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M^2 parameter transformation in real axially symmetric optical systems — a quasi-geometrical approach*

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It is proposed to estimate lens quality for multi mode beam transformation by factor Q as a ratio of output to input beam parameter M^2. Quasi-geometrical method of M^2 calculation is presented, and several examples are discussed. It is shown that for high M^2 beams, the requirements on lens quality are considerably lower than for diffraction limited systems.

1. Introduction

Theoretical approaches describing light propagation in optical systems are usually restricted to two contrasting cases, *i.e.* incoherent illumination and fully coherent beam propagation. For incoherent illuminatiom, the well known MTF formalism was developed, while in this case resolution is, as a rule, limited by numerical aperture and aberration of a lens. In the latter case, caustic sizes are determined mostly by phase and amplitude distributions of entrance beam. Lens aberrations are neglected in beam propagation problems in majority of practical applications because paraxial approximation conditions are fulfilled. Even in this case, the state of spatial coherence of entrance light should be considered. Only for a few light sources (*e.g.* He-Ne or CO_2 lasers), the condition of full coherence is fulfilled to a sufficient degree.

2. Method of calculations

In terms of laser technique, state of coherence of light beam is usually described by M^2 parameter (see, *e.g.* [1]). This parameter can be defined for our purposes as

$$\langle r^2\rangle\langle\sin^2(\theta)\rangle = (M^2\lambda/\pi)^2 \tag{1}$$

where $\langle r^2\rangle$ denotes mean square of beam radius in the waist, and $\langle\sin^2(\theta)\rangle$ denotes mean square sinus of half divergence angle of the beam, λ denotes the wavelength. The parameters $\langle r^2\rangle$, $\langle\sin^2(\theta)\rangle$ have a clear theoretical and experimental definition. To calculate the above parameters in terms of partial coherence theory, the

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averaging should be done over phase space taking intensity as the weight function. Experimentally, we have to measure simply the waist diameter and the divergence of beam in a well known way (see, for instance, [2], [3]). Knowing both the quantities, M^2 parameter can be determined from (1). This unidimensional parameter, proportional to the number of transverse modes of the beam, can be interpreted as a square of quotient of the beam radius to the radius of beam coherence.

Propagation of partially coherent beam in optical systems can be described for paraxial and aberration free case using the generalized Fresnel transform (see *e.g.* [4], [5]). For Gaussian-Schell sources, the formulas for the beam transformation are the same as the well known Gaussian beam transformatiom formulas (see, *e.g.* [4], [6]).

Our purpose is to find the method of M^2 transformation including both the effects of aberrations of lens and the state of coherence of the entrance beam. Let the entrance beam be determined by: W_0 – radius of waist, θ_0 – half angle of divergence. From the above data, we can determine M_0^2 in the following way:

$$M_0^2 = W_0 \sin(\theta_0)\pi/\lambda. \tag{2}$$

To calculate the same parameters after passing through any lens, it is proposed to generalize the well known geometrical construction of Gaussian beam transformation (see *e.g.* [7]) above the paraxial limit in the following way. Let us define the family of entrance rays $\{r_i, \theta_i, g_i\}$, where r_i denotes height of intersection of i-th ray with waist plane, θ_i denotes skew angle of this ray with respect to the meridian plane (see Fig. 1), g_i denotes the appropriate weight coefficient.

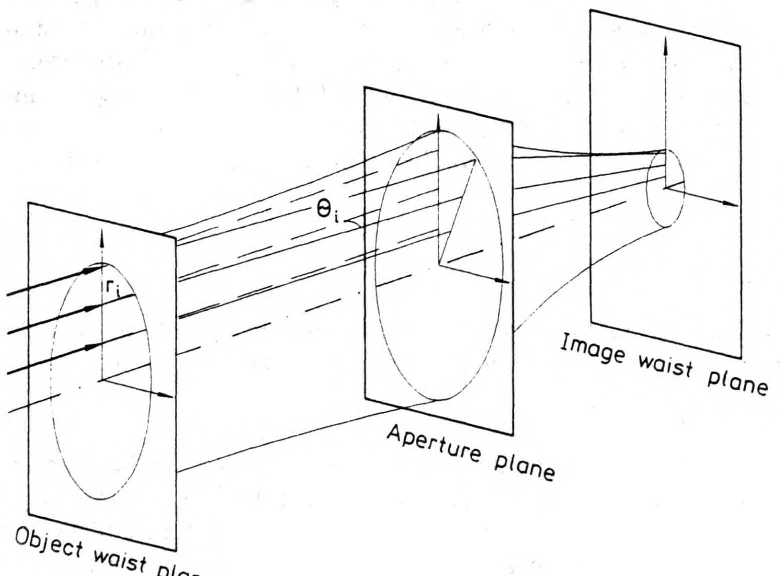


Fig. 1. Scheme of "quasi-geometrical" construction of multimode beam transformation in axially symmetric optical systems

