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Quick measurement of parameters Δ and α of the refractive index profile in the preforms and waveguides. Generalized shearing methods*

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A quick method of measurement of parameters Δ and α describing the refractive index profile for preforms and light waveguides has been presented. The interference methods of both transversal and radial shearing have been described as far as being related to the problem. The proposed method makes it possible to apply a very simple scanning of the interferograms and the calculations are reduced to determination of the coefficients to the given equations.

1. Introduction

In many application and, in particular, when large series of measurements on similar objects (mass production) are performed, the measurement results as described by analytic formulae with parameters Δ and α determined experimentally are often more useful than the detailed tabelarized results of sampling. Interference methods to determine the parameters Δ and α of both preforms and optical waveguides have been exploited since a long time [1], [2]. However, any improvements are worth noting, while an increment of both the measurement accuracy and reliability is very wellcome.

Our approach is more general than that in the papers cited above, while the assumed cylindric symmetry of the objects is used only for the sake of exemplification. This work is the forth one of the five-paper series of publications [3]–[5] intended by the author. The first one, [3], describes the method as employing the interference with plane reference wave. In works [4], [5], the specific relations are exploited which appear in the methods of radial and transversal shearing, respectively. They show the possibilities of the measurement of the sought parameters Δ and α as far as deformed interference fringes are concerned for two determined normed coordinates of $\hat{r}$.

The aim of this paper is to generalize the shearing methods to enable the measurement of the fringe deformation for two arbitrary values of $\hat{r}$. The next paper, being now elaborated, will present a further generalization of all the described

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methods being also some specific generalization of the methodices of the interference measurements.

The interferometric measurements of the phase objects consist in the registration of the orders of interference fringes on the plane interferograms in which the information taken from the whole volume of the object is coded by passing the testing light beams. The calculation of the spatial distribution of the refractive index in the examined object is possible if the suitable general data concerning the object are known. In the case of preforms and light waveguides the needed information is that the geometry of both the object and the refractive index distribution are of rotational symmetry. The transition from the data space to the space of results is a classic one and it is realized by applying the whole theory with the suitable mathematical apparatus. The results of sampling are usually obtained on the base of sampling by carrying out a full time-consuming cycle of calculations for each sampling of results. In the calculations, the general information about the object examined are also exploited.

In the present paper, another approach is attempted. The aim was to reduce the data from the interferogram on the one hand, and to obtain the maximally detailed results achievable under these circumstances. It has been assumed that in addition to the general information about axial symmetry of both the object shape and the refractive index distribution, the general form of the function (1) (see down) of the object is known which describes the real continuous distribution of the refractive index with a sufficient accuracy. This function may be uniquely described by using only two numbers (parameters Δ and α). It has been also assumed that only those two values and not the very detailed data, which sometime may obliterate the general properties of the examined objects, are the subject of interest.

In order to fully explain the further treatment, let us note that in this work the computing simultation examinations were used. They gave the same effect as the measurement data treated classically which would fill the space of results.

We want to show that, if the data from the interferograms are even drastically reduced, the measurement of Δ and α parameters is still possible with the accuracy sufficient for many practical purposes.

The increase of measurement rate became possible due to the following assumptions [3]:

– Continuous function

$$n(\hat{r}) = n_o\sqrt{1-2\Delta\hat{r}^{\alpha}}, \quad n_o = n_p/\sqrt{1-2\Delta} \tag{1}$$

is a sufficiently good approximation of the real profile of the core, where $\hat{r} = r/r_o$ is a normalized argument, $r_o = d/2$ is the core radius, and n_p is the refractive index in the core.

– Visual or automatic measurement of the fringe deformations is sufficiently intensive to the microdiscontinuities of the fringes.

The following additional assumptions, which influence the numerical values of the coefficients reported in this work, have been also accepted:

i) The sought parameters are contained within the limits $\Delta \in \langle 0.009, 0.015 \rangle$ or

$\alpha\langle 1.6, 2.4\rangle$, the shearing parameters: $b \in \langle 0.995, 0.98\rangle$ or $s \in \langle 0.005, 0.02\rangle$.

ii) The accuracy of the zero order approximation in calculation of the wavefront is sufficient.

iii) Calculations are carried out for the wavelength $\lambda = 632.8$ nm.

The simulation computer examinations have been performed, which consisted in:

1. Calculations of the wavefront $g(\hat{r})$ within the zero approximation for given Δ and α (the latter being changed within the ranges given above) and for different values of the core diameter d.

2. Calculations of the shape of the interference fringes for different Δ, α and d as well as different shearing parameters b or s.

3. Analysis of the changes of obtained interference orders (fringe shapes) to establish which variables and in which way influence the interference orders resulting in reduction of the number of variables.

4. Expressing the interference order through the parameters Δ and α.

5. Finding new functions and new numerical coefficients which enable a direct calculation of Δ and α from the measured interference order $\delta m(\hat{r})$.

The symbol δ denotes the difference operator. The relative interference orders $\delta m(\hat{r}) = m(\hat{r}) - m_p$ are given by the difference between the interference orders in the region of core and coat, respectively.

2. Radial shearing interference

Radial shearing interference is produced by the shearing element giving two similar wavefronts, with one of them being slightly evidenced with respect to the other [6]–[8], Fig. 1. This interference is characterized by the shearing parameter $b = r_2/r_1$. The optical path difference δz is usually measured with respect to the so-called reference level (PO in Fig. 1**b**). In the most cases, the position of the chosen fringe in the coat of the preform or light waveguide is accepted as a reference level. When the fringes in the coat are positioned perpendicularly to the axis of the preform or light waveguide, respectively, then the optical path difference may be determined from the interferogram (Fig. 1**c**) by using the method of fringe deviation from the straight line (by measuring $y(x)$ and c)

$$\delta z(x) = g_1(x) - g_2(x) = g(x) - g(bx) = y(x)/c = \delta m(x)\lambda \tag{2}$$

where c is the interfringe distance. When knowing the light wavelength used to produce the interference, the difference of interference orders $\delta m(x)$ may be determined in a arbitrary point x.

