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Letter to the Editor

Island bistability caused by approximation to slowly varying amplitude

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It is shown that approximation of slowly varying amplitudes could lead to non-physical shape of a theoretical bistability curve in a ring resonator.

It is very difficult to solve the Maxwell equations or their simpler version – the Helmholtz equations – in nonlinear media. The most popular method of solving them is called "approximation of slowly varying amplitudes" and it is based on the assumption which claims that the light wave amplitude varies slowly enough to neglect its second derivatives [1]–[3]. In nonlinear ring resonators that assumption is used together with another one which declares that, there is no backward wave inside the nonlinear crystal [2], [3]. In paper [4] they are both used to simplify the mathematical model applied for describing, how a static magnetic field influences the course of dispersive bistability in a nonlinear ring resonator. The received results (presented in [4]) are based on nonlinear polarization phenomena caused by the following equations of the resonator state:

$$
\begin{aligned}
\frac{2T_{\|}}{|\Omega|^2} I_0 &= I_1[R_{\perp}^2+1-2R_{\perp}\cos(\Delta\Phi_2+\Phi_{\perp}-k_2L)]^{-1} \\
&= I_2[R_{\perp}^2+1-2R_{\perp}\cos(\Delta\Phi_1+\Phi_{\perp}-k_1L)]^{-1}, \\
\Delta\Phi_j &= \gamma_{j1}I_1+\gamma_{j2}I_2, \quad j=1,\ 2, \\
k_j &= \frac{\omega}{c}\sqrt{\varepsilon_r+(-1)^j\kappa B_0}, \quad j=1,\ 2.
\end{aligned}
\tag{1}
$$

I_0 denotes intensity of the incident light, I_1, I_2 are the intensities of the right- and, respectively, left-circularly polarized waves inside the nonlinear crystal, B_0 is the induction of the constant, external magnetic field parallel to direction of light propagation, L is the length of the nonlinear crystal, $R_{\perp}$ ($T_{\perp}=1-R_{\perp}$) and $R_{\|}$ ($T_{\|}=1-R_{\|}$) are the intensity reflectivities of the mirrors, $\Phi_{\perp}$, $\Phi_{\|}$ are phase increases around the cavity for two light polarizations – having the electric vector normal and, respectively, parallel to the mirrors surface, ε_r is the electric permittivity of the medium, κ characterizes girotropical properties of the medium, γ_{jk}, j, $k=1$, 2. are constants depending on the properties of the cavity and of the medium.

Either Ω or I_{tr} (the intensity of the transmitted light) are nonlinear functions of I_1, I_2 (for details see [4]).

The parameters selected for numerical analysis in [4] (referring to CS_2) give classical bistability curves of dependence of I_{tr} on I_0 [1]–[4]. However, interdependence between I_{tr} and I_0 is multistable and the analyses of almost all papers [1]–[4] are devoted to the first hysteresis cycle. A very good example of such approach is constituted by the so-called "medium field approximation".

The results presented in this paper refer to the second hysteresis cycle and they are made for the following cavity parameters: $B_0 = 0.02$ T, $L = 0.05$ m, $\Phi_{\perp} = 0$, $\Phi_{\parallel} = \Pi$ (compare with [4]). The received curve of the interdependence of I_0 and I_{tr} (Fig. 1) is completely different from the typical ones. The appearance of the closed