This family of rays should satisfy the following formulas:

$$W_0^2 = N_r^{-1} \sum_{i=1}^{N} g_i r_i^2, \quad \sin^2(\theta_0) = N_r^{-1} \sum_{i=1}^{N} g_i \sin^2(\theta_i) \tag{3}$$

where $N_r = \sum_{i=1}^{N} g_i$ and N denotes number of rays.

Then, each ray is transformed in a centred optical system by the exact ray tracing formulas (see, for instance, [8]). It is assumed that the aperture diameter is large enough not to intercept any ray. None of the rays in the image space cross the optical axis. Moreover, by a revolution of this ray trace around optical axis, a conic surface (hyperbolid) limiting the beam power at a given level is determined.

The waist plane in the image space of a lens is defined from the criterion of minimization of $\langle r'^2 \rangle$ – mean square of distances of all i-th rays to optical axis. Then we calculate beam parameters in image space of a lens similarly as for the entrance beam:

$$W_0'^2 = N_r^{-1} \sum_{i=1}^{N} g_i r_i'^2, \quad \sin^2(\theta_0') = N_r^{-1} \sum_{i=1}^{N} g_i \sin^2(\theta_i') \tag{4}$$

where W_0', θ_0', r_i' denote parameters of an exit beam. The lens quality parameter Q is proposed to be defined in the following way:

$$Q = M_0'^2 / M_0^2 \tag{5}$$

where $M_0'^2$ denotes exit beam parameter defined from (4) and (1).

2.1. Properties of the lens quality factor Q

Similarly as W_0, θ_0, M^2, the factor Q is a statistical estimation of "quality" of beam transformation by a lens. Its properties result from properties of M^2 parameter. Physical properties of M^2 parameter are analogous to the entropy function in statistical physics as follows:

i) It is impossible to lower the M^2 parameter without beam power losses, $Q \geqslant 1$ – for lossless systems.

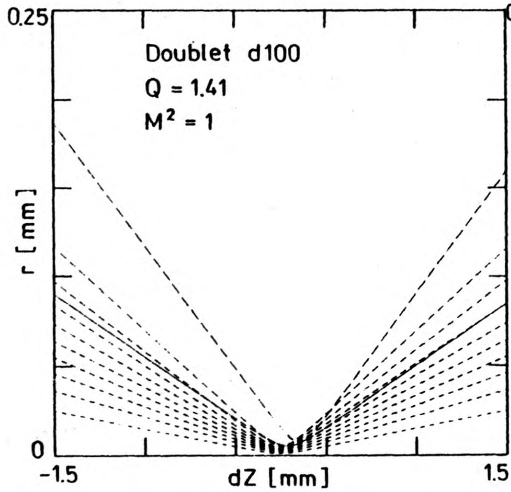
ii) For paraxial approximation M^2 becomes an invariant of transformation, $Q = 1$ – for paraxial approximation.

iii) For aberration-free, *i.e.* diffraction limited and lossless systems M^2 also becomes an invariant even beyond the paraxial approximation limit, $Q = 1$ – for diffraction limited system.

2.2. Caustics plots

Applying the "quasi-geometrical" method proposed above, it is possible to draw the caustics plots for any beam/lens configuration. These plots illustrate very well an influence of both lens aberrations and partial coherence of entrance beam on caustics shape.

Let us show caustics plots for cases of fully and partially coherent beams transformed by two lenses in terms beyond the paraxial limit. The F-number of lens/beam system is defined here as a ratio of the focal length f to the beam diameter $2W_0$. Dashed curves on plot represent each i-th ray tracing, while the continuous curve represents the trace $\langle r'^2 \rangle$ in the image space in the vicinity of the image focal point.



