplitude transformation of p- and s-components of the polarized light will be presented in a simple matrix expression, according to R. C. JONES [3, 4, 5, 6]. In the assumed orthogonal coordinate system x, y, z the z-axis lies in the direction of the beam travelling, and the y-axis is located in the plane of incidence.

where:

$R_\perp$ – reflectivity od dividing layer for radiation with vibration plane in the plane of incidence,

$R_\parallel$ – reflectivity of dividing layer for radiation with vibration plane perpendicular to the plane of incidence,

δ_2 – difference of the phase shift of the dividing layer for reflection $\delta_2 = \delta_\perp - \delta_\parallel$,

E_1 – electric field vector of the entering light, polarized linearly in the vibration plane yz.

In general case $R_\perp \neq R_\parallel$ and $\delta_2 \neq 0$; see e.g. [7].

The result shows that the form of polarization is changed after reflection.

During further reflections the elipticity will be still changing, unless the correction, which is directly bound on the specific design of the divider, is used.

Everything is simplier when using the light linearly polarized in one of basic directions, e.g. for the case

$$\varepsilon_1 = E_1\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

we get after reflection

$$\varepsilon_2 = E_1\begin{bmatrix} \sqrt{R_\perp}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & \sqrt{R_\parallel}\exp\left(-i\frac{\delta_2}{2}\right) \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= E_1\sqrt{R_\parallel}\exp\left(-i\frac{\delta_2}{2}\right)\begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The form of polarization is preserved at any number of reflections in basic directions. That is why a linearly polarized light is employed.

3. A laser measurement system with approximately circularly polarized light

It is known that in the case of interference two plane waves of the same frequency and intensity, polarized in the same vibration plane, yield the maximum interference signal, i.e. interference fringes are produced with zero minimum in the interference pattern; this means that the contrast given by the formula

$$c = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

is equal to unity; this is the maximum contrast.

However, the sense of movement of the movable part of the interferometer cannot be determined.

3.1. Interference of two circularly polarized beams of the same handedness

Using the circularly polarized light of the same intensity and the same handedness we get conditions similar to those of linearly polarized light.

Let us suppose that the first circularly polarized beam is given by:

$$E_{1x} = A_1 \sin \omega t; \; E_{1y} = A_1 \cos \omega t,$$

where ω – angular frequency, t – time, ψ – phase shift, A_1 – amplitude.

Analogically, the second circularly polarized beam is expressed by:

$$E_{2x} = A_1 \sin(\omega t + \psi); \; E_{2y} = A_1 \cos(\omega t + \psi).$$

Adding the instantaneous amplitudes of the same vibration direction we have:

$$E_{xc} = 2A_1 \cos\frac{\psi}{2} \sin\left(\omega t + \frac{\psi}{2}\right),$$

$$E_{yc} = 2A_1 \cos\frac{\psi}{2} \cos\left(\omega t + \frac{\psi}{2}\right).$$

After the interference without analyser (polarization filter), but before entering the detector the resulting intensity is

$$I_c = 4A_1^2 \cos^2\frac{\psi}{2} = 2A_1^2(1+\cos\psi).$$

It is evident that we have the maximum interference signal or maximum contrast. The intensity of the light leaving the interferometer changes from zero to $4A_1^2$ depending on the phase shift ψ.

When the analyser is inserted before entry in the detector with orientation given by the azimuth β, then the electric field vector $\boldsymbol{E}_x$ is projected in the direction β by the factor $\cos\beta$, and the electric vector $\boldsymbol{E}_y$ by the factor $\sin\beta$.

Behind the analyser we have:

$$E_{\beta c} = 2A_1 \cos\frac{\psi}{2}\left[\sin\left(\omega t + \frac{\psi}{2}\right)\cos\beta + \cos\left(\omega t + \frac{\psi}{2}\right)\sin\beta\right]$$

$$= 2A_1 \cos\frac{\psi}{2} \sin\left(\omega t + \frac{\psi}{2} + \beta\right).$$

The resulting intensity of the signal is given by the expression:

$$I_{\beta c} = 2A_1^2 \cos^2\frac{\psi}{2} = A_1^2(1+\cos\psi);$$

it changes from zero to $2A_1^2$ depending on the phase shift ψ, but it does not depend on the azimuth β of the analyser. Hence it follows that the choice of azimuth does not make it possible to get two signals in quadrature. The way of cho sing the intensities I_x and I_y by means of the analysers, and transmission of the corresponding signals to x and y plates of an osciloscope is shown in fig. 1. It is obvious that both the signals are in phase.

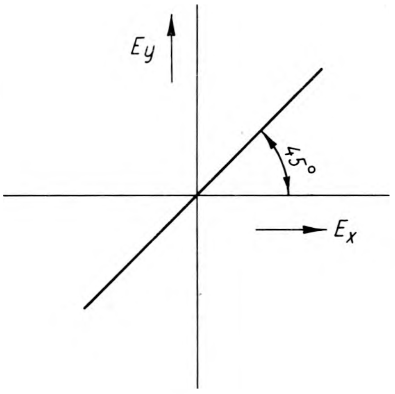

Fig. 1. Interference of two circulary polarized beams of the same handedness and intensity, with an analyser inserted in front of each detector

The interference of two circularly polarized beams of the same handedness and the same intensity, without an analyser before entry in the detector, is presented in fig. 2. A nearly

