Talbot effect (self-imaging phenomenon) in holography

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Talbot effect in holography is being studied. The study has shown that in monochromatic light illumination of a bleached off-axis recorded hologram of a periodic grating, self-images of the grating are reconstructed in the real image region not only in the rays of the ± 1st orders, but also in the zero diffraction order.

1. Introduction

So far the Talbot effect in holography has been studied rather poorly. Holographical recording and object reconstruction were considered theoretically in paper [1]. The object was 2-D periodical structure consisting of identical elements. The study has shown that real and virtual images are reconstructed in the Fourier approximation and two series of images are formed by one and the same wavefront in front of and behind the hologram reconstructed in the Fresnel approximation. The possibility of self-imaging of a periodic grating from its holograms has been tested experimentally in papers [2], [3]. Holograms of one-dimensional grating were recorded off-axis in these experiments. The hologram plane coincided with one of the planes of the self-imaging. Paper [3] differs from paper [2] in studying self-imaging of a periodic grating from its holograms not only in the virtual image region, but also in the region of the real image. For this purpose, a second reference beam conjugated with the first one is introduced into the hologram recording and reconstruction scheme. As the intensity of the reconstructed images was very low, the measured signal was 1000 times amplified by an optoelectronic transducer, and then recorded at a photoplate. The positions of the grating self-images reconstructed from their holograms in the regions of virtual and real images were compared with the positions of the planes of the self-images of the grating itself. These measurements have shown the equality of the self-image periods produced by the original grating, those reconstructed from the hologram and the theoretical ones.

This paper presents the results of theoretical and experimental studies of a more generalized case of Talbot effect in holography. Holograms of a periodic grating were recorded according to the traditional off-axis scheme, but unlike the situation assumed in papers [2], [3], the distance from the object to the hologram plane was chosen arbitrarily. Self-images of the grating were found to be reconstructed

simultaneously not only in the region of the real image in the ± 1st orders of diffraction, but also in the direction of the zero diffraction order.

2. Theory

Let us consider a hologram recording of a one-dimensional amplitude grating disposed arbitrarily with respect to the hologram plane (x_1, y_1) as an interaction of two plane waves — object wave U_0 and reference wave A, which are incident at an angle Θ to the axis and perpendicular to the hologram plane, respectively. The transmission coefficient of such a grating with slit and interval transmittances equal to 1 and 0, respectively, can be represented by [4]

$$t(y_0) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)}{(2n+1)} \cos[(2n+1)\omega y_0] = t_0 + t_n \tag{1}$$

where t_0 is a constant component of the transmission coefficient, t_n is a diffraction component of a non-zero spatial frequency, $\omega = 2\pi/d$ (d — the grating period measured in units of length). The object wave directly behind the grating is equal to

$$U^1(y_0) = U_0 t(y_0). \tag{2}$$

At any distance z from the grating, it is defined by the expression

$$U_0(y) = U_0 t_n + U_0 t_n \exp[i(2n+1)^2 \Phi] = a_0 \exp(i\varphi_0) + a_n \exp(i\varphi_n)$$

where: $a_0 = U_0 t_0$, $a_n = U_0 t_n$, $\varphi_n = (2n+1)^2 \Phi$, $\Phi = \omega z^2/2k$, $k = 2\pi/\lambda$, a_0, φ_0 are the amplitude and phase of the coherent background, a_n and φ_n are the amplitude and phase of the diffracted waves.