2.1. Results of simulation examination

The measured interference order δm is a function of variables: $\hat{r}$, d, Δ, α and b. It has been examined how the interference order depends on those parameters. The following conclusions have been formulated:

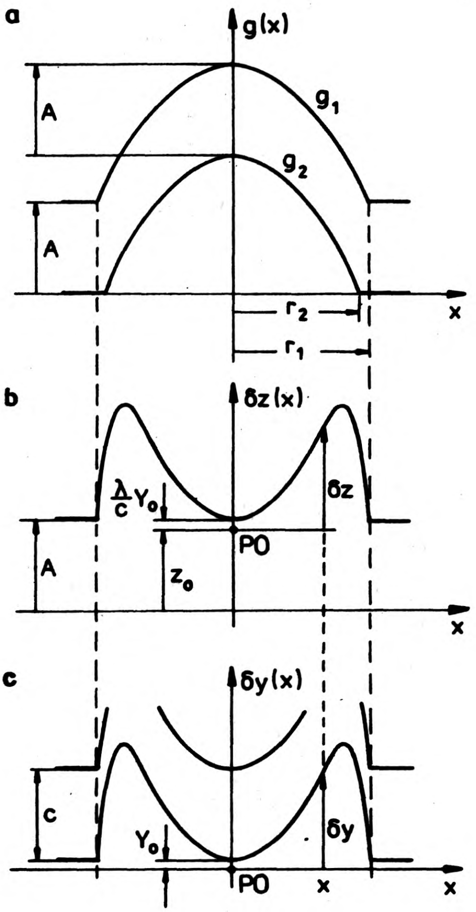

Fig. 1. Radial shearing interferometry: **a** – mutually interfering wavefronts g_1 and g_2; **b** – optical path difference δz; **c** – interference fringes on the interferogram

i) As shown in [4], the normed magnitude

$$\delta\hat{m}(\hat{r}, \alpha) = \frac{\delta m(\hat{r}, d, \Delta, \alpha, b)}{d\Delta(1-b)} \tag{3}$$

is constant in the first approximation for various values of d, Δ, b.

ii) The normed interference order $\delta\hat{m}$ depends nonlinearly on $\hat{r}$ and α (Fig. 2). Nonlinear dependence of $\delta\hat{m}$ on α (for constant $\hat{r}$) may be seen distinctly in Fig. 3.

iii) It follows from the accurate study that the normed order of interference $\delta\hat{m}$ depends linearly on Δ for fixed $\hat{r}$ (Fig. 4). This is correction to the point i).

iv) Normed order of interference $\delta\hat{m}$ depends almost linearly on the shearing parameter b (Fig. 5). This is a correction to the point i).

It has been shown [4] that the parameters Δ and α may be calculated by

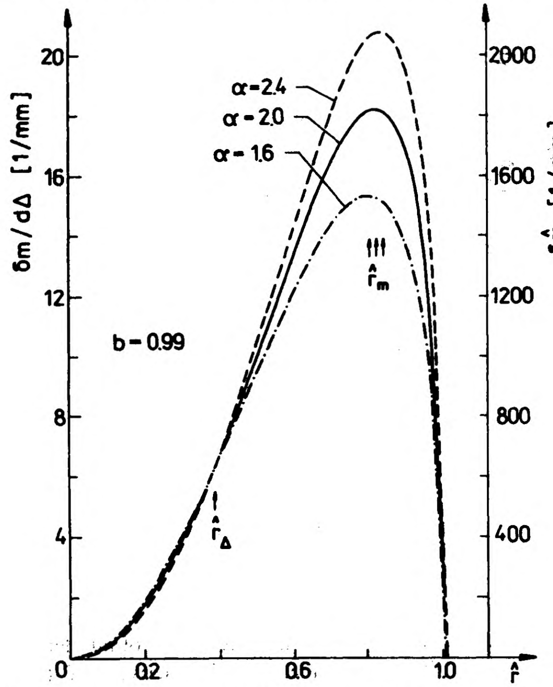

Fig. 2. Ratio of the relative order of interference δm to the product of the core diameter d and the parameter Δ related to α. Normed order of interference $\delta \hat{m}$ depending on α

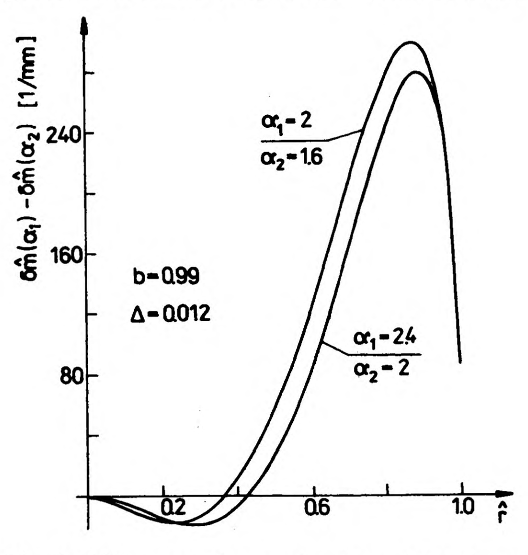

Fig. 3. Differences of normed orders of interference as depending on different values of α

measuring δm in two locations on the core radius, while in order to calculate Δ it is sufficient to measure δm for one exactly defined value of the radius. We show that this problem may be solved more universally (Sect. 2.2) by means of general method given in [3].

