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Fiber gyroscope operation – numerical modelling

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The analytical description of all-fiber gyroscope based on the Jones calculus is presented. This calculus allows on an uniform description of the single-mode fiber elements from gyro minimal configuration. The numerical simulation allows to calculate the influence of element parameters on gyro sensitivity and bias-offset.

1. Introduction

In the research and development of fiber gyros the capabilities to analyse and simulate the optical system are required. One of the methods allowing a simple description of gyro set is that of the Jones matrix. By using this method ULRICH [1] described the role of polarization problem in gyro operation. The block structure of this method allows us to investigate the influence of the parameters of system elements on the output signal. In this way, the influence of polarizer [2]–[4], polarization control unit [5], broad band source with high-birefringence fiber loop [3], [6], [7], and phase modulator [8], [9] on gyro operation was described. On the other hand, the Jones matrix method can be directly used to numerical modelling of the set working. Such a way is used in paper [10], but for bulk optics elements in Jones matrices.

Below, the numerical simulation of all-fiber gyro operation using Jones matrix method is presented.

2. Numerical simulation algorithm

Analysing gyro set and its equivalent lumped element representation (ELER) scheme in Jones matrix notation is shown in Fig. 1. The fiber coil lies in the X–Z plane; the widely applied Sagnac phase equation has the following form:

$$\Delta\Phi = K\Omega \tag{1}$$

where K – scale factor, Ω – rotation rate. We assume an additional nonreciprocal phase shift of $\pi/2$ for propagation in the counter-clockwise direction in order to

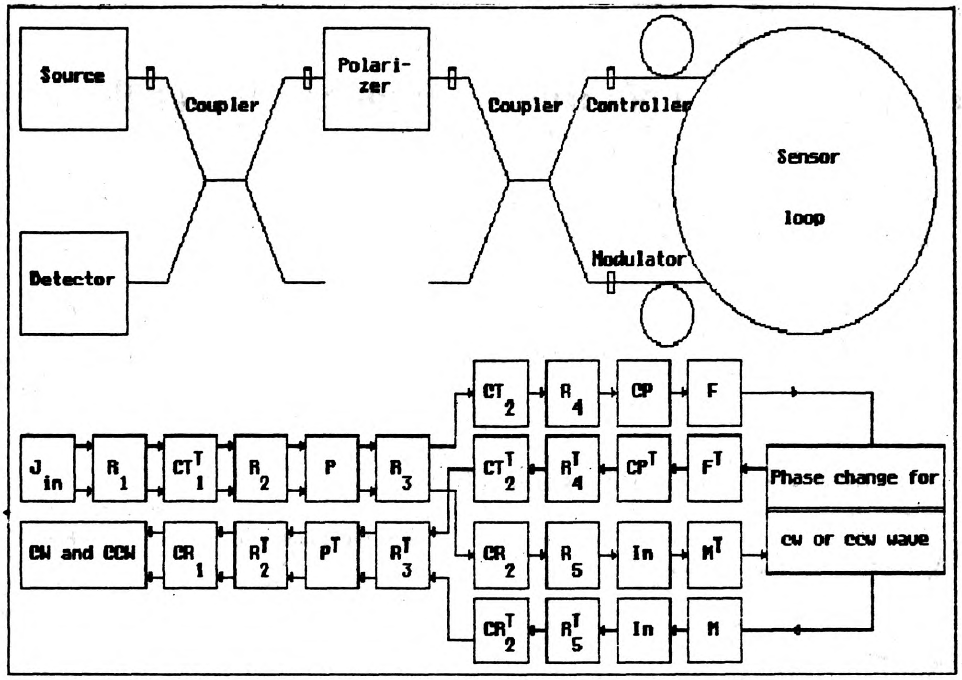

Fig. 1. Gyro arrangement for analysis and its ELER scheme. $\boldsymbol{J}_{in}$ – matrix of source, $\boldsymbol{R}_i$ – matrix of adjusting i connector, $\boldsymbol{CT}$, $\boldsymbol{CR}$ – matrices of transmission and reflection distances in coupler, $\boldsymbol{P}$ – matrix of polarizer, $\boldsymbol{M}$ – matrix of modulator, $\boldsymbol{CP}$ – matrix of polarization controller, $\boldsymbol{F}$ – matrix of sensor loop, $\boldsymbol{T}$ – (in upper index) matrix transposition, $\boldsymbol{In}$ – matrix of inversion, $\boldsymbol{CW}$ and $\boldsymbol{CCW}$ – matrices for cw and ccw wave of gyro