The intensity of the light incident on the photoplate is defined by the equation

$$\begin{aligned}
J(x_1, y_1) &= |A + a_0 \exp[i(\varphi_0 + \gamma)] + a_n \exp[i(\varphi_n + \gamma)]|^2 = A^2 + a_n^2 + a_0^2 \\
&\quad + 2a_0 a_n \cos(\varphi_n - \varphi_0) + 2A a_n \cos(\gamma + \varphi_n) + 2A a_0 \cos(\gamma + \varphi_0)
\end{aligned} \tag{3}$$

where $\varphi_n = (2n+1)^2 \Phi_1$, Φ_1 is the phase of the object wave recorded on the hologram, $\gamma = k y_1 \sin \Theta$. In this expression, there exist three interference terms: the first one, $2a_n a_0 \cos(\varphi_n - \varphi_0)$ describes the Gabor hologram, the second term, $2A a_n \cos(\gamma + \varphi_n)$ corresponds to the Fresnel hologram, the third term, $2A a_0(\gamma + \varphi_0)$ represents an interference pattern of parallel lines (one-dimensional periodic grating) localized at the hologram itself with the spatial frequency $a = \sin \Theta/\lambda$. If the linearity condition is met both for recording and treatment of the hologram, the transmission coefficient τ is proportional to $J(x_1, y_1)$ and, consequently, expression (3) describes the function of the amplitude transmission of the hologram of the object with a precision of up to the constant factor

$$\tau(x_1, y_1) = \tau_0 + \tau_1 + \tau_2 + \tau_3$$

where: $\tau_0 = A^2 + a_0^2 + a_n^2$,

$\qquad \tau_1 = C_1\{\exp[-i(\varphi_n - \varphi_0)] + \exp[i(\varphi_n - \varphi_0)]\}$,

$$\tau_2 = C_2\{\exp[-i(\gamma+\varphi_n)]+\exp[i(\gamma+\varphi_n)]\} = \tau_2^- + \tau_2^+,$$
$$\tau_3 = C_3\{\exp[-i(\gamma+\varphi_0)]+\exp[i(\gamma+\varphi_0)]\} = \tau_3^- + \tau_3^+.$$

Such a hologram can be bleached. Background images (A^2, a_0^2, a_n^2) are eliminated, amplitudes of the interference terms $C_1 = a_0 a_n$; $C_2 = A a_n$; $C_3 = A a_0$ take the constant values over the whole hologram plane.

Let us consider the reconstruction process of such a hologram. When it is illuminated with the initial reference beam A, the wave behind the hologram H is distributed among three directions — the straightforward one and two others at the
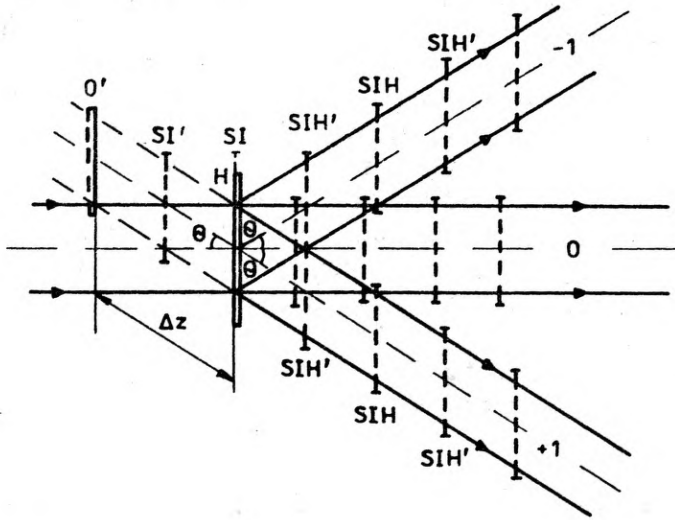

Fig. 1. Reconstruction scheme of self-images from the hologram of a periodical grating $z_1 = \Delta z$; O^1 is the grating position during its hologram recording

angles of $\pm \Theta$ (see Fig. 1). The field intensities in the directions of the zero and $\pm$1st diffraction orders are expressed, respectively, by:

$$J_0 = |A(\tau_0+\tau_1)|^2 = A^2\tau_0^2 + A^2\tau_1\tau_1^* + A^2(\tau_0\tau_1^* + \tau^*_0\tau_1), \tag{4}$$
$$J_{+1} = |A(\tau_2^+ + \tau_3^+)|^2 = A^2\tau_2^+\tau_2^+ + A^2\tau_3^-\tau_3^+ + A^2(\tau_2^+\tau_3^- + \tau_2^-\tau_3^+), \tag{5}$$
$$J_{-1} = |A(\tau_2^- + \tau_3^-)|^2 = A^2\tau_2^+\tau_2^- + A^2\tau_3^+\tau_3^- + A^2(\tau_2^+\tau_3^- + \tau_2^+\tau_3^-), \tag{6}$$

where symbol $\cdot$ denotes a complex conjugate value.