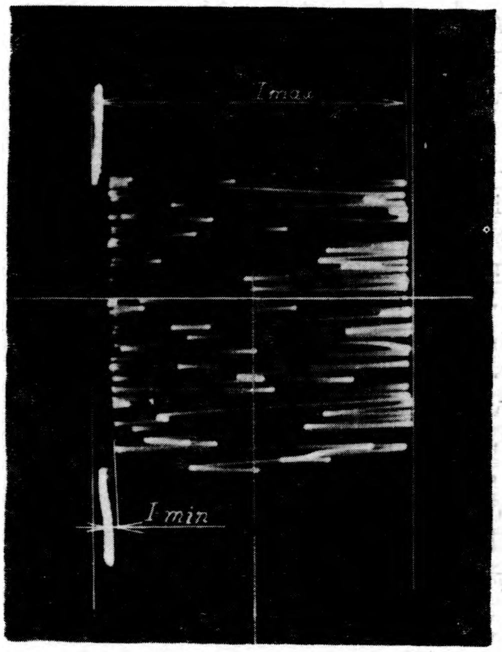

Fig. 2. Interference of two circularly polarized beams of the same handedness and intensity, without the analyser before the detector entry

maximum contrast ($c > 0.95$) (a little deviation from unity is caused mainly by unsufficient setting of wavefronts of both interferring beams) confirms the interference of two circularly polarized beams; since in the case of elliptic polarization with various azimuths or ellipticity the maximum contrast would not be obtained.

3.2. Principle of obtaining two signals in quadrature

Using the circularly polarized light of the same intensity and opposite handedness we get for the first beam:

$$E_{1x} = A_1 \sin \omega t; \; E_{1y} = A_1 \cos\omega t,$$

and for the second beam:

$$E_{2x} = A_1 \sin(\omega t + \psi); \; E_{2y} = -A_1 \cos(\omega t + \psi).$$

The sum of instantaneous amplitudes of the same vibration direction gives:

$$E_{xc} = 2A_1 \cos\frac{\psi}{2} \sin\left(\omega t + \frac{\psi}{2}\right),$$

$$E_{yc} = 2A_1 \sin\frac{\psi}{2} \sin\left(\omega t + \frac{\psi}{2}\right).$$

The resulting total intensity detected without an analyser is:

$$I_c = 2A_1^2\left(\cos^2\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right) = 2A_1^2 = \text{const}.$$

We get a D. C. signal or a zero contrast. This case is shown in fig. 3, where a little residual contrast ($\Delta c \sim \Delta I$) is caused by little deviations of the intensities, or by slight ellipticity of both beams.

By dividing the output beam of the interferometer into two beams, and inserting in their parts one analyser at a time two signals with maximum contrast are obtained.

We assume the orientation of the analyser with the azimuth β and determine the projections of the electric field vectors in the direction β. The sum of both components is:

$$E_{\beta c} = E_{xc}\cos\beta + E_{yc}\sin\beta = 2A_1\left[\cos\frac{\psi}{2}\cos\beta + \sin\frac{\psi}{2}\sin\beta\right],$$

$$\sin\left(\omega t + \frac{\psi}{2}\right) = 2A_1 \cos\left(\frac{\psi}{2} - \beta\right)\sin\left(\omega t + \frac{\psi}{2}\right),$$

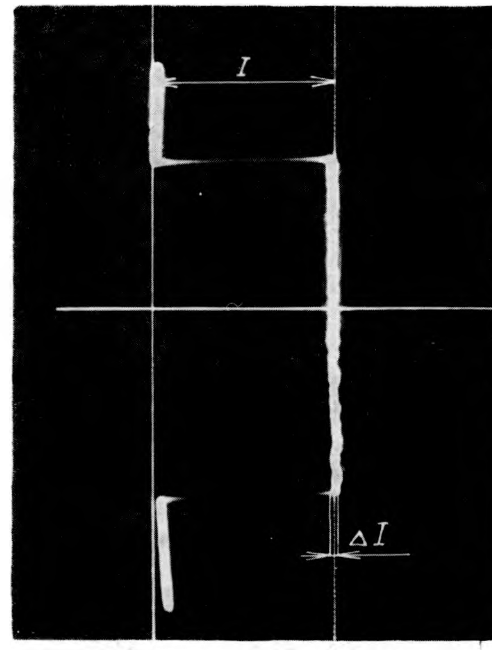

Fig. 3. Interference of two circularly polarized beams of the opposite handedness and the same intensity, without the analyser before the detector entry

and the intensity:

$$I_{\beta c} = 2A_1^2 \cos^2\left(\frac{\psi}{2} - \beta\right) = A_1^2[1 + \cos(\psi - 2\beta)].$$

Choosing, by the aid of analysers, the vibration directions we get for $\beta = 0$ and $\beta = \pi/4$:

$$I_{0c} = A_1^2[1 + \cos\psi],$$

$$I_{\pi/4c} = A_1^2\left[1 + \cos\left(\psi - \frac{\pi}{2}\right)\right] = A_1^2[1 + \sin\psi].$$

It is seen that both interference signals have the maximum contrast and are in quadrature.

The principles introduced above are known, their precise realization is however rather difficult. The light in the interferometer must be first split into two beams: reference and measuring beams, and then recombined. Besides the dividing element the interferometer involves also other optical components, which possess certain polarization properties. By respecting all the possible effects of these components, and taking account of real properties of particular elements it is possible to design an interferometer, which fulfills the necessary requirements.

3.3. The first design of the polarization optics of the interferometer

Requirements:

1. Circularly polarized light of the reference beam on the output of the interferometer.

2. The intensity of the reference beam $I_r = 1/4\, I_1$.

3. Circularly polarized light of the measuring beam at the output of the interferometer with opposite handedness with respect to the reference beam.