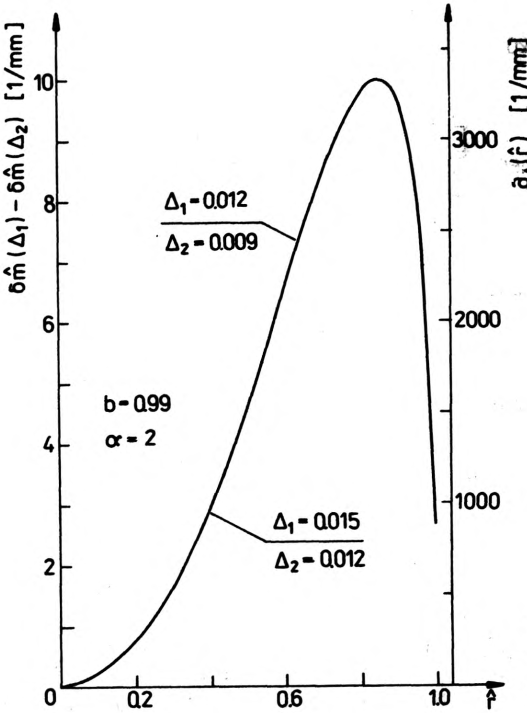

Fig. 4. Differences of normed orders of interference with different parameters Δ. The derivative $\partial_\Delta(\hat{r})$ of the normed order of interference with respect to the parameter Δ

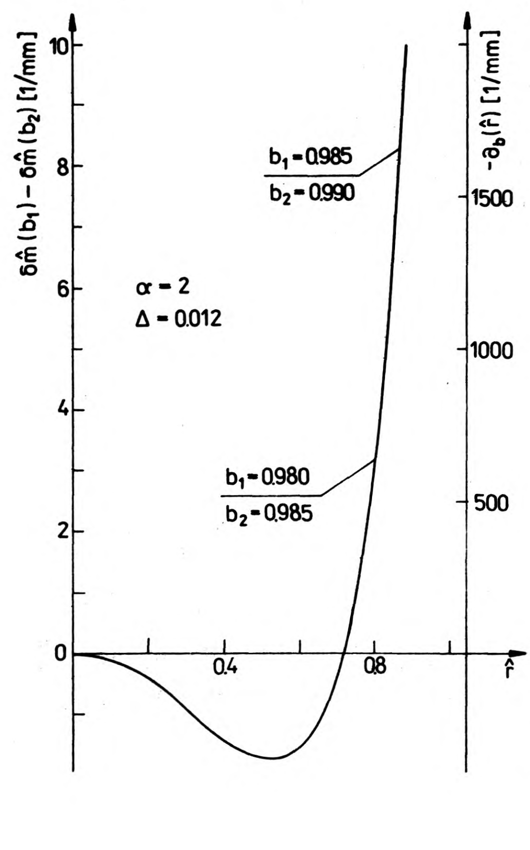

Fig. 5. Differences of normed orders of interference with different shearing parameters b. The derivative $\partial_b(\hat{r})$ of the normed order of interference with respect to the parameter b

2.2. Generalized method

The conclusions presented in Sect. 2.1 will be helpful for derivation of the mathematical relations given below. It follows that the normed order of interference is a function of $\hat{r}$, Δ, α and b, which is continuous in the whole range of the examined arguments.

2.2.1. Three-point method

To describe the normed order of interference $\delta\hat{m}$ the following conclusions have been taken into considerations:

1. Nonlinear dependence $\delta\hat{m}(\hat{r},\ \alpha)$ with respect to $\hat{r}$ and α (Fig. 3)

$$\delta\hat{m}(\hat{r},\ \alpha) = a_0(\hat{r})+a_1(\hat{r})\alpha+a_2(\hat{r})\alpha^2 \tag{4}$$

where a_h are the coefficients dependent on $\hat{r}$.

2. Linear dependence of $\delta\hat{m}$ on Δ

$$\delta\hat{m}(\hat{r},\ \Delta) = \partial_\Delta(\hat{r})(\Delta-0.012) \tag{5}$$

where $\partial_\Delta(\hat{r}) = \partial\delta\hat{m}/\partial_\Delta$ is partial derivative (Fig. 4).

3. Almost linear dependence $\delta\hat{m}$ on the shearing parameters b (Fig. 5)

$$\delta\hat{m}(\hat{r},\ b) = \partial_{b1}(\hat{r})(b-0.99)+\partial_{b2}(\hat{r})(b-0.99)^2 \tag{6}$$

where ∂_{b1} abd ∂_{b2} describe the partial derivative $\partial_b(\hat{r}) = \partial\delta\hat{m}/\partial b$. In many cases, we may restrict ourselves to the linear dependence

$$\delta\hat{m}(\hat{r},\ b) = \partial_b(\hat{r})(b-0.99), \tag{6a}$$

$\partial_b(\hat{r})$, $\partial_{b1}(\hat{r})$, $\partial_{b2}(\hat{r})$ are given in Tab. 1.

Equation (4) describes the dependence for $\Delta = 0.012$ and $b = 0.99$, while Eqs. (5) and (6) describe the relative changes evolved by Δ and b in relation to the Eq. (4), so the general dependence takes the form

$$\begin{aligned}\delta\hat{m}(\hat{r},\ \Delta,\ \alpha,\ b) = {} & a_0(\hat{r})+a_1(\hat{r})\alpha+a_2(\hat{r})\alpha^2+\partial_\Delta(\hat{r})(\Delta-0.012)\\ & +\partial_{b1}(\hat{r})(b-0.99)+\partial_{b2}(\hat{r})(b-0.99)^2,\end{aligned} \tag{7}$$

or, when using Eq. (6a) instead of (6), the form

$$\delta\hat{m}(\hat{r},\ \Delta,\ \alpha,\ b) = a_0(\hat{r})+a_1(\hat{r})\alpha+a_2(\hat{r})\alpha^2+\partial_\Delta(\hat{r})(\Delta-0.012)+\partial_b(\hat{r})(b-0.99). \tag{7a}$$

The respective coefficients a_h, ∂_Δ, ∂_b for some values of $\hat{r}$ are given in Tab. 1. The values of these coefficients (without ∂_{b1} and ∂_{b2}) for arbitrary $\hat{r}$ may be calculated from the series of coefficients given in Tab. 2.

