Optica Applicata, Vol. XX, No. 3, 1990

Sphero-chromatic aberration of holographic lens*

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Holographic lens is burdened with considerable chromatic aberration. The dependence of the image position on the used light wevelength creates the possibility of using the holo-lens as a spectral device. For this aim the holo-lens recording and imaging geometries should be chosen suitably to minimize the spherical aberration for as wide a range of the light wavelengths as possible. In the paper, the helpful formulae are derived, and their application is illustrated with an example.

1. Introduction

The specific optical properties of holographic optical elements have resulted in their use in a variety of optical systems [1], [2]. The holographic lens (holo-lens), being a particular kind of holographic optical element, is also a subject of special interest nowadays. The specific feature of the holo-lens is its great chromatism [3], following from the fact that its operating principle is based on the diffraction of light on a quasiperiodic structure.

The application of holo-lens is therefore limited usually to the operating in monochromatic or quasimonochromatic light; the chromatic aberration being treated generally as an undesired difficulty. The influence of chromatic aberration can be minimized by designing a multi-element optical system composed of two or more holo-lenses [4]–[8].

The specific dependence of the imaging characteristic on the used light wavelength can be, however, a profitable property of such optical element. Namely, it is worth trying to build-up a spectral device (a monochromator or scanning spectroscope) using a single holo-lens as a dispersive and focusing element simultaneously [9], [10]. It seems therefore necessary to consider the possibility of correction of the non-monochromatic aberrations as a preliminary step.

2. Correction of sphero-chromatic aberration

Let us consider a holographic lens recorded according to geometry shown in Fig. 1a. Symbols P_α and P_β denote point sources of spherical waves creating the holo-lens; z_α and z_β are the respective distances from these points to the holo-lens plane. The light

* The research was supported by the Ministry of National Education, Fund for Recognition Research; grant No. 33000–4.

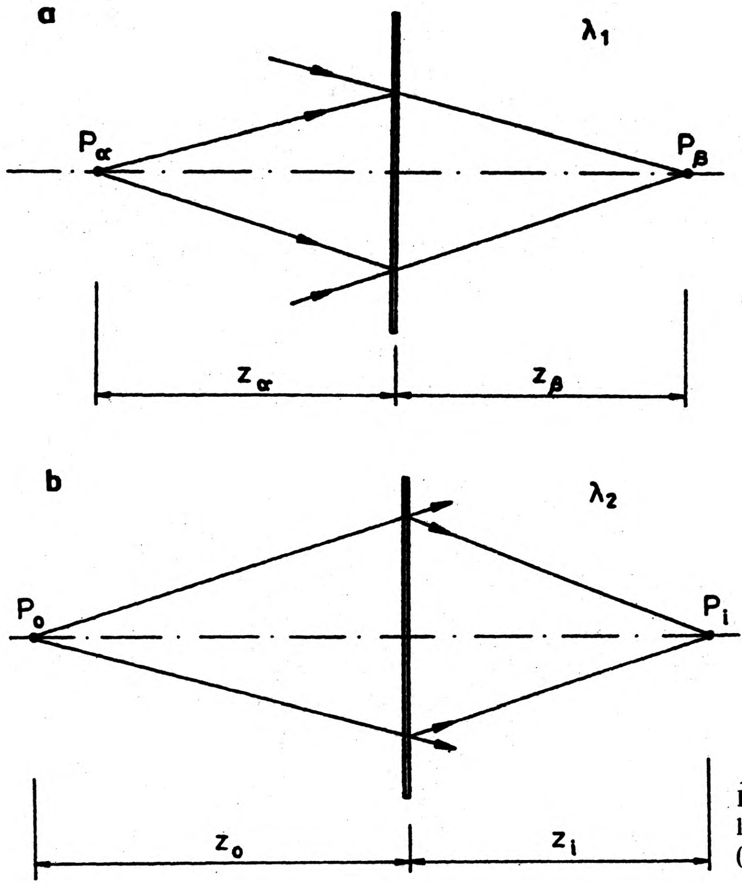

Fig. 1. Geometry of the holo-lens recording (a), and imaging (b)

wavelength used during the holo-lens recording is λ_1.