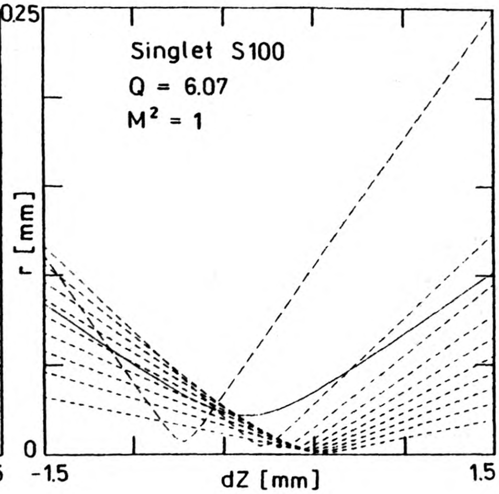


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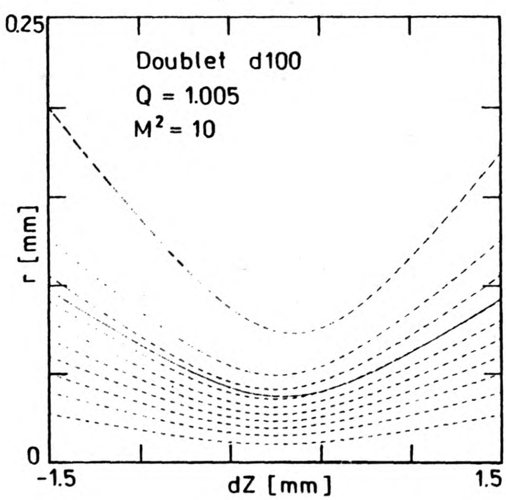
Fig. 2. Caustics plot for a well corrected lens, $Q = 1.41$, $M_0^2 = 1$, $F = 6.67$
Fig. 3. Caustics plot for an aberrated lens, $Q = 6.07$, $M_0^2 = 1$, $F = 6.67$



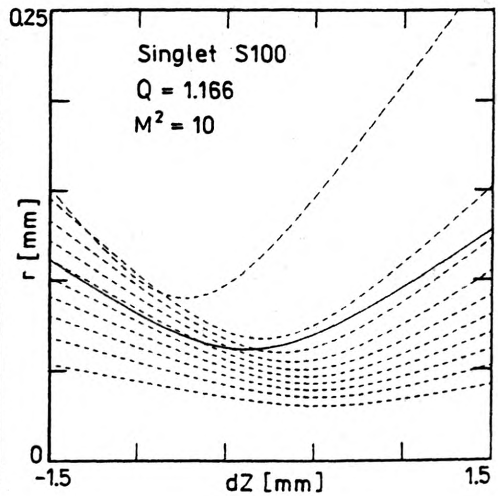


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Fig. 4. Caustics plot for a well corrected lens, $Q = 1.005$, $M_0^2 = 10$, $F = 6.67$
Fig. 5. Caustics plot for an aberrated lens, $Q = 1.17$, $M_0^2 = 10$, $F = 6.67$

As shown in Figures 2 and 3, there are considerable differences between caustics plots for a well corrected lens and lens with considerable aberrations. The conic surfaces determined for each *i*-th ray crosses one another for the aberration case while the caustics sizes differ considerably in both the cases.

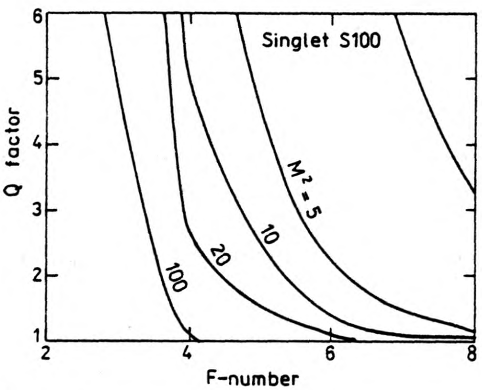
Let us show the caustics plots of the same lenses but for large M^2 parameter (see Figs. 4, 5). The differences of caustics are negligible. The waist parameters in the image space can be now estimated from paraxial formulas (see [4]) in spite of the fact that the paraxial approximation conditions are not fulfilled.