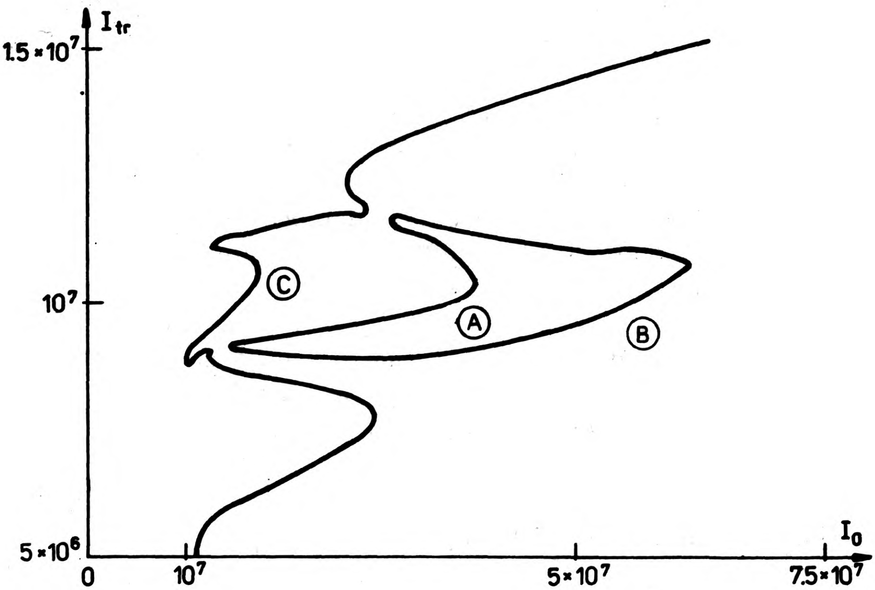

Fig. 1. Interdependence of I_{tr} and I_0

subcurve (denoted by the letters A and B) has never been reported in the literature. The shape of the closed subcurve suggests "island bistability". It is strictly connected with the shape of the curve illustrating the interdependence of I_1 and I_2 (Fig. 2). The parts of that curve denoted by the letters A, B and C refer to the parts denoted by A, B and C in Fig. 1. It shows that the state of the wave field inside the nonlinear ring cavity is not unique (the parts A, B, C) for a given value of the transmitted light intensity I_{tr}. Hence I_0 is not a function (in the mathematical meaning of that notion) of I_{tr}, which enables classical interpretation [2], [3] of the hysteresis curve.

There are two possible explanations of "island bistability" presented in Fig. 1:

i) It is a new, unexpected physical phenomenon.

ii) The parts denoted by B and C are non-physical and their appearance is caused by the mathematical assumptions described at the beginning of the paper.

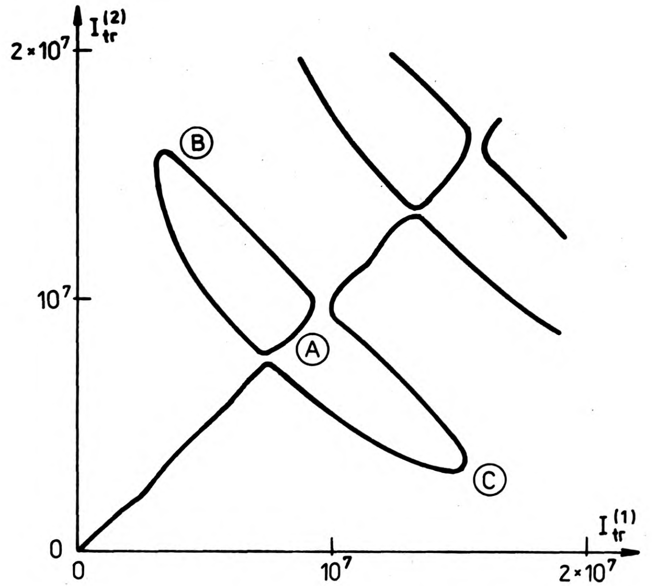

Fig. 2. Interdependence of I_1 and I_2

CS_2 is a weekly nonlinear and girotropic medium what implies that it should be $|I_1 - I_2| \ll I_1, I_2$ and, consequently, only the part of the (I_1, I_2)-curve (and of the $(I_0 I_{tr})$-curve) denoted by A should be real. Hence, the explanation i) is not realistic and gives no interpretation of the real course of bistability. It implies that the explanation ii) should be accepted as correct.

The problem of the shape of the (I_0, I_{tr})-curve can be solved by experiments or by analysis of dynamical states of the cavity.

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Островая бистабильность, вызванная аппроксимацией медленно изменяющейся амплитуды

Было показано, что аппроксимация медленно изменяющейся амплитуды может привести к нефизическому виду теоретической кривой бистабильности в кольцевом резонаторе.

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