This holo-lens is used to image a point object P_o located in the distance z_o in front of it as it is shown in Fig. 1**b**. The light wavelength λ_2 used in this step is, in general, different than λ_1 and $\lambda_2/\lambda_1 = \mu$. The image P_i is observed in Gaussian plane situated in the distance z_i behind the holo-lens. This z coordinate of the Gaussian image can be calculated from the Meier formula

$$\frac{1}{z_i} = \frac{1}{z_o} \pm \mu\left(\frac{1}{z_\alpha} - \frac{1}{z_\beta}\right) \tag{1}$$

where the upper sign corresponds to the primary image, and the lower sign – to the secondary one.

A focal length f can be ascribed to such holo-lens

$$f = \left[\pm\mu\left(\frac{1}{z_\alpha} - \frac{1}{z_\beta}\right)\right]^{-1}. \tag{2}$$

If the object point P_o is located on axis, than all the third-order aberration coefficients except the one describing the spherical aberration S are equal to zero, while

$$S = \frac{1}{z_o^3} \pm \mu\left(\frac{1}{z_\alpha^3} - \frac{1}{z_\beta^3}\right) - \frac{1}{z_i^3}. \tag{3}$$

To simplify the notation the following unimensional parameters describing the holo-lens recording and imaging geometries are introduced:

$$\begin{aligned} z_\alpha &= z, \\ z_\beta &= z/r, \\ z_o &= z/p, \\ z_i &= z/t, \\ f &= z/q. \end{aligned} \tag{4}$$

Formulae (1)–(3) can be now rewritten in the from:

$$q = \pm\mu(1-r), \tag{5}$$

$$t = p \pm \mu(1-r), \tag{6}$$

$$S = [p^3 \pm \mu(1-r^3) - t^3]/z^3. \tag{7}$$

In the last equation the constant factor $1/z^3$ has no essential meaning and will be ignored.

For the fixed values of parameters: r – describing the holo-lens recording geometry, and p – bearing information about imaging conditions, the spherical aberration depends only on the light wavelength used while imaging, characterized by the coefficient μ. The dependence of S on μ can be obtained by substituting the formula (6) into (7). After suitable rearranging the function $S(\mu)$ can be expressed as

$$S = \pm(1-r^3)\mu^3 - 3p(1-r)^2\mu^2 \pm [1-r^3-3p^2(1-r)]\mu,$$

or

$$S = \pm\mu\left\{\mu + \frac{3p+\sqrt{4(1-r)^3/(1-r)-3p^3}}{2(1-r)}\right\}\left\{\mu + \frac{3p-\sqrt{4(1-r)^3/(1-r)-3p^3}}{2(1-r)}\right\}. \tag{8}$$

The function $S(\mu)$ has a form of a third order polynomial and its graphs for some exemplary values of parameters r and p (and for $\mu > 0$) are presented in Fig. 2**a** and **b**. The first root of this function is $\mu_1 = 0$ and is of no meaning to us. The other ones (if they exist) fall on the values:

$$\mu_2 = \frac{3p+\sqrt{4(1-r)^3/(1-r)-3p^3}}{2(1-r)}, \tag{9}$$

$$\mu_3 = \frac{3p-\sqrt{4(1-r)^3/(1-r)-3p^3}}{2(1-r)}. \tag{10}$$

If the primary image is considered, the function $S(\mu)$ has a local maximum for

