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# Experimental determination of the refractive index of solids by reflection

E. M. Georgieva

Department of Physics of Condensed Matter, Faculty of Physics, University of Sofia, 5 A. Ivanov Blvd., Sofia, BG-1126, Bulgaria.

A simple method and a set-up for experimental determination of the real part of the refractive index and its dispersion by reflection with an accuracy of $\pm 0.005$ are described.

## 1. Introduction

The optical properties of an isotropic material are characterized by its complex index of refraction, $\tilde{n} = n - ik$, related to the complex dielectric constant $\tilde{\varepsilon} = \varepsilon_1 - i\varepsilon_2$, $\varepsilon_1 = n^2 - k^2$ and $\varepsilon_2 = 2nk$, where $k$ is the extinction coefficient. In many practical applications, there are also used the absorption coefficient $\alpha = 4\pi k/\lambda$ ($\lambda$ is the wavelength of the light), and the reflectance for normal incidence

$$R = [(n-1)^2 + k^2]/[(n+1)^2 + k^2].$$

The transmittance methods used in practical determination of $n$ and $k$ fail in the cases when $\alpha > 10^4\ \mathrm{cm}^{-1}$ and reflectance measurements are preferred. Some of the described reflectance measurements require Kramers–Kronig analysis [1]–[3] and uncertain extrapolations or approximations. For simultaneous determination of $n$ and $k$, the ellipsometric measurements are preferred [4]–[6], but they require special equipment.

## 2. Description of the method

This paper describes a simple practical method for determining $n$ by means of ordinary optical equipment. The method requires uniformity and isotropy of the sample and neglects the existence of a surface layer, different from the bulk material (it should be used for fresh or well-cleaned surface). The expression for $n$ is easily derived from the well-known Fresnel equation [7]

$$\frac{E_s''}{E_s} = \frac{\sin(\varphi - \varphi')}{\sin(\varphi + \varphi')}, \quad \frac{E_p''}{E_p} = \frac{\tan(\varphi - \varphi')}{\tan(\varphi + \varphi')} \tag{1}$$

where $\varphi$ is the angle of incidence, $\varphi'$ denotes the angle of refraction ($\sin\varphi/\sin\varphi' = n$), $E_s$ and $E_p$ are the amplitudes of the incident electric vectors, perpendicular and parallel to the plane of incidence, and $E_s''$ and $E_p''$ are the corresponding reflected amplitudes.

From (1) for $\varphi = 45°$, taking $E_s = E_p$,

$$\frac{E''_p}{E''_s} = \frac{(2n^2-1)^{1/2}-1}{(2n^2-1)^{1/2}+1}\,.$$

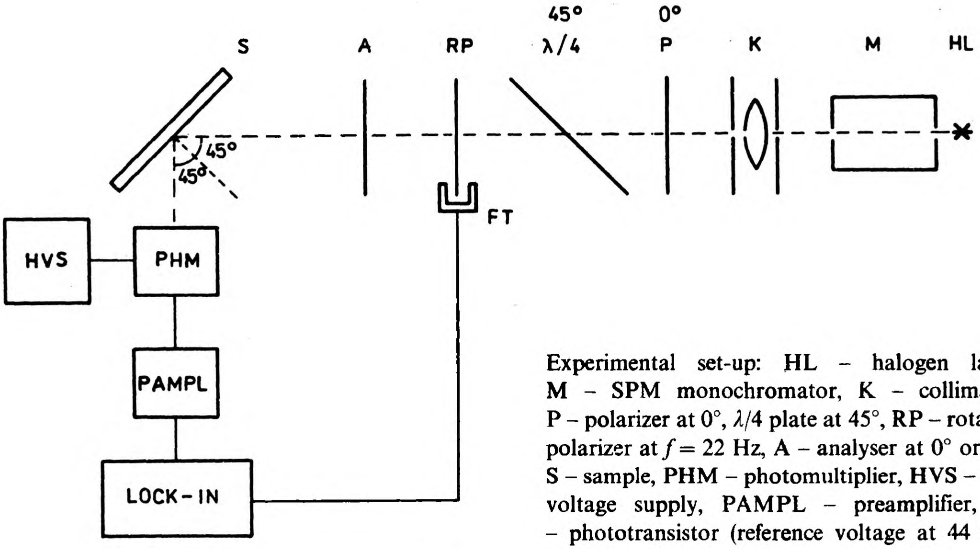
If we denote $a$ and $b$ as follows:

$$a = \frac{I''_p}{I''_s} = \frac{(E''_p)^2}{(E''_s)^2} \quad \text{and} \quad b = \frac{1-a}{1+a} = \frac{I''_s - I''_p}{I''_s + I''_p}\,,$$

then a straightforward calculation of the real part of the refractive index leads to the formula

$$n = \frac{[1+(1-b^2)^{1/2}]^{1/2}}{b}\,. \tag{2}$$

The experimental set-up in the Figure presents an ordinary optical equipment.