In these expressions $A^2\tau_0^2$ describes the background; $A^2\tau_1\tau_1^*$ defines the virtual and real images of the grating in the way they are defined in the Gabor hologram. Component $A^2\tau_2^-\tau_2^+$ describes the usual virtual and real images of the object, which are reconstructed in the $\pm$1st diffraction orders. Component $A^2\tau_3^+\tau_3^-$ describes the zonal Fresnel grating localized on the hologram itself. As a consequence, a rainbow image of the object is reconstructed on the hologram iteself, when it is illuminated with white light.

Components $A^2(\tau_0\tau_1^* + \tau^*_0\tau_1)$ in (4), and $A^2(\tau_2^+\tau_3^- + \tau_2^-\tau_3^+)$ in expressions (5) and (6)

are responsible for occurrence of the Talbot holographic effect. They are described with a precision up to constant factors by the expression

$$C \sum_{n=0}^{\infty} \frac{(1)^n}{(2n+1)} \cos[(2n+1)^2 \Phi y_0]\{\exp[-i(2n+1)^2 \Phi_{\pm}] + \exp[i(2n+1)^2 \Phi_{\pm}]\} \quad (7)$$

where C is a constant, $\Phi_{\pm} = \Phi \pm \Delta\Phi$, $\Phi = \omega^2 z_1 / 2k$ is the state of the phase of the object wave recorded in the hologram, $\Delta\Phi$ is the variation of the reconstructed wave phase with respect to the hologram. The signs "+" and "−" refer to the +1st and −1st diffraction orders, respectively.

In the planes where the condition $\Phi_{\pm} = p2\pi$ is fulfiilled, in the real (behind the hologram) and virtual (in front of the hologram) space regions, expression (7) with the precision of up to the constant factors coincides with expression (2) describing the grating images located directly behind this grating. In this case, the phase of the reconstructed waves $\Phi_{\pm}$ is evaluated from the hologram, while taking account of the state of the object wave phase recorded on the hologram. Hence, the positions of the planes of the grating self-images reconstructed from the hologram depend on the relative positions of the object and the hologram during its recording, and, also on the direction of the diffracted ray, along which the grating images are carried over. The wave phase change of 2π is equivalent to its passing a distance equal to the self-imaging period $\Delta z = 2d^2/\lambda$.

In the real region, the distance from the hologram to the self-imaging planes z_p for the +1st and −1st orders is defined by the following respective expressions:

$$z_p^+ = p\Delta z - z_1, \quad z_p^- = p\Delta z + [z_1 - E(z_1/\Delta z)\Delta z] \quad (8)$$

where z_1 is the distance from the object to the hologram during its recording, $E(a)$ is the integer of number a. $p = 0, 1/2, (1+1/2), \ldots (j+1/2), j = 0, 1, 2, 3, \ldots$ — the integral number, integral and semi-integral values of P indicate the planes of the positive and negative self-images, respectively. Note that the distances from the hologram to the self-image planes in the direction of the zero order are equal to $z_p^0 = z_p^- \cos\Theta$.