$$\mu_0 = 3p/2(1-r). \tag{11}$$

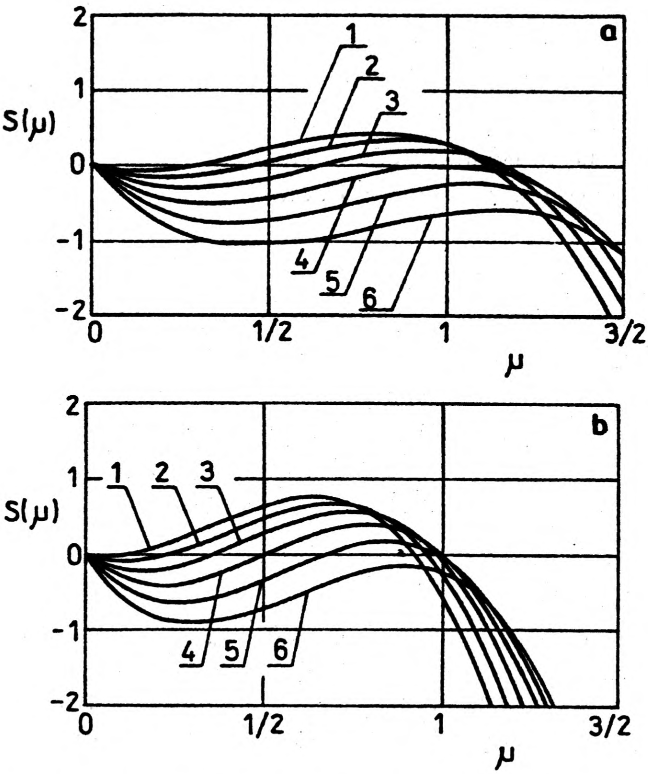

Fig. 2. Spherical aberration coefficient S versus the relative light wavelength μ for different values of the parameter p and selected values of the paremeter $r = -1/2$ (**a**), and $r = -1$ (**b**). 1 – $p = -0.7$, 2 – $p = -0.8$, 3 – $p = -0.9$, 4 – $p = -1.0$, 5 – $p = -1.1$, 6 – $p = -1.2$

There is no possibility for the spherical aberration to vanish for more than 2 values of the parameter μ greater than 0, and consequently it is impossible to obtain the aberration-free image for the wider spectrum of the light wavelengths. By proper choice of the parameters r and p the aberration free imaging can be obtained for the desired value of μ; hovever, any change of the imaging light wavelength will cause the appearence of spherical aberration and image blurring. It can be expected that for the given position of a point object and observation plane (fixed p and t) even if the polychromatic light is used for imaging a sharp image will be formed only in one particular wavelength; the other wavelengths giving smeared and low intensity images.

On the other hand, any shift observation plane (changing of the parameter t only) corresponds to the selection of the other light wavelength (other μ), for which the Gaussian image relation (5) is fulfilled

$$\mu = (t-p)/(1-r). \tag{12}$$

This fact is a base for an idea of exploiting a holo-lens as a monochromator or a scanning spectroscope [10]. It seems to be possible to select the light wavelength by shifting the observation plane along the axis perpendicular to the holo-lens. By placing an opaque screen with a pinhole in different distances z_i from the holo-lens a spectral device (simple monochromator) can be built-up. It cannont be expected, however, that the image remains aberration free, as before, because the parameters μ, r, p, and t no longer fulfill the conditions (9) nor (10).

The form of the expression (8) as well as the shape of the curves in Figs. 2**a** and **b** suggest that it is possible to have $S(\mu) = 0$ for some value of $\mu = \mu_0$ and simultaneously $dS/d\mu = 0$ for the same $\mu = \mu_0$. This means that a slight change of the light wavelength should not cause observable increase of spherical aberration, and therefore a better correction of sphero-chromatic aberration can be achieved. The necessary condition is

$$p = p_0 = -\frac{2}{\sqrt{3}}\sqrt{\frac{1-r^3}{1-r}} \tag{13}$$

(the "$-$" sign is chosen to assure the image to be real). Then:

$$\mu = \mu_2 = \mu_3 = \mu_0 = \sqrt{3\frac{1-r^3}{(1-r)^3}}, \tag{14}$$

$$t = t_0 = \frac{-2\pm 3}{\sqrt{3}}\sqrt{\frac{1-r^3}{1-r}}, \tag{15}$$

$$q = q_0 = \sqrt{3\frac{1-r^3}{1-r}} \tag{16}$$

where the upper sign corresponds to the primary image, and the lower sign — to the secondary one.