4. Approximately the same (or slightly higher) intensity of the measuring beam.

On the entrance of interferometer, there is a linear retardation element RP_1 (a phase retardation plate), with a retardation δ_1, its fast axis being oriented at the angle θ_1 to the axis x. This phase retardation plate distributes the intensities and assures the circularly polarized light of the reference beam with suitable intensity at the output of the interferometer.

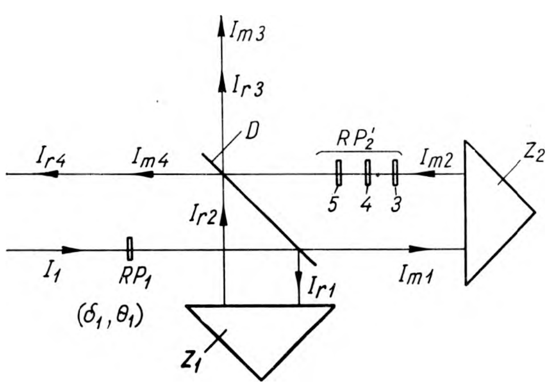

Fig. 4. First design of the interferometer presented schematically

A scheme of such an interferometer is presented in fig. 4. The linearly polarized light from a single-frequency laser I_1, after being expanded goes through a linear phase retardation plate RP_1, falls on the dividing layer D, where it becomes split into reference I_{r1} and measuring I_{m1} beams. The reference beam I_{r1}, after being reflected by the retroreflector Z_1, goes back as a beam I_{r2}, paralled to the beam I_{rl}, and falls again on the dividing layer D. It is partially reflected back constituting the output reference beam I_{r4}. A similar measuring beam I_{m1}, after being reflected by the retroreflector Z_2, passes through the retardation element RP_2, it partially passes again the dividing layer D, and at the interferometer output it represents the measuring beam I_{m4}. The beams I_{r3} and I_{m3} interfere and can be used to a visual checking of interferometer setting. Unified beams I_{r4} and I_{m4} also interfere yielding the output from the interferometer.

For the reference beam it has been assumed that the properties of optical parts of the interferometer given in matrix form are the following:

For the dividing layer (divider) D at twofold reflection:

$$\begin{bmatrix} R_{\perp}\exp(i\delta_2) & 0 \\ 0 & R_{\parallel}\exp(-i\delta_2) \end{bmatrix}.$$

For the linear phase retardation plate RP_1:

$$\begin{bmatrix} C_1^2\exp\left(i\dfrac{\delta_1}{2}\right)+S_1^2\exp\left(-i\dfrac{\delta_1}{2}\right) & C_1S_12i\sin\dfrac{\delta_1}{2} \\ C_1S_12i\sin\dfrac{\delta_1}{2} & C_1^2\exp\left(-i\dfrac{\delta_1}{2}\right)+S_1^2\exp\left(i\dfrac{\delta_1}{2}\right) \end{bmatrix},$$

where

$$C_1 = \cos\theta_1 \text{ and } S_1 = \sin\theta_1.$$

Little influence of the retroreflector on the polarization properties is neglected [7].

There is a symmetrical arrangement of the interferometer with two retroreflectors, each of them in one path, so that the lasers in both paths are the same and in fact do not influence the result of calculation. The polarization properties of retroreflectors can be neglected due to special coatings deposited on them. The measurement results of the optical elements are submitted for publication [7].

At the output of the interferometer, when requirements 1. and 2. are satisfied, we have for the reference beam:

$$\varepsilon_r = \begin{bmatrix} R_{\perp}\exp(-i\delta_2) & 0 \\ 0 & R_{\parallel}\exp(-i\delta_2) \end{bmatrix} \times$$

$$\times \begin{bmatrix} C_1^2\exp\left(i\dfrac{\delta_1}{2}\right)+S_1^2\exp\left(-i\dfrac{\delta_1}{2}\right) & C_1S_12i\sin\dfrac{\delta_1}{2} \\ C_1S_12i\sin\dfrac{\delta_1}{2} & C_1^2\exp\left(-i\dfrac{\delta_1}{2}\right)+S_1^2\exp\left(i\dfrac{\delta_1}{2}\right) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ E_1 \end{bmatrix} = \begin{bmatrix} i \\ 1 \end{bmatrix}\exp(i)\varkappa k E_1.$$

For $\varkappa = \delta_2$ and $k = \dfrac{\sqrt{2}}{4}$ we get:

$$R_{\perp} = \frac{\sqrt{2}}{8}\frac{1}{C_1 S_1 \sin\frac{\delta_1}{2}},$$

$$R_{\parallel} = \frac{\sqrt{2}}{4}\frac{\sin\left(\frac{\delta_1}{2}+2\delta_2\right)}{S_1^2 \sin\delta_1},$$

$$\tan\theta_1 = \sqrt{\frac{\sin\left(\frac{\delta_1}{2}+2\delta_2\right)}{\sin\left(\frac{\delta_1}{2}-2\delta_2\right)}}.$$

For practical values of the dividing layers the results were processed on a table calculator *HP*, and an example of computed and registrated graph is shown in fig. 5. This graph allows to determine the phase shift δ_1 of the phase retardation plate RP_1, and the angle of its azimuth θ_1 for the given properties of the layers $R_{\perp}$ and $R_{\parallel}$ and δ_2. The phase retardation plate of these properties assures at the output the circular polarization of the reference beam of a desired intensity.