By measuring $\delta m(\hat{r})$, for three different values of $\hat{r}$, and inserting these values to the system of three equations of the form (7), we obtain after suitable transformations the quadratic equation with respect to α

$$A\alpha^2+B\alpha+C = 0 \tag{8}$$

where:

Table 1. Coefficients $a_i(\hat{r})$ and derivatives $\partial_\Delta(\hat{r})$ and $\partial_b(\hat{r})$ describing $\delta\dot{m}(\hat{r}, \Delta, \alpha, b)$

$\hat{r}$	$a_0(\hat{r})$	$a_1(\hat{r})$	$a_2(\hat{r})$	$\partial_\Delta(\hat{r})$	$\partial_b(\hat{r})$	$\partial_{b1}(\hat{r})$	$\partial_{b2}(\hat{r})$
0.4	444.05824	233.749373	−56.7912156	1003.7673	276.5489	269.9647	−1316.853
0.44	438.79074	333.010552	−73.2524313	1223.2535	307.2772	305.0825	−438.94
0.48	420.94615	440.749872	−89.5080413	1419.3262	333.6147	331.4197	−439.0133
0.52	393.75232	551.69845	−104.323125	1668.0717	342.394	342.394	0
0.56	358.39413	662.399697	−116.874687	1919.7417	335.8087	332.5163	−658.4667
0.6	318.36029	767.504785	−126.134156	2168.4967	311.6673	307.2777	−877.9333
0.64	272.46602	865.696372	−132.101437	2428.945	241.4313	239.2357	−439.1333
0.68	224.5314	950.965047	−133.747437	2648.43	131.6893	118.5207	−2633.733
0.72	178.17651	1017.05695	−130.249437	2885.4717	−24.1433	−25.2407	−219.4666
0.76	133.53403	1060.92576	−121.812968	3107.8833	−278.745	−290.816	−2414.333
0.8	92.44601	1077.71702	−108.644187	3233.72	−625.525	−635.402	−1975.267
0.84	58.73369	1057.96324	−90.1252187	3353.7	−1150.09	−1154.48	−878.2
0.88	29.49774	998.621325	−68.5200625	3301.0283	−1953.4	−1990.71	−7462.4
0.92	12.11502	879.853222	−43.622375	3078.615	−3228.59	−3293.34	−12949.47

Table 2. Polynomial coefficients describing coefficients $a_i(\hat{r})$ and derivatives $\partial_\Delta(\hat{r})$ and $\partial_b(\hat{r})$

j	$a_i(\hat{r}) = \Sigma a_{ij}\hat{r}^j$			$\partial_\Delta(\hat{r}) = \Sigma a_{\Delta j}\hat{r}^j$	$\partial_b(\hat{r}) = \Sigma a_{bj}\hat{r}^j$
	$i = 0$	$i = 1$	$i = 2$		
		a_{ij}		$a_{\Delta j}$	a_{bj}
0	−368.751971	−4278.60453	99.366183	−28066.1897	36993.5877
1	4047.838702	43250.372	−715.846552	293524.1688	−472714.0084
2	−3978.5638	−186941.2209	2874.7285	−1254392.42	2569103.05
3	−5769.8499	434400.8039	−8772.3331	2847693.54	−7676389.49
4	8944.235265	−541774.7676	10738.0122	−3570406.05	13652488.06
5	−2868.526998	348744.9385	−4216.977809	2359759.68	−14453911.32
6		−93006.6461		−646661.8529	8432873.67
7					−2097095.26

$$
\begin{aligned}
A &= a_{2ij}/b_{ij} - a_{2kl}/b_{kl}, \\
B &= a_{1ij}/b_{ij} - a_{1kl}/b_{kl}, \\
C &= (a_{0ij}/b_{ij} - a_{0kl}/b_{kl}) + (c_{1ij}/b_{ij} - c_{1kl}/b_{kl})(b-0.99) \\
&\quad + (c_{2ij}/b_{ij} - c_{2kl}/b_{kl})(b-0.99)^2.
\end{aligned}
\tag{9}
$$

The indices i, j, k and l, taking the values: 1, 2, 3, refer to the corresponding three equations (each for another value of $\hat{r}$), while

$$
\begin{aligned}
a_{hmn} &= a_h(\hat{r}_m)\delta m(\hat{r}_n) - a_h(\hat{r}_n)\delta m(\hat{r}_m), \\
b_{mn} &= \partial_\Delta(\hat{r}_m)\delta m(\hat{r}_n) - \partial_\Delta(\hat{r}_n)\delta m(\hat{r}_m), \\
c_{pmn} &= \partial_{bp}(\hat{r}_m)\delta m(\hat{r}_n) - \partial_{bp}(\hat{r}_n)\delta m(\hat{r}_m).
\end{aligned}
\tag{10}
$$

The pair of coefficients m, n takes the values of either the pair i, j or k, l. Such notation has been accepted due to the fact that the unique determination of the solution of Eq. (8). Consequently, if the coefficients 1, 2, 3 describe the measured interference orders $\delta_m(\hat{r})$ (for the growing values of $\hat{r}_1$, $\hat{r}_2$, $\hat{r}_3$) the relation (8) may be expressed by various differences, for example:

$$\begin{aligned} &\text{a) } i = 3,\ j = 2,\ k = 2,\ l = 1,\\ &\text{b) } i = 3,\ j = 1,\ k = 3,\ l = 2,\\ &\text{c) } i = 3,\ j = 1,\ k = 2,\ l = 1. \end{aligned} \tag{11}$$

For the coefficients given in Tab. 1, there exists only one true solution of Eq. (8). For a) and c) from (11) we get

$$\alpha = \frac{-B-\sqrt{B^2-4AC}}{2A}, \tag{12}$$

and for b) from (11) – the similar equation with the sign "+" in front of the square root.