3. Experiment

Our experiment has been carried out with a one-dimensional amplitude grating having a period $d = 5000 \pm 7$ μm produced by a photographic method. The grating self-image period Δz was equal to 79.2 cm ($\lambda = 0.63$ μm). A hologram of this grating was recorded at a photoplate LOI-2, using the plane object and reference waves in an off-axis scheme. The hologram was bleached by using a due photochemical treatment. During the hologram recording, in contrast to paper [3], the distance between the grating and the hologram plane was chosen arbitrarily. In this experiment, the following distances were choosen: $z_1 = \Delta z$, $\Delta z + \Delta z^1/2$, $\Delta z + 3\Delta z^1/4$, $\Delta z + \Delta z/2$, $(\Delta z^1 = \Delta z/2)$. The holograms produced were illuminated by a plane monochromatic wave incident normally to the hologram (Figs. 1 and 2), as well as by white light. The three cases are discussed below.
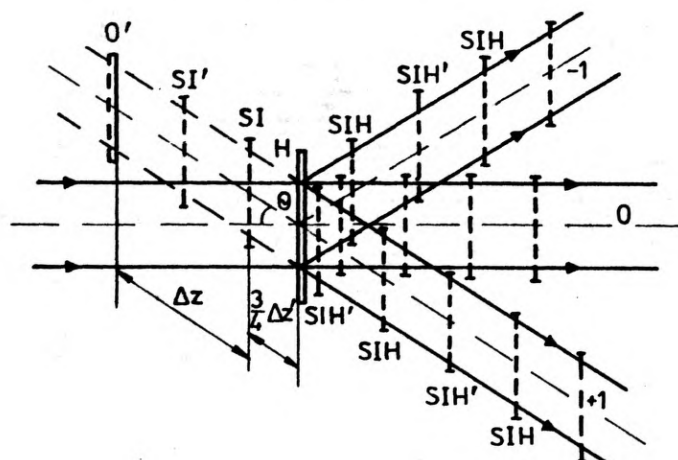
Fig. 2. Scheme of reconstruction of self-images from the periodic grating holograms: $z_1 = \Delta z + (3/4)\Delta z^1$

1. During hologram illumination with a plane monochromatic wave, besides the traditional real and virtual images of the grating (they are not shown in the figures), the grating self-images were reconstructed simultaneously at certain distances not only in the beams of the +1st and −1st diffraction orders, but also along the zero order direction. In the figures, the positions of the self-image planes of the positive and negative images of the grating itself are denoted by SI and SI^1, while those reconstructed from the holograms are denoted by SIH and SIH^1, respectively (O^1 is the grating position during the hologram recording).

The positions of the self-image planes z_p^+ and z_p^-, the images being reconstructed from the holograms in the directions of the +1st and −1st orders according to formulae (8), are listed in the Table for different values of z_1.

z_1	z_p^+	z_p^-
Δz	$\Delta z/2$, $(3/2)\Delta z$, $2\Delta z$, ...	$\Delta z/2$, Δz, $(3/2)\Delta z$, $2\Delta z$, ...
$\Delta + \Delta z^1/2$	$\Delta z/4$, $(3/4)\Delta z$, $(5/4)\Delta z$, ...	$\Delta z/4$, $(3/4)\Delta z$, $(5/4)\Delta z$, ...
$\Delta z + (3/4)\Delta z^1$	$\Delta z/8$, $(5/8)\Delta z$, $(9/8)\Delta z$, ...	$(3/8)\Delta z$, $(7/8)\Delta z$, $(11/8)\Delta z$, ...
$\Delta z + \Delta z^1$	$\Delta z/2$, Δz, $(3/2)\Delta z$, $2\Delta z$, ...	$\Delta z/2$, Δz, $(3/2)\Delta z$, $2\Delta z$, ...

These distances were determined by the positions of the frosted screen with sharp grating images placed behind the hologram on the way of respective waves. The distances were defined more precisely by the measuring microscope MIR-2.

The positions of the self-image planes in the zero-order beam are defined by the formula $z_p^0 = z_p^- \cos\Theta$.