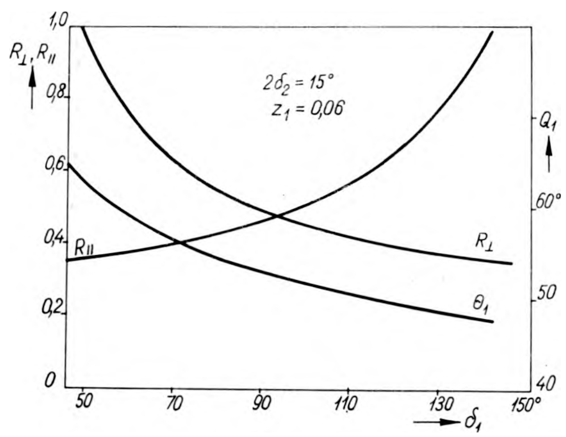

Fig. 5. Graph determining the values δ_1 and θ_1 of the first phase retardation plate RP_1 depending on the dividing layers properties $2\delta_2$, $R_{\parallel}$, $R_{\perp}$, Z_1

The first design of the polarization optics of the interferometer represents an approximative solution, in which the suitable properties of the divider D were specified by a step method with the use of graphs. In this case e.g.

$$R_{\parallel} = 0.40;\ R_{\perp} = 0.54;\ 2\delta_2 = 15°.$$

Then the properties of RP_1 are:

$$\delta_1 = 78°,\ \theta_1 = 55°.$$

The measuring beam

A solution by the aid of there linear retardation elements is proposed. The properties of particular elements of the measuring path:

The dividing layer D for a single transmission:

$$\begin{bmatrix} T_{\perp}^{1/2}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & T_{\parallel}^{1/2}\exp\left(-i\frac{\delta_2}{2}\right)\end{bmatrix}$$

The linear phase retardation plate No. 3:

$$\begin{bmatrix} \exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & \exp\left(-i\frac{\delta_2}{2}\right)\end{bmatrix}.$$

The linear phase retardation plate No. 4:

$$\frac{1}{2}\begin{bmatrix} 2\cos\frac{\delta_4}{2} & 2i\sin\frac{\delta_4}{2} \\ 2i\sin\frac{\delta_4}{2} & 2\cos\frac{\delta_4}{2}\end{bmatrix}.$$

The linear phase retardation plate No. 5:

$$\begin{bmatrix} \exp\left(-i\frac{\delta_2}{2}\right) & 0 \\ 0 & \exp\left(i\frac{\delta_2}{2}\right)\end{bmatrix}.$$

Jones vector of the light beam behind the phase retardation plate RP_1:

$$\varepsilon_1 = \frac{\sqrt{2}}{4}E_1\exp(i\delta_2)\begin{bmatrix} i\frac{1}{R_{\perp}}\exp(-i\delta_2) \\ \frac{1}{R_{\parallel}}\exp(i\delta_2)\end{bmatrix}.$$

At the output of the interferometer we have for the measuring beam:

$$\varepsilon_m = \begin{bmatrix} T_{\perp}^{1/2}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & T_{\parallel}^{1/2}\exp\left(-i\frac{\delta_2}{2}\right)\end{bmatrix}\times$$

$$\times\begin{bmatrix} \exp\left(-i\frac{\delta_2}{2}\right) & 0 \\ 0 & \exp\left(i\frac{\delta_2}{2}\right)\end{bmatrix}\times$$

$$\times\begin{bmatrix} 2\cos\frac{\delta_4}{2} & 2i\sin\frac{\delta_4}{2} \\ 2i\sin\frac{\delta_4}{2} & 2\cos\frac{\delta_4}{2}\end{bmatrix}\begin{bmatrix}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & \exp\left(-i\frac{\delta_2}{2}\right)\end{bmatrix}\times$$

$$\times\begin{bmatrix} T_{\perp}^{1/2}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & T_{\parallel}^{1/2}\exp\left(-i\frac{\delta_2}{2}\right)\end{bmatrix}\times$$

$$\times\begin{bmatrix} i\frac{1}{R_{\perp}}\exp(-i\delta_2) \\ \frac{1}{R_{\parallel}}\exp(i\delta_2)\end{bmatrix}\times$$

$$\times\frac{\sqrt{2}}{8}E_1\exp(i\delta_2) = \frac{\sqrt{2}}{4}E_1\exp(i\delta_2)\begin{bmatrix}1\\ i\end{bmatrix}.$$

The solution results in:

$$\tan\frac{\delta_4}{2} = \frac{1}{\sqrt{T_{\parallel}T_{\perp}}}\,\frac{T_{\parallel}R_{\perp}-T_{\perp}R_{\parallel}}{R_{\perp}+R_{\parallel}}.$$

Three linear retardation elements RP_2 in the measuring beam can be replaced by one non-linear retarding elements RP_2' with a matrix:

$$\begin{bmatrix} C_r^2\exp\left(-i\frac{\delta}{2}\right)+S_r^2\exp\left(-i\frac{\delta}{2}\right) \\ C_rS_r\left(2i\sin\frac{\delta}{2}\right)\exp(-i\gamma) \\ C_rS_r\left(2i\sin\frac{\delta}{2}\right)\exp(i\gamma) \\ C_r^2\exp\left(i\frac{\delta}{2}\right)+S_r^2\exp\left(i\frac{\delta}{2}\right)\end{bmatrix}$$

having the following values:

$$\delta = \delta_4,\ \gamma = \delta_2,\ C_r = S_r = \frac{\sqrt{2}}{2}.$$

3.4. The second design of the polarization optics of the interferometer

A scheme of the interferometer arrangement is shown in fig. 6. At the input of the interferometer there is the linear phase retardation plate RP_1, the plate with dividing layer D serves as a beam divisor, two retroreflectors Z_1 and Z_2 are employed for retroreflection of the reference and measuring beams, respectively. The phase retardation plate RP_2 is inserted in the path of the reference beam, RP_3 and RP_4 being inserted in the paths of the measuring beams. It is assumed that the difference between the phase shifts of the dividing plate for reflection is δ_2.