Experimental set-up: HL – halogen lamp, M – SPM monochromator, K – collimator, P – polarizer at 0°, $\lambda/4$ plate at 45°, RP – rotating polarizer at $f = 22$ Hz, A – analyser at 0° or 90°, S – sample, PHM – photomultiplier, HVS – high voltage supply, PAMPL – preamplifier, PT – phototransistor (reference voltage at 44 Hz)

A $\lambda/4$ plate is used after the polarizer for obtaining circularly polarized light, which eliminates the possible polarization effects from the monochromator, thus assuring that $|E_s| = |E_p|$. For measuring $n = n(\lambda)$, a set of $\lambda/4$ plates for different wavelengths (or a compensator) is used. The rotating polarizer, whose frequency $f$ or revolution in our set-up is 22 Hz, continuously changes the position of the plane of the polarization at a frequency $2f$. A signal of frequency $2f$ from the phototransistor is applied to the reference of the lock-in nanovoltmeter. With the analyser and the help of a micrometer screw there are selected the incident $I_s$ (corresponding to maximum $I''_s$ signal of the lock-in) and the incident $I_p$ (corresponding to minimum $I''_p$ signal of the lock-in).

There is a preferential reflection for those waves in which the electric vector is perpendicular to the plane of incidence.

Only $I_s''$ and $I_p''$ are measured, when the angle of incidence is 45°.

Reflections from the back surface have been eliminated by making the sample wedge-shaped or suppressed by grinding the second surface and blackening it. When this procedure is not suitable, correction for additional reflection from the second surface is necessary for thin crystals and the intensity of the reflected light is

$$RI_0 = \frac{1+(1-R)^2 e^{-2\alpha d}}{1-R^2 e^{-2\alpha d}}$$

where $I_0$ is the incident intensity, $R$ denotes the reflectance, and $d$ – the thickness of the crystal. This correction needs a knowledge of $R$ and $\alpha$. For transparent crystals ($k = 0$), the reflected intensity becomes

$$RI_0 = \frac{1+(1-R)^2}{1-R^2} .$$

## 3. Error analysis

Differentiation of Equation (2) leads to

$$\frac{\Delta n}{n} = \frac{b^2 n^2}{2(1-n^2 b^2)} \frac{\Delta b}{b}$$

where

$$\frac{\Delta b}{b} = \frac{\Delta(I_s'' - I_p'')}{I_s'' - I_p''} + \frac{\Delta(I_s'' + I_p'')}{I_s'' + I_p''} .$$

For example, for $Bi_{12}GeO_{20}$: $I_s'' \sim U_s'' = 21$ mV, $I_p'' \sim U_p'' = 6.6$ mV, $|\Delta U_s''| = |\Delta U_p''| = 16 \times 10^{-3}$ mV, and $\Delta n = \pm 0.0035$.

Taking into account the error of angle adjustment (in our experiment it is less than 0.1°), the overall error of the experiment increases to $\pm 0.005$.

A single measurement of $I_s''$ and $I_p''$ determines the second decimal integer of the index (see the Table). Several independent measurements of $I_s$ and $I_p$ allow the determination of the third decimal integer. The magnitude of $n$ also depends on the quality of the polished surface of the sample.

Index of refraction of some specimens for $\lambda = 589$ nm

| | |
|---|---|
| $z$ cut quartz | $1.550 \pm 0.005$ |
| $x$ cut calcite | $1.650 \pm 0.005$ |
| $Bi_{12}GeO_{20}$ | $2.551 \pm 0.005$ |

$|\Delta U_s''| = |\Delta U_p''| = 16 \times 10^{-3}$ mV is for the best polished surface we obtained in our conditions. This error is due to the overall noise of the electronics used and the quality of the sample surface. For fixed angle adjustment it is reproducible. It was independent of the several Zeiss $\lambda/4$ plates used.

This set-up is practically useful for exact vertical and horizontal positioning of a polarizer (maximum $I_s''$ signal and minimum $I_p''$ signal) without any reference to another coordinate system.

## 4. Summary

We described a simple practical method for determining the real part of the refractive index by reflection. The determination of $n$ and $k$ by transmittance methods fails in the cases when $\alpha > 10^4$ cm$^{-1}$ and reflectance measurements are preferred. The method does not need a special and expensive equipment. On the other hand, it is suitable for exact determination of the vertical or horizontal position, without reference to any coordinate system, which allows one to carry out the proper orientation of different optical components ($\lambda/4$ plates, compensators, crystals, photoelastic modulator, and so on), which is so necessary in polarimetry and ellipsometry.

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## Экспериментальное определение коэффициента преломления твердых тел методом отражения

В статье описаны простой метод и установка для экспериментального определения реальной части коэффициента преломления и его дисперсии точностью в ±0,005 с использованием отражения.

*Проверил Станислав Ганцаж*