The lateral magnification in such a situation does not depend on the parameter r and amounts

$$\frac{t_0}{p_0} = \frac{2\pm 3}{2}. \tag{17}$$

The exemplary curves $S(\mu)$ for selected values of the parameter r and corresponding values of parameter p (according to the formula (13)) are presented in Fig. 3.

It can be seen in this picture, that as the value of parameter r comes closer to zero then the curve $S(\mu)$ becomes more flat. This means that the increase of spherical aberration accompanying the shift of the image plane is less significant if absolute value of r is smaller.

While choosing the geometry of the holo-lens recording the formulae (13)–(15) are the most important. According to the value of μ_0 for which exact correction is desired the other parameters should be established, having in mind, however, the technological limitations of r and p. For easy evaluation of these parameters the curves illustrating $\mu_0(r)$ and $p_0(r)$ presented in Fig. 4 can be useful.

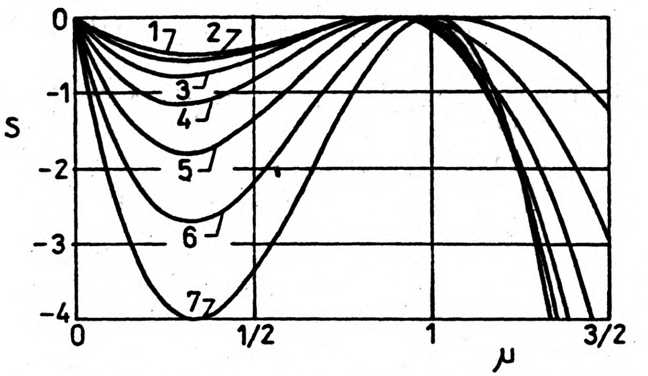

Fig. 3. Spherical aberration coefficient S versus the relative light wavelength μ for selected pairs of the parameters r and p. 1 – $r = -0.5$, $p = -1.0$; 2 – $r = -0.75$, $p = -1.0408$; 3 – $r = -1$, $p = -1.1547$; 4 – $r = -1.25$, $p = -1.3229$; 5 – $r = -1.5$, $p = -1.5275$; 6 – $r = -1.75$, $p = -1.7559$; 7 – $r = -2$, $p = -2$

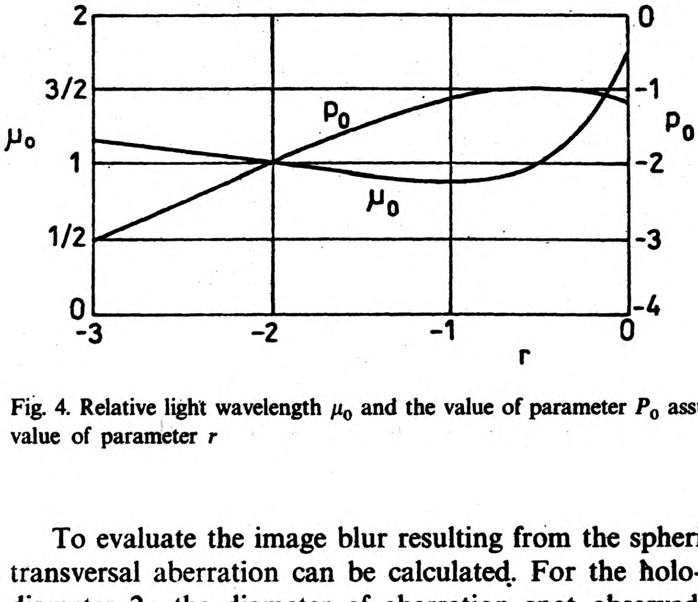

Fig. 4. Relative light wavelength μ_0 and the value of parameter P_0 assuring the full correction versus the value of parameter r