When knowing α from one of the three equations of (7), the parameter Δ may be calculated

$$A'\Delta^2+B'\Delta+C' = 0 \tag{13}$$

where:

$$\begin{aligned} A' &= d(1-b)\partial_\Delta(\hat{r}),\\ B' &= d(1-b)[a_0(\hat{r})+a_1(\hat{r})\alpha+a_2(\hat{r})\alpha^2-\partial_\Delta(\hat{r})\cdot 0.012\\ &\qquad +\partial_{b1}(\hat{r})(b-0.99)+\partial_{b2}(\hat{r})(b-0.99)^2],\\ C' &= -\delta m(\hat{r}). \end{aligned} \tag{14}$$

To only true solution is

$$\Delta = \frac{-B'+\sqrt{B'^2-4A'C'}}{2A'}. \tag{15}$$

2.2.2. Two-point method

If we decide to reduce slightly the accuracy of the measurement, the expression $\partial_\Delta(\hat{r})(\Delta-0.012)$ in (7) may be neglected. Then the parameters Δ and α may be determined from the system of two equations modified in these way. Therefore, only $\delta m(r_1)$ and $\delta m(r_2)$ are measured on the core radius. The solution of the system of two equations is the equation of the form (8), where:

$$\begin{aligned} A &= a_2(\hat{r}_2)/\delta m(\hat{r}_2)-a_2(\hat{r}_1)/\delta m(\hat{r}_1),\\ B &= a_1(\hat{r}_2)/\delta m(\hat{r}_2)-a_1(\hat{r})/\delta m(\hat{r}_1), \end{aligned}$$

$$C = [a_0(\hat{r}_2)/\delta m(\hat{r}_2) - a_0(\hat{r}_1)/\delta m(\hat{r}_1)]$$
$$+ [\partial_{b1}(\hat{r}_2)/\delta m(\hat{r}_2) - \partial_{b1}(\hat{r}_1)/\delta m(\hat{r}_1)](b-0.99)$$
$$+ [\partial_{b2}(\hat{r}_2)/\delta m(\hat{r}_2) - \partial_{b2}(\hat{r}_1)/\delta m(\hat{r}_1)](b-0.99)^2. \tag{16}$$

The only solution of Eq. (8) for the coefficients taken from Tab. 1 is

$$\alpha = \frac{-B+\sqrt{B^2-4AC}}{2A}. \tag{17}$$

When knowing α from one modified Eq. (7), we obtain

$$\Delta = \delta m(\hat{r})/[d(1-b)(a_0(\hat{r}) + a_1(\hat{r}_1)\alpha + a_2(\hat{r})\alpha^2 + \partial_{b1}(\hat{r})(b-0.99)$$
$$+ \partial_{b2}(\hat{r})(b-0.99)^2]. \tag{18}$$

As may be seen, in the method of two-points the expression for Δ and α are less complex than those in the three-points method. These relations may be simplified even more replacing Eq. (6) by Eq. (6a). Then, instead of Eq. (7) we have Eq. (7a) and the components in the sums containing the factor $(b-0.99)^2$ in Eqs. (16) and (18) vanish, while $\partial_{b1}(\hat{r})$ should be replaced by $\partial_b(\hat{r})$, in the solution for the coefficient C.

3. Transversal shearing interference

Transversal shearing interference is produced by a shearing element giving two identical wavefronts slightly shifted transversally with respect to each other [6], [9], Fig. 6. This interference is characterized by the shearing parameters $s = s'/r_0$, where s' is the transversal shift, and r_0 – the radius of the wavefront. The optical path difference δz is measured usually with respect to the so-called reference level (PO in Fig. 6**b**), which in most cases is defined by the position of the fringe in the coat of the preform or light waveguide, respectively. When the fringes in the coat are positioned perpendicularly to the axis of either the preform or the light waveguide the optical path difference may be determined from the interferogram (Fig. 6c) by using the method of the fringe deviation from the straight line (by measuring $y(x)$ and c)

$$\delta z(x) = g'(x) - g(x) = g(x-s') - g(x) = y(x)/c = \delta m(x)\lambda \tag{19}$$

where c is the interfringe distance. When knowing the light wavelength for the light used to the interference, the difference of the interference orders $\delta m(x)$ may be determined in an arbitrary point x.

3.1. Results of simulation examination

The measured interference order δm is a function of variables $\hat{r}$, d, Δ, α and s. The dependence of the interference order on these parameters has been examined which led to the following conclusions:

i) As it has been shown in [5], the normed magnitude

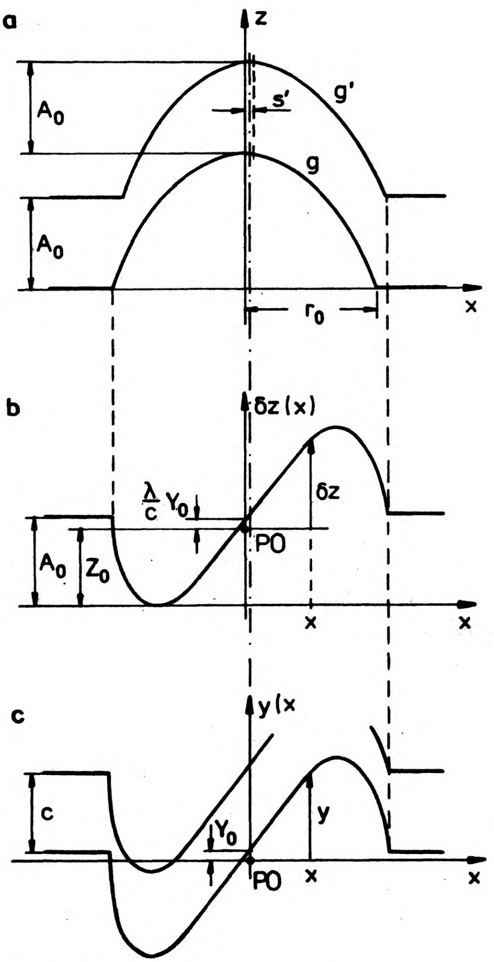

Fig. 6. Transversal shearing interference of: **a** – mutually interfering wavefronts g_1 and g_2; **b** – optical path difference δz; **c** – interference fringes on the interferogram

$$\delta\hat{m}(\hat{r}, \alpha) = \delta m(\hat{r}, d, \Delta, \alpha, s)/(d\Delta s) \tag{20}$$

is, in the first approximation, constant for different values of d, Δ, s.

ii) The normed order of interference $\delta\hat{m}$ depends nonlinearly on $\hat{r}$ and α (Fig. 7). Nonlinear dependence of $\delta\hat{m}$ on α (for fixed $\hat{r}$) may be seen distinctly in Fig. 8.

iii) From the more accurate examinations, it follows that the interference order $\delta\hat{m}$ depends linearly on Δ (for fixed $\hat{r}$), Fig. 9. This is the correction to the point i).

iv) Normed order of interference $\delta\hat{m}$ depends almost linearly on the shearing parameter s (Fig. 10). This is the correction to the point i).