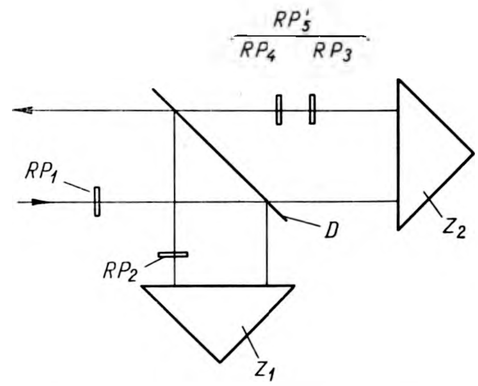

Fig. 6. Second design of the interferometer presented schematically

Assuming that the phase retardation δ, and orientation θ of the phase retardation plates, are:

for RP_1:

$$\delta_1 = \frac{\pi}{2};\ \theta_1 = \frac{\pi}{4};$$

for RP_2:

$$\delta_{20} = \pi;\ \theta_{20} = \frac{\pi}{4};$$

and neglecting the little influence of the polarization properties of the retroreflector we can write for the reference beam:

$$\varepsilon_r = \begin{bmatrix}\sqrt{R_{\perp}}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & \sqrt{R_{\parallel}}\exp\left(-i\frac{\delta_2}{2}\right)\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\times$$

$$\times\begin{bmatrix}\sqrt{R_{\perp}}\exp\left(i\frac{\delta_2}{2}\right) & 0 \\ 0 & \sqrt{R_{\parallel}}\exp\left(-i\frac{\delta_2}{2}\right)\end{bmatrix}\frac{1}{\sqrt{2}}$$

$$\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}0\\ E_1\end{bmatrix} = \sqrt{\frac{R_{\perp}R_{\parallel}}{2}}E_1\begin{bmatrix}1\\ i\end{bmatrix}.$$

Similarly, we can derive the expression for the measuring beam. On the assumption that the phase retardation δ and orientation θ of the phase retardation plates are

for RP_3:

$$\delta_3 = \pi;\ \theta_3 = \frac{\pi}{4},$$

for RP_4:

$$\delta_4 = \pi;\ \theta_4 = 0,$$

and the difference of the phase shifts of the dividing plate for transmission is δ_2', we can write:

$$\varepsilon_m = \begin{bmatrix} \sqrt{T_\perp}\exp\left(i\frac{\delta_2'}{2}\right) & 0 \\ 0 & \sqrt{T_\parallel}\exp\left(-i\frac{\delta_2}{2}\right) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times$$

$$\times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{T_\perp}\exp\left(\frac{\delta_2}{2}\right) & 0 \\ 0 & \sqrt{T_\parallel}\exp\left(-i\frac{\delta_2}{2}\right) \end{bmatrix}$$

$$\frac{E_1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix} = \sqrt{\frac{T_\perp T_\parallel}{2}}\, E_1 \begin{bmatrix} i \\ 1 \end{bmatrix}.$$

It is evident that this solution yields in both beams the circularly polarized light of the opposite handedness. If in both beams the same intensity is desired, the following condition must be fulfilled:

$$R_\perp R_\parallel = T_\perp T_\parallel.$$

As the realization of an ideal dividing layer without polarizing effects, i.e. $R_\perp = T_\perp$; $R_\parallel = T_\parallel$, is very difficult, it is sufficient that

$$R_\perp = T_\parallel;\ R_\parallel = T_\perp$$

hold.

The dividing layers which satisfy the stated conditions were developed for our application in research laboratories of MEOPTA WORKS Prerov.

The problems of the dividers with small polarization effects at the oblique incidence are treated in [8].

Two linear retardation elements RP_3 and RP_4 can be replaced by a single rotation element RP_5 with the matrix

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

i.e. by values:

$$\delta = \pi;\ \xi = \pm 90°.$$

The values obtained from an experimental interferometer arrangement are shown in figs. 7 and 8. Assuming that the beam entering the interferometer is precisely circularly polarized the outgoing light of the reference beam

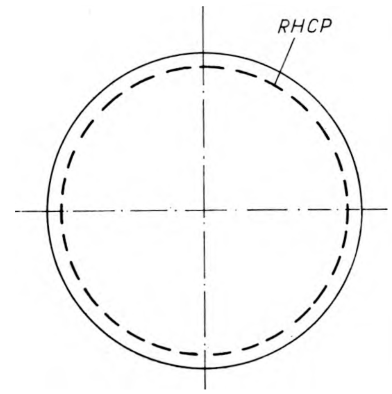

Fig. 7. Form of the nearly circularly polarized light of the reference beam [*RHCP*] outgoing from the experimental interferometer

presented in fig. 7 is approximately circularly polarized with deviation ~2%. Fig. 8 presents the form of the outgoing measuring beam; its deviation is ~3%. The total intensities of reference and measuring beams were the same,