To evaluate the image blur resulting from the spherical aberration the value of transversal aberration can be calculated. For the holo-lens with circular pupil of diameter 2ρ the diameter of aberration spot observed in Gaussian plane is

$$\delta_s = \rho^3 z_i S \tag{18}$$

or, after introducing parameters r, p, t, and omitting constant factors,

$$\delta_s = \frac{p^3 \pm \mu(1-r^3) - t^3}{p \pm \mu(1-r)}. \tag{19}$$

Sometimes it is useful to compare the aberration spot size to the respective diffraction spot whose diameter is

$$\delta_D = \lambda_2 z_i / \rho. \tag{20}$$

The ratio of the above values is:

$$\delta_{SD} = \delta_S/\delta_D = (p^3 - t^3)/\mu \pm (1 - r^3) \tag{21}$$

(constant scaling factor $K = \rho^4/z^3\lambda_1$ is omitted here).

3. Conclusions

It seems that it is possible to exploit the formulae presented above for designing the simple, single-element monochromator or scanning spectroscope. With this aim it is necessary to find the proper holo-lens recording geometry as well as imaging geometry, assuring the minimization of the spherical aberration. It can be done with the help of Fig. 5 which presents the curves illustrating the spherical aberration coefficient S, the parameter t describing the Gaussian image location and the relative size of the aberration spot δ_{SD} versus the parameter μ for a selected holo-lens defined by the parameter r. These curves constitute the most important characteristic of the holo-lens used as a monochromator. They enable us to estimate the range of the image observation plane shift (parameter t) for which the aberration spot (δ_{SD} ratio) does not exceed the admissible value, as well as the corresponding change of the light wavelength (parameter μ). It should be noted, however, that the value of t must not be negative or equal to zero if only the real image is of interest. Therefore the minimum light wavelength for which a real image can be observed is limited to the parameter $\mu_{min} = 0.75\ \mu_0$.

Following the above calculations an example of a holo-lens is chosen. For the value of $r = -1$ we have: $\mu_0 = 0.8660$, $p_0 = -1.1547$, $t_0 = 0.5777$. Assuming that the light wavelength used to record this holo-lens $\lambda_1 = 514.5$ nm and its focal length for the wavelength coresponding to the full correction ($\lambda_0 = 445.6$ nm) is $f_0 = 100$ mm, the respective dimensions are: $z_\alpha = z_\beta = 193.2$ mm, $z_o = -150$ mm.

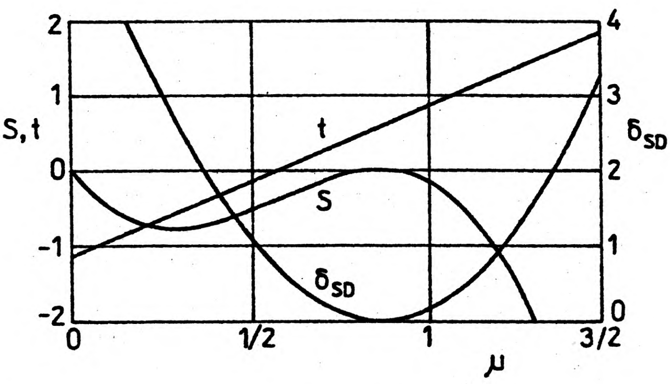

Fig. 5. Spherical aberration coefficient S, the value of parameter t and the relative size of aberration spot δ_{SD} versus the relative light wavelength μ_0 for the exemplary holo-lens characterized by $r = -1$ and $p = -1.1547$