It has shown [5] that the parameters Δ and α may be calculated by measuring δm in two positions on the core radius, while to calculate Δ it is sufficient to measure δm for one strictly determined value of the ray. In the generalized method, we show that this problem may be solved more generally (see Sect. 2, in the case of radial shearing).

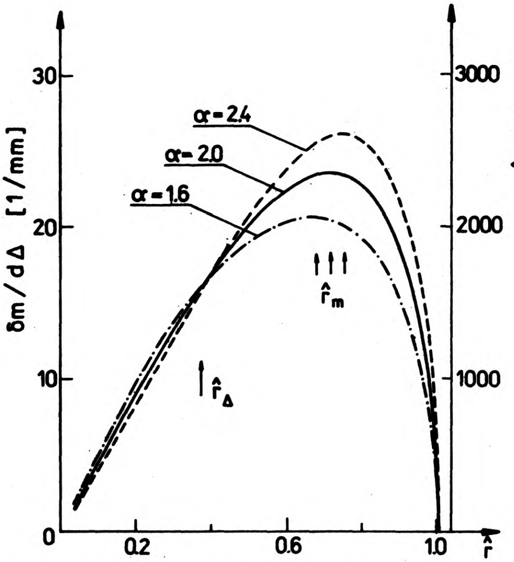

Fig. 7. Ratio of the relative order of interference δm to the product of the core diameter d and the parameter. Δ. Normed order of interference $\delta \hat{m}$ depending on α

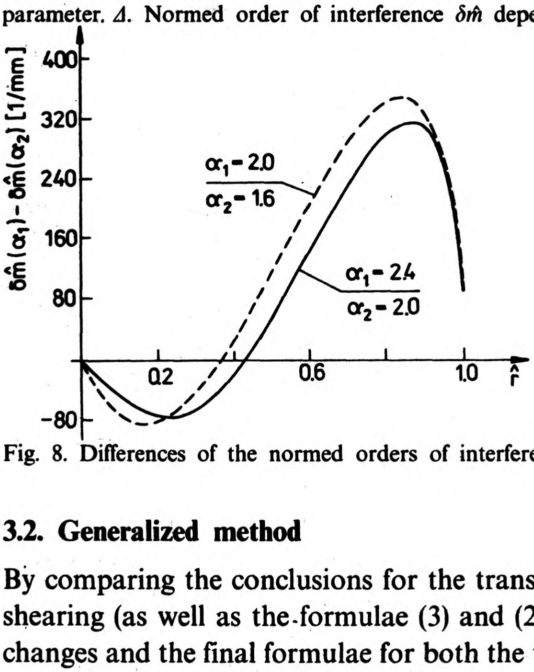

Fig. 8. Differences of the normed orders of interference as depending on different values of α

3.2. Generalized method

By comparing the conclusions for the transversal shearing with those for the radial shearing (as well as the formulae (3) and (20)), it may be seen that the character of changes and the final formulae for both the types of shearing are similar in form. The only difference consists in replacing the expression $(1-b)$ in formulae (3)–(18) for s, and ∂_{b1}, ∂_{b2}, ∂_b, $b-0.99$ for ∂_{s1}, ∂_{s2}, ∂_s, and $s-0.01$, respectively (homologue of formula (4) describes the relation for $s = 0.01$).

When taking account of the above differences all the formulae (3)–(18) are true also for the transversal shearing. This is also valid for the uniqueness of the solutions for Δ and α. And, thus, the homologue Eq. (7) is

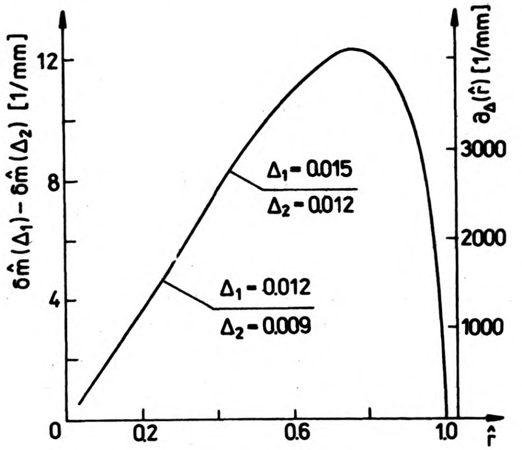

Fig. 9. Differences of normed orders of interference with different parameters Δ. The derivative $\partial_\Delta(\hat{r})$ of the normed order of interference with respect to parameter Δ

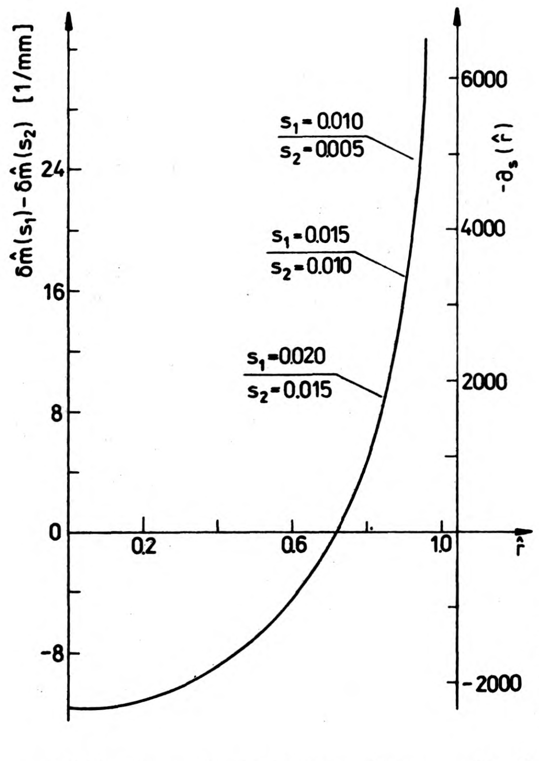

Fig. 10. Differences of normed orders of interference with different shearing parameters s. The derivative $\partial_s(\hat{r})$ of the normed order of interference with respect to the shearing parameter s

$$\delta\hat{m}(\hat{r},\ \Delta,\ \alpha,\ s) = a_0(\hat{r})+a_1(\hat{r})\alpha+a_2(\hat{r})\alpha^2+\partial_\Delta(\hat{r})(\Delta-0.012)$$
$$+\partial_{s1}(\hat{r})(s-0.01)+\partial_{s2}(\hat{r})(s-0.01)^2. \tag{7b}$$