In the figures, this is illustrated for two values only: $z_1 = \Delta z$ and $z_1 = \Delta z + (3/4)\Delta z^1$.
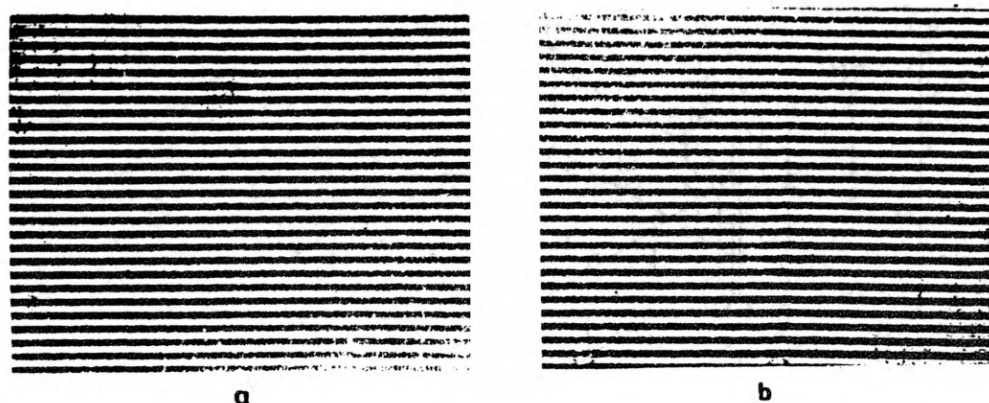
Fig. 3. Photo of a negative (a) and a positive (b) self-images reconstructed from the hologram

Figures 3a, b show the photos of the positive and negative images of the grating self-images in the +1st order ray, placed at the distances of 49.5 and 89 cm, respectively, from the hologram ($z_1 = \Delta z + (3/4)\Delta z^1$), these distances correspond to those of $z_p^+ = (5/8)\Delta z$ and $z_p^+ = (9/8)\Delta z$. As can be seen, these images are identical and differ only in background distribution.
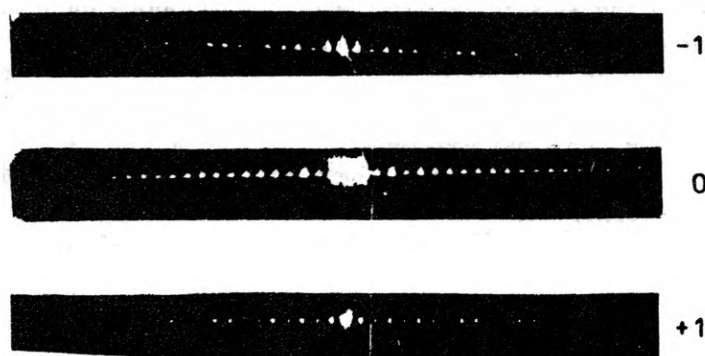

Fig. 4. Spatial spectra in the rays of the +1st, 0th and −1st diffraction orders

Figure 4 shows the spatial spectra of the reconstructed grating images in the directions of the +1st and −1st diffraction orders. These spectra were produced according to the Abbe scheme [4].

A uniform illumination is observed in the planes meeting the condition $\Phi_\pm = \Phi_1 \pm \Delta\Phi = (j + 1/2)\pi$.

2. A band of a bright contrast rainbow image, both in transmitted and reflected light, is reconstructed on the hologram itself, when the hologram is illuminated with white light. The whole image field can be looked through if the observation angle is being changed. The sharpness of the reconstructed grating image depends on the conditions of hologram recording — the image is sharp when the hologram plane during its recording coincides with any of two self-image planes, SI and SI^1.

3. If the hologram is illuminated with diffusely dispersed white light directly over the whole field of the hologram, one will see a monotonic and contrasting image of the grating, whose sharpness, as in the previous case, depends on the hologram recording conditions.

4. Conclusions

1. It has been shown both theoretically and experimentally that the grating self-images are reconstructed in the real image region simultaneously along the rays of the $\pm$ 1st diffraction orders and in the zero diffraction order when a bleached hologram of a periodic grating is illuminated with monochromatic light.

2. The grating image is reconstructed also when such a hologram is illuminated with white light.

References

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