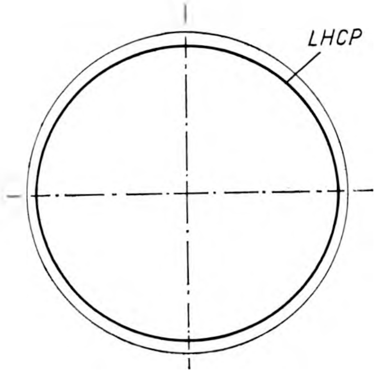

Fig. 8. Form of the nearly circularly polarized light of the measuring of the beam [*LMCP*] outgoing from the experimental interferometer

the differences were 2%. This deviation was mainly caused by unequal total reflectance of the retroreflector. When the intensities were equalized, the maximum contrast of the interference phenomenon could be obtained. From the theoretical polarization optics standpoint there are the preconditions for obtaining the maximum contrast ($c = 1$). In order to obtain two signals in quadrature, polarization filters were employed. Both signals were transmitted to the osciloscope: one to x-plate, and the second to y-plate. On the display a circle appears; the contrast can be determined in both channels by using the graph and reading the

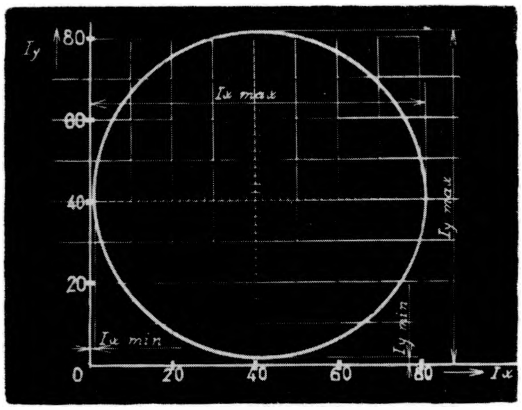

Fig. 9. Contrast of two signals in quadrature obtained at the output of the experimental interferometer: *LI* — laser interferometer; *1, 2, 3* ... dividing layers, *4, 5, 6, 7* ... polarization filters; *8, 9, 10, 11* ... detectors

minimum and maximum values of the signal. As seen from fig. 9, the following values have been obtained:

$$c_x = 0.98 \quad \text{and} \quad c_y = 0.97.$$

The contrast approaches really the unity, the deviations are due to inaccuracy of coincidence of the wavefronts of both beams.

3.5. Universal detection unit

All the types of interferometers are supposed to be equipped with a built-in or a remote universal detection unit. In all cases the beam entering the detection unit has the same form of the circularly polarized light, which is the unified output from all interferometer types.

The detection unit should yield: 1) at last two signals in quadrature, and 2) the signal for signalization of beam interruption.

This detection unit when suitably designed may, moreover, enable the elimination of *DC* component of the signal and allow independent function at a low contrast. To obtain at least three $(2+1)$ signals of different properties the beam must be divided again. But in contradiction to the dividers used for the beam splitting in a greater number of coordinates, the circularly polarized light of the reference and measuring beams of opposite handedness must be divided.

A layout of the detection unit arrangement is shown in fig. 10. The usage of a "symmetric dividing layer" with properties: $R_\perp = T_\parallel$; $R_\parallel = T_\perp$; $\delta_\perp - \delta_\parallel = \sigma_\perp - \sigma_\parallel = \delta_2$ — where $\delta_\perp - \delta_\parallel$ is the phase shift for reflection and $\sigma_\perp - \sigma_\parallel$ is the phase shift for transmission — is supposed.

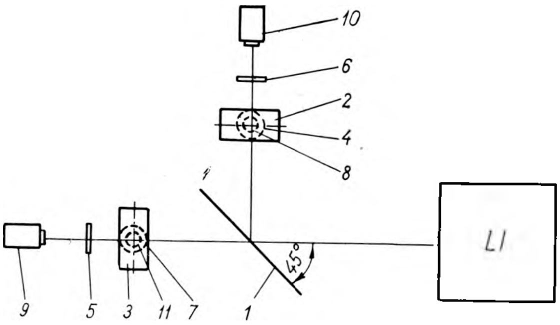

Fig. 10. Scheme of the detection unit

The situation for detectors 8 and 9 before incidence on the polarization filters is as follows:

For the right circularly polarized light — let it be e.g. the reference beam coming from the interferometer *LI*:

$$\varepsilon_{r1} = E_1 k_4 \frac{\sqrt{2}}{2} \begin{bmatrix} -i \\ 1 \end{bmatrix},$$

where k_4 is a constant, we have:

$$\varepsilon_{r8} = \varepsilon_{r9} = \begin{bmatrix} R_\perp^{1/2} \exp\left(i\,\frac{\delta}{2}\right) & 0 \\ 0 & R_\parallel^{1/2} \exp\left(-i\,\frac{\delta}{2}\right) \end{bmatrix} \times$$

$$\times \begin{bmatrix} R_\parallel^{1/2} \exp\left(-i\,\frac{\delta}{2}\right) & 0 \\ 0 & R_\perp^{1/2} \exp\left(i\,\frac{\delta}{2}\right) \end{bmatrix}$$

$$E_1 k_4 \frac{\sqrt{2}}{2} \begin{bmatrix} -i \\ 1 \end{bmatrix} = E_1 k_4 \frac{\sqrt{2}}{2} \sqrt{R_\perp R_\parallel} \begin{bmatrix} -i \\ 1 \end{bmatrix}.$$

It can be seen from the result that a perfect transformation of the type and form of polarization is accomplished only when the intensity is reduced. Thus in the detectors 8 and 9 neither type nor form of polarization is changed at splitting the entering light. The same is valid for the left circularly polarized light — say e.g. for the measuring beam. Consequently, for the detectors 8 and 9 the preconditions for preserving the maximum contrast of the interference signal have been created.