Clearly, for the transversal shearing the coefficients for the solution of the system of Eqs. (7b) are different than those for the radial shearing, and are given in Tab. 3 and 4. In the case of transversal shearing, Eq. (8) has the coefficients:

Table 3. Coefficients $a_i(\hat{r})$ and derivatives $\partial_\Delta(\hat{r})$ and $\partial_s(\hat{r})$ describing $\delta\hat{m}(\hat{r}, \Delta, \alpha, s)$

$\hat{r}$	$a_0(\hat{r})$	$a_1(\hat{r})$	$a_2(\hat{r})$	$\partial_\Delta(\hat{r})$	$\partial_s(\hat{r})$	$\partial_{s1}(\hat{r})$	$\partial_{s2}(\hat{r})$
0.4	1116.4669	572.68575	−140.331875	2510.89	1755.865	−1733.92	−4389.467
0.44	1001.7646	748.82141	−165.846906	2739.15	1624.175	−1602.23	−4389.866
0.48	885.35094	907.672122	−185.188812	2961.56	1496.874	−1499.07	439
0.52	764.19597	1053.18972	−200.004	3189.82	1308.121	−1299.34	−1755.667
0.56	645.80723	1177.47156	−208.646031	3406.3767	1093.025	−1060.1	−6584.533
0.6	532.8169	1278.211478	−211.115343	3581.9633	904.27	−899.879	−878.2
0.64	428.78191	1351.30263	−207.000031	3763.405	610.1627	−581.628	−5706.867
0.68	333.83372	1397.06467	−197.12325	3915.5783	320.4447	−305.08	−3072.867
0.72	247.44517	1415.50122	−182.308125	4032.6333	−17.558	21.947	−877.8
0.76	176.02727	1398.7403	−161.4435	4071.44	−447.879	467.8303	−3990.333
0.8	116.19288	1349.32511	−136.566406	4074.54	−963.179	988.7253	−5109.333
0.84	70.58216	1262.0229	−108.043187	3976.9133	−1605.91	1634.71	−5759.2
0.88	36.94688	1134.243	−77.6764375	3761.9433	−2484.94	2525.391	−8089.8
0.92	15.393226	956.17524	−47.1408437	3374.6533	−3794.6	3870.014	−15082.87

Table 4. Polynomial coefficients describing coefficients $a_i(\hat{r})$ and $\partial_\Delta(\hat{r})$ and $\partial_s(\hat{r})$

j	$a_i(\hat{r}) = \Sigma a_{ij}\hat{r}^j$			$\partial_\Delta(\hat{r}) = \Sigma a_{\Delta j}\hat{r}^j$	$\partial_s(\hat{r}) = \Sigma a_{sj}\hat{r}^j$
	$i=0$	$i=1$	$i=2$		
		a_{ij}		$a_{\Delta j}$	a_{sj}
0	788.272942	475.6018	334.209969	5619.0525	195639.1
1	7233.195555	−11753.1317	−1681.254471	−42276.1136	−2310108.68
2	−24369.6211	57633.9605	1690.132402	168207.0086	11666021.58
3	23833.8392	−92959.3544	−2649.4027	−292438.9268	−32320011.46
4	−7481.958501	68040.4324	4777.488792	256389.0588	53049354.11
5		−21006.3065	−2464.938837	−93578.2118	−51657655.95
6					27660963.86
7					−6293828.17

$$A = a_{2ij}/b_{ij} - a_{2kl}/b_{kl},$$

$$B = a_{1ij}/b_{ij} - a_{1kl}/b_{kl},$$

$$C = [a_{0ij}/b_{ij} - a_{0kl}/b_{kl}] + [c_{1ij}/b_{ij} - c_{1kl}/b_{kl}](s-0.01) + [c_{2ij}/b_{ij} - c_{2kl}/b_{kl}](s-0.01)^2 \tag{9b}$$

where:

$$a_{hmn} = a_h(\hat{r}_m)\delta m(\hat{r}_n) - a_h(\hat{r}_n)\delta m(\hat{r}_m),$$

$$b_{mn} = \partial_\Delta(\hat{r}_m)\delta m(\hat{r}_n) - \partial_\Delta(\hat{r}_n)\delta m(\hat{r}_m),$$

$$c_{pmn} = \partial_{sp}(\hat{r}_m)\delta m(\hat{r}_n) - \partial_{sp}(\hat{r}_n)\delta m(\hat{r}_m). \tag{10b}$$

Similarly, Eq. (13) has the coefficients:

$$\begin{aligned} A' &= ds\partial_\Delta(\hat{r}), \\ B' &= ds[a_0(\hat{r})+a_1(\hat{r})\alpha+a_2(\hat{r})\alpha^2-\partial_\Delta(\hat{r})0.012 \\ &\quad +\partial_{s1}(\hat{r})(s-0.01)+\partial_{s2}(\hat{r})(s-0.01)^2], \\ C' &= -\delta m(\hat{r}). \end{aligned} \tag{14b}$$