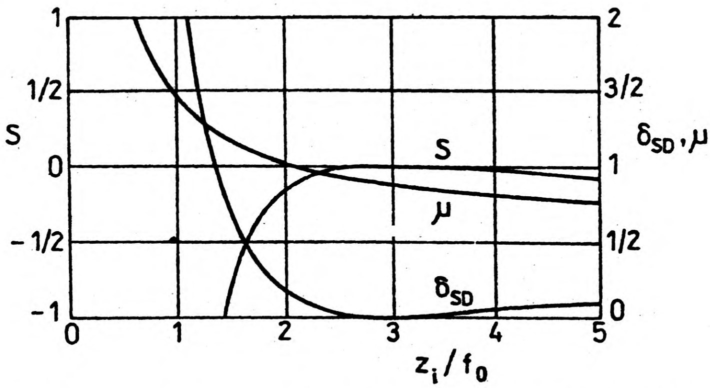

Fig. 6. Relative light wavelength μ, the spherical aberration coefficient S and the relative size of aberration spot δ_{SD} versus the relative location of image observation plane z_i/f_0 for the exemplary lens characterized by the parameters $r = -1$, $p = -1.1547$, $\mu_0 = 0.886$

Spherical aberration coefficient (S) and the relative size of the aberration spot (δ_{SD}) in relation to the location of image plane (t, z_i) and light wavelength (μ, λ_2) for the exemplary holo-lens recorded with $z_\alpha = -z_\beta = 173.2$ mm and $\lambda_1 = 514.5$ nm; used for imaging with $z_o = -150$ mm

μ[−]	λ_2[nm]	t[−]	z_i[mm]	S*[−]	δ_{SD}**[−]
0.6	308.7	0.0453	3823.4	−0.340	0.56
0.7	360.0	0.2453	706.1	−0.151	0.22
0.8	411.6	0.4453	389.0	−0.028	0.04
0.866	445.6	0.5773	300.0	0.000	0.00
0.9	463.0	0.6453	268.4	−0.008	0.01
1.0	524.5	0.8453	204.9	−0.144	0.14
1.1	566.0	1.0453	165.7	−0.482	0.44
1.2	617.4	1.2453	139.1	−1.071	0.89
1.3	669.0	1.4453	119.8	−1.959	1.51
1.4	720.3	1.6453	105.3	−3.367	2.28
1.5	771.8	1.8453	93.9	−4.823	3.22
1.6	832.0	2.0453	84.7	−6.896	4.31

* To obtain real values of S the values in the Table should be multiplied by $z^{-3} = 1.925 \times 10^{-7}$.

** To obtain real values of δ_{SD} the values in the Table should be multiplied by $\rho^4/z^3\lambda_1 = 3.741$ for the holo-lens of diameter $2\rho = 20$ mm.

By shifting the plane of image detection z_i from about 15000 mm to 88.6 mm the wavelength λ_2 belonging to the interval 300 nm to 800 nm is selected. The exact values of these quantities as well as the spherical aberration coefficient S and the relative diameter of the aberration spot δ_{SD} are collected in the Table. The same information is shown in Fig. 6. It seems that the quality of the images observed in distances z_i from the holo-lens ranging from 700 mm to 100 mm corresponding to the light wavelength interval from 360 nm may be assumed as satisfactory.

The present work is to be continued with the aim to design a monochromator or scanning spectroscope with a single holo-lens acting as a dispersive and imaging element simultaneously. The obtained results will be reported in the near future.

Acknowledgements – The author wishes to express his thanks to Doc. Dr J. Nowak and Dr B. Dubik for inspiration and many helpful discussions.

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Received March 9, 1990

Сферо-хроматическая аберрация голографической линзы

Голографическая линза обладает значительной хроматической аберрацией. Зависимость расположения изображения от использованной длины волны создает возможность использовать голографическую линзу в качестве спектрального прибора. Для этого надо так подобрать геометрию записи голо-линзы и изображения, чтобы уменьшить в наиболее широком, по мере возможности, диапазоне длин волн. В настоящей работе приведены пригодные для этой цели формулы и представлены примеры для проиллюстрирования их приложения.