The situation for detectors 10 and 11 before incidence on the polarization filters is as follows:

For the right circularly polarized light — let it be e.g. the reference beam — we have:

$$\varepsilon_{r10} = \varepsilon_{r11} = \begin{bmatrix} T_{\parallel}^{1/2}\exp\left(-i\frac{\delta}{2}\right) & 0 \\ 0 & T_{\perp}^{1/2}\exp\left(i\frac{\delta}{2}\right) \end{bmatrix} \times$$

$$\times \begin{bmatrix} R_{\perp}^{1/2}\exp\left(i\frac{\delta}{2}\right) & 0 \\ 0 & R_{\parallel}^{1/2}\exp\left(-i\frac{\delta}{2}\right) \end{bmatrix},$$

$$E_1 k_4 \frac{\sqrt{2}}{2}\begin{bmatrix} -i \\ 1 \end{bmatrix} = E_1 k_4 \frac{\sqrt{2}}{2}\begin{bmatrix} -iR \\ R \end{bmatrix}.$$

For the left circularly polarized light — let it e.g. the measuring beam — we have:

$$\varepsilon_{m10} = \varepsilon_{m11} = \begin{bmatrix} T_{\parallel}^{1/2}\exp\left(-i\frac{\delta}{2}\right) & 0 \\ 0 & T_{\perp}^{1/2}\exp\left(i\frac{\delta}{2}\right) \end{bmatrix} \times$$

$$\times \begin{bmatrix} R_{\perp}^{1/2}\exp\left(i\frac{\delta}{2}\right) & 0 \\ 0 & R_{\parallel}^{1/2}\exp\left(-i\frac{\delta}{2}\right) \end{bmatrix},$$

$$E_1 k_4 \frac{\sqrt{2}}{2}\begin{bmatrix} i \\ 1 \end{bmatrix} = E_1 k_4 \frac{\sqrt{2}}{2}\begin{bmatrix} i R_{\perp} \\ R_{\parallel} \end{bmatrix}.$$

It is evident that both the (right and left polarized) beams have at the output (thus before entering the filters of the detectors 10 and 11) the same ellipticity, azimuth and intensity of the elliptic polarized light. The preconditions for obtaining the maximum contrast in the interference signal are also given.

The experimental checking of the dividing system of the detection unit is shown in figs. 11 and 12. The circular polarization and the same intensity for left and right circularly polarized reference and measuring beams before entering the polarization filters of the detectors 8 and 9 are shown in fig. 11.

Deviations from the precise circularity are due to unhomogeneity of the layers, as well as to deviations from both symmetry and the angle 45°, respectively. Elliptically polarized light with the same ellipticity, azimuth and intensity for the reference left polarized beam and the measuring right polarized beam in

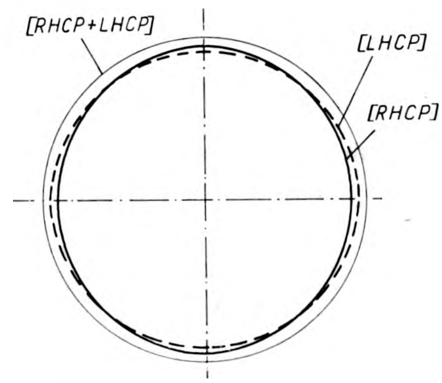

Fig. 11. Experimental checking of the dividing system of the detection unit for the detectors *8* and *9*. $[RHCP+LHCP]_1$ — entering right and left circularly polarized light, $[RHCP]_2$, $[LHCP]_2$ — outgoing right and left circularly polarized light

detectors 10 and 11 are presented in fig. 12. $[RHCP+LHCP]_1$ is the designation of the input right and left circularly polarized beams $[RHCP]_2$, $[LHCP]_2$ is the designation of the corresponding output beams.

The proposed solution makes it also possible to eliminate the *DC* component by use of two pairs of detectors, e.g. 8+9 and 10+11 with polarization filters turned geometrically to make

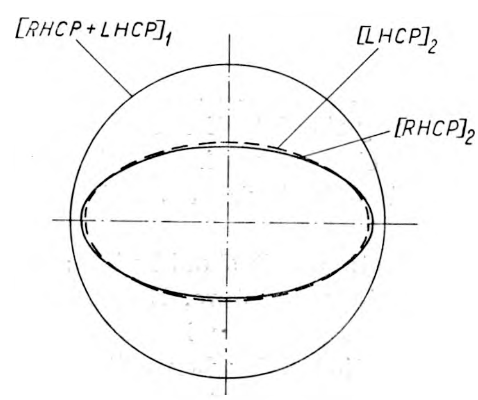

Fig. 12. Experimental checking of the dividing system of the detection unit for the detectors *10* and *11*. $[RHCP+LHCP]_1$ — entering right left circularly polarized light, $[RHPC]_2$, $[LHCP]_2$ — outgoing right and left elliptically polarized light

90°. Hence, the interference signals coming from the detectors 8+9 and 10+11 are in opposite phases and the following differential amplif er suppresses (do not amplify) the *DC* component. It is possible to modify the configuration of dividing in one plane.

Fig. 13 shows the arrangement for elimination of the reference beam. In this arrangement only two detectors, e.g. 9 and 10 are used for two signals in quadrature, while the dividing plate 2 is omitted. The beam, after being reflected from the dividing plate 1 and transmitted through the linear phase retardation plate 2 and the polarization filter 3, falls on the detector 4 for signalization of the beam interruption.