For the two-point method, the coefficients are also slightly different:

$$\begin{aligned} A &= a_2(\hat{r}_2)/\delta m(\hat{r}_2)-a_2(\hat{r}_1)/\delta m(\hat{r}_1), \\ B &= a_1(\hat{r}_2)/\delta m(\hat{r}_2)-a_1(\hat{r}_1)/\delta m(\hat{r}_1), \\ C &= [a_0(\hat{r}_2)/\delta m(\hat{r}_2)-a_0(\hat{r}_1)/\delta m(\hat{r}_1)] \\ &\quad +[\partial_{s2}(\hat{r}_2)/\delta m(\hat{r}_2)-\partial_{s1}(\hat{r}_1)/\delta m(\hat{r}_1)](s-0.01) \\ &\quad +[\partial_{s2}(\hat{r}_2)/\delta m(\hat{r}_2)-\partial_{s1}(\hat{r}_1)/\delta m(\hat{r}_1)](s-0.01)^2, \end{aligned} \tag{16b}$$

and the expression of Δ has the form

$$\Delta = \delta m(\hat{r})/[ds(a_0(\hat{r})+a_1(\hat{r})\alpha+a_2(\hat{r})\alpha^2+\partial_{s1}(\hat{r})(s-0.01)+\partial_{s2}(\hat{r})(s-0.01)^2)]. \tag{18b}$$

The simplifications in the solution of two Eqs. (16b) and (18b) due to neglecting the respective components of sums in two above equations comprising the factor $(s-0.01)^2$ are also possible and result in only slight worsening of the reconstruction accuracy of the parametres Δ and α.

4. Measurement accuracies

Due to almost identical mathematical dependences for the methods of radial and transversal shearing a common analysis of the measurement accuracy is possible. In this analysis, it has been assumed that the relative error in determination of the relative interference order is equal to $\sigma(\delta m(\hat{x})/\delta m(\hat{x}) = 0.05$, in the case of visual measurements, and to 0.01 in the case of a scanning device, respectively. Besides, it has been also assumed that the relative error of the core diameter measurement is equal to $\sigma(d)/d = 0.005$, while the error of the normed coordinate is equal to $\sigma(\hat{x}) = \sigma(\hat{r}) = 0.001$. It has been also assumed that the errors of the shearing parameters fulfill the conditions $\sigma(1-b)/(1-b) = \sigma(s)/s = 0.01$, which for typical values $b = 0.99$ and $s = 0.01$ gives $\sigma(1-b) = \sigma(s) = 0.0001$.

The analysis of the accuracy has been carried out partially by computer simulation of the respective dependences as well as by analysis the suitable equations analogically as it was the case in works [3]–[5].

The main conclusions following from this analysis as follows:

1. Errors due to reconstruction of the results on the basis of the given coefficients are practically avoidable.

2. The biggest measurement errors are introduced by: the approximation of the

zero order used to calculate the coefficients given in the tables, the accuracy of the interference order determination $\sigma(\delta m(\hat{x}))$, the accuracy of the coordinate measurement $\sigma(\hat{x})$, and the accuracy of the shearing parameters $\sigma(b)$ and $\sigma(s)$.

3. In practice, the sufficient accuracy is provided by the two-point method.

4. The measurement accuracies of Δ and α determination for the visual method are equal to 4.7% and 5.5%, respectivelly, while those for the method with scanning device are 3.6% and 4%, respectively.

The above errors have been calculated under an assumption that the measurement of the fringe deformation in two (or three) positions on the core radius may truly reconstruct the profile course along the whole radius. This is true only to such degree to which the real profile may be described by the dependence (1). All the deviations from this dependence may increase the calculated errors. In order to diminish such errors the deformation may be measured in three or four positions on the radius (preferably for its left and right hand sides) and the calculations should be performed for the combination of different pairs of the coordinates to calculate the averaged parameters Δ and α.

The above accuracies may be improved (so that the errors diminish by about 1.5%), if instead of the zero order approximation the accurate calculation reported in [10] are applied. Obviously, the whole cycle of calculations presented in the present paper should be repeated. Then all the dependences and formulae stay the same and only the values of the coefficients in Tables 1–4 will change.

5. Conclusions

It has been shown that for drastically reduced data extracted from interferogram the results obtained are only slightly less accurate. It has been also shown that by studying the space of results the analytic shape of the function which describes the results may be found and the time consuming transitions from the data space to the space of results may be replaced by a simple calculation of parameters (Δ and α) describing the function (1) of the examined object.

The proposed method requires the function of results (7) to be found earlier, being expressed by the sought parameters (Δ and α) which describe the object functions (1). This is, in general, the problem of multidimensional approximations function as defined on the parameters of the object function (Δ and α) general object properties (d, $\hat{r}$) and the kind of interference (b or s). The approximation concerns the data $(\delta m(\hat{r}, d, \Delta, \alpha, b))$ obtained either by the method of computer simulation or from the direct measurements. The simulation is made with respect to the object function (1) and the kind of interference (2) or (19). Clearly, the object function and its general properties generate the suitable wavefront, which creates the basis for calculations. In order to find the connection of the function of results with the sought parameters, the system of (generally nonlinear) Eqs. (7) must be solved. Such a method of solving the problem (as one of many possible) has been presented in this paper. In the case of carrying out approximation procedure with respect to the measurement data (when (1) is not known), the suitable form of the object function must be found. This function should recover the real distribution of the refractive index in the examined type of the object with sufficient accuracy, while the number of the sought parameters describing this function should be possibly small. The finding of such general form of the object function for the difinite type of the object is possible (at the expense of the suitable extention of the softwave) by introducing to the computer the detailed information from several interferograms of the objects of the same type. Thus, the measuring system may be "taught" how to measure different types of the objects for the reduced number of data. The works in this direction (as shown in this paper) are encouraging and will be continued. This may be useful when constructing the "intelligent" measuring system.

The method suggested is especially advantageous when many measurements are carried out for similar objects. The reduction of the data allows us to replace the time-consuming analysis of the interferogram as well as the expensive apparatus needed by a very simple scanning device to analyse only few lines. The simplified method of calculations allows obtaining the results in the real time, for instance, during the preform or the light waveguide through the interferometer.

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Быстрое измерение параметров Δ и α профиля коэффициента преломления в световодовых преформах. Обобщенные методы типа shearing

В работе представлен метод быстрого изменения параметров Δ и α, описывающих профиль коэффициента преломления преформ и световодов. Описаны интерференционные методы поперечный shearing и радиальный shearing. Предлагаемый метод дает возможность применения очень простого сканинга интерферограмм, а расчеты сводятся к вычислениям готовых уравнений с данными коэффициентами.

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