We always have to consider the lens quality factor Q for given input beam parameters. The Q factor can be a useful merit function for problems concerning laser beams propagation in optical systems, especially for multimode beam cases. Heuristic foundations rather than any exact grounds enable proposing the above presented method. However, we decided to work out this "quasi-geometrical" model, for its simplicity and practically unavailable exact information of phase and amplitude distributions in wave front of partially coherent beam. It should be noticed that the calculated W_0, θ_0, M^2, Q are only the simplest statistical estimations of the real measurable quantities characterizing any partially coherent light beam.

3. Application of Q factors to the beam focusing problem

As an example of the above method, the beam focusing problem was analysed. The purpose was to determine the dependence of the entrance beam and lens parameters on the Q factor of typical well corrected singlets and doublets destined for laser beam focusing.

The waist is placed always in the front focal plane of a lens. Four lenses (singlets and doublets with focal lengths $f = 100$ mm and $f = 8.6$ mm, respectively) were chosen to be analysed.



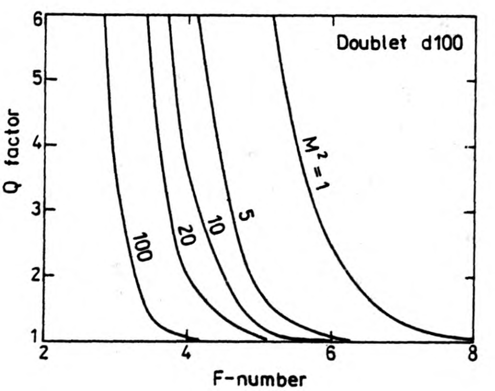


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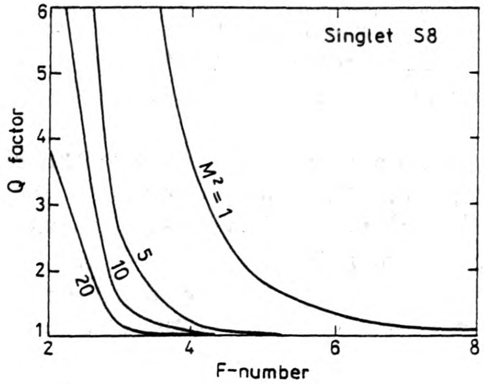
Fig. 6. Dependence of F-number on Q factor for a singlet with the focal length $f = 100$ mm for several M^2

Fig. 7. Dependence of F-number on Q factor for a well corrected doublet with the focal length $f = 100$ mm for several M^2



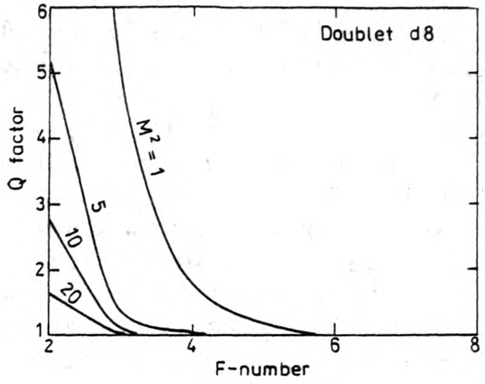


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Fig. 8. Dependence of F-number on Q factor for a singlet with the focal length $f = 8.6$ mm, for several M^2

Fig. 9. Dependence of F-number on Q factor for a well corrected doublet with the focal length $f = 8.6$ mm, for several M^2

As is shown in Figures 6–9, lens quality factor Q as a rule lowers with F-number, but also significantly depends on M^2 parameter. With rises of M^2 the F-number limit of a good performance of the lens defined at a given level of Q factor (*e.g.* $Q = 1.5$) lowers significantly.

This limit depends also on the lens correction state for a given wavelength. However, the general conclusion of the above analysis is that for high M^2 beams it is sufficient to use simple singlets instead of well corrected lenses.

4. Final remarks

In spite of the limitations of the "quasi-geometrical" method, it can be recommended as a complementary tool in lens design, especially for laser beam focusing problems. It should be emphasized that this method is not a simple application of ray tracing technique. Additional information about the coherence state of source is introduced in the starting plane of rays, and through it, the beam parameters in image space are estimated more exactly. For very large M^2, the results of this model are the same as when applying the traditional ray tracing and spot diagrams technique, while for near fully coherent case the "quasi-geometrical" approach shows its evident advantage.

The extension of this method to the nonsymmetric beams and systems is possible and will be presented in following papers.

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