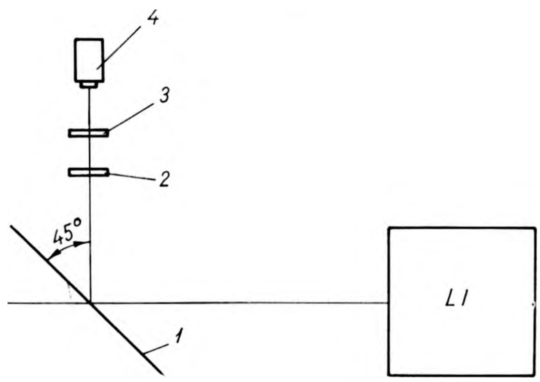

Fig. 13. Layout of the arrangement for elimination of the reference beam as a part of the system for signalization of the beam interruption

The reference beam, after reflection from the dividing plate 1 and transmission through the linear phase retardation plate 2 with retardation ϱ can be described by

$$\varepsilon_{r2} = \begin{bmatrix} \exp\left(\frac{i\varrho}{2}\right) & 0 \\ 0 & \exp\left(-i\frac{\varrho}{2}\right) \end{bmatrix} \times$$

$$\times \begin{bmatrix} R_{\perp}^{1/2}\exp\left(i\frac{\delta}{2}\right) & 0 \\ 0 & R_{\parallel}^{1/2}\exp\left(-i\frac{\delta}{2}\right) \end{bmatrix} E_1 k_4 \frac{\sqrt{2}}{2} \begin{bmatrix} -i \\ 1 \end{bmatrix}.$$

Assuming:

$$\delta + \varrho = \frac{\pi}{2}$$

we have:

$$\varepsilon'_{r2} = E_1 k_4 \frac{\sqrt{2}}{2} \exp\left(-i\frac{\pi}{4}\right) \begin{bmatrix} \sqrt{R_{\perp}} \\ \sqrt{R_{\parallel}} \end{bmatrix},$$

i.e. a linearly polarized light, which can be eliminated by the filter 3.

The reference beam behind the filter is:

$$\varepsilon_{r3} = E_1 k_4 \frac{\sqrt{2}}{2} \exp\left(-i\frac{\pi}{4}\right) \begin{bmatrix} C_1^2 & C_1 S_1 \\ C_1 S_1 & S_1^2 \end{bmatrix} \begin{bmatrix} \sqrt{R_{\perp}} \\ \sqrt{R_{\parallel}} \end{bmatrix}.$$

Assuming that for the setting of the filter angle η_a is valid:

$$-\tan\eta_1 = \sqrt{\frac{R_{\perp}}{R_{\parallel}}}$$

we get

$$\varepsilon_{r3} = 0.$$

Thus the reference beam is eliminated.

Then the measuring beam is given by

$$\varepsilon_{m3} = E_1 k_5 \exp\left(-i\frac{\pi}{4}\right) \frac{\sqrt{R_{\perp} R_{\parallel}}}{R_{\perp} + R_{\parallel}} \begin{bmatrix} \sqrt{R_{\parallel}} \\ \sqrt{R_{\perp}} \end{bmatrix},$$

and the intensity of the measuring beam behind the filter 3 is:

$$I_{m3} = \frac{1}{2} E_1^2 k_5^2 \frac{R_{\perp} R_{\parallel}}{R_{\perp} + R_{\parallel}}.$$

In this way we obtain a very reliable possibility of checking the measuring beam interruption.

Acknowledgement

At the end of this paper the author would like to emphasize that the solution of the presented problems of the laser measurement system was possible through the kindness of many persons and Institutions. The author highly appreciates the cooperation with the Laboratory of thin films led by Dr. Zdeněk Knittl CSc., to whome the author is also indebted for careful reading and correction of the manuscript, further the cooperation with working group of Monokrystaly Turnov led by Dipl. Ing. Zdeněk Cuchý CSc., and the working group ČSAV Turnov led by Dr. Ivan Šolc CSc. They all have worked on the elements of the polarization optics. In the laser measurement system development, measurement of respective elements and realization many workers in the Division of Quantum Electronics of the Institute of Scientific Instruments ČSAV have taken their part. The author wishes to express his gratitude to all of them.

Лазерная измерительная система с поляризованным светом в приближении по кругу

Во вступительной части объясняется принцип, дающий возможность получить два сигнала в квадратуре для целей лазерной интерферометрии с лазером единичной частоты. В этой статье затронуты следующие вопросы: проблема деления света в лазерных измерительных системах для

измерения длины, конструкция поляризационной оптики в некоторых типах интерферометров, а также конструкция комплекта детектирования для лазерных систем, предназначенных для измерения длины, плоскостнсти и динамических процессов.

References

[1] Petrů F., Popela B., Kršek J., Stejskal A., *Univerzální laserový interferometr*, Jemná Mechanika a Optica XVIII, 1973, 10, pp. 261–269.

[2] Baldwin R. R., Gordon G. B., Rudé A. F., *Remote Laser Interferometry*, Hewlett-Packard Journal **23**, 1971, 4, pp. 14–19.

[3] Jones R. C., *A New Calculus for the Treatment of Optical Systems*, I. *Description and Discussion of the Calculus*, Journal of the Optical Society of America **31**, 1941, 7, pp. 488–493.

[4] Jones R. C., *A New Calculus for the Treatment of Optical Systems*, II. *Proof of Three General Equivalence Theorems*, Journal of the Optical Society of America **31**, 1941, 7, pp. 493–499.

[5] Jones R. C., *A New Calculus for the Treatment of Optical Systems*, III. *The Šohncke Theory of Optical Activity*, Journal of the Optical Society of America **31**, 1941, 7, pp. 500–503.

[6] Shurcliff W. A., *Polarized Light*, Harward University Press, 1962.

[7] Petrů F., *Měření polarizačních vlastností optických členů laserového interferometru*, Jemná Mechanika a Optica (to be published).

[8] Knittl Z., *Optics of thin films*, John Wiley and Sons Ltd, 1976.

Received, November 24, 1976