obtain output equation with the maximum sensitivity at zero-rotation rate. The possibility of the element birefringence axis misalignment in connections is included in rotator matrix

$$\boldsymbol{R}_i \equiv \boldsymbol{R}(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix}, \quad i = 1, \ldots, 5 \tag{2}$$

where θ_i – angle between misalignment axis. Inversion matrix

$$\boldsymbol{In} \equiv \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

(Fig. 1) resulting from the fiber birefringence axis is subject to a reflection about the Y–Z plane as the loop is traversed [3].

In the case when Jones matrices of set elements are known, the gyro matrix is

$$\boldsymbol{Z} = \boldsymbol{CW}e^{j\Delta\Phi/2} + \boldsymbol{CCW}e^{j(\pi/2 - \Delta\Phi/2)} \tag{4}$$

where

$$\boldsymbol{CW} = \boldsymbol{CR}_1^T \boldsymbol{R}_2^T \boldsymbol{P}^T \boldsymbol{R}_3^T \boldsymbol{CR}_2^T \boldsymbol{R}_5^T \boldsymbol{In}\, \boldsymbol{M}\, \boldsymbol{F}\, \boldsymbol{PC}\, \boldsymbol{R}_4\, \boldsymbol{CT}_2\, \boldsymbol{R}_3\, \boldsymbol{P}\, \boldsymbol{R}_2\, \boldsymbol{CT}_1\, \boldsymbol{R}_1 \tag{5a}$$

is Jones matrix for clockwise (cw) direction, and

$$CCW = CR_1^T R_2^T P^T R_3^T CT_2^T R_4^T CP^T F^T M^T In\, R_5\, CR_2\, R_3\, P\, R_2\, CT_1\, R_1 \qquad (5b)$$

Jones matrix for counter-clockwise (ccw) direction.

If the source is represented by its coherence matrix [10]

$$J_{\text{in}} = \frac{1}{2}\begin{bmatrix} 1-P\cos 2\vartheta \cos 2\varepsilon & P(\sin 2\vartheta \cos 2\varepsilon - j\sin 2\varepsilon) \\ P(\sin 2\vartheta \cos 2\varepsilon + j\sin 2\varepsilon) & 1+P\cos 2\vartheta \cos 2\varepsilon \end{bmatrix} \qquad (6)$$

where P, ϑ, ε, – degree azimuth and ellipticity of polarization, respectively, then gyro signal coming back to the source is

$$I = Tr(J_{\text{out}}), \qquad J_{\text{out}} = ZJ_{\text{in}}\dagger Z \qquad (7)$$

where J_{out} – coherence matrix for output wave, and $\dagger$ – Hermitian conjugate.

Equations (1)–(7) describe the algorithm of numerical simulation of gyro operation equivalent to signal

$$I(\Omega) = A + B/2\ \sin(\Delta\Phi + \Phi_0) \qquad (8)$$

where Φ_0 is the phase bias-offset, A – mean intensity level, and B – intensity scale factor.

3. Jones-matrix representation of gyro elements

According to the equivalence theorem of Hurwitz and Jones [11], any optical unit (gyro elements) may be represented by partial polarizers, rotators and retardation plates. Thus, it is possible to construct ELER for gyro elements including its

WAVELENGTH	:	0.85 [um]
FIBER COIL RADIUS	:	0.10 [m]
FIBER LENGTH	:	500.00 [m]
SYSTEM LOSS	:	12.00 [dB]
FIBER LOSS	:	2.00 [dB/km]
SOURCE POWER	:	2.00 [mW]
FIBER TYPE	:	High-Birefringent
COUPLERS TYPE	:	Untwisted
CORE ELLIPTICITY	:	0.40
FIBER TWIST RATE	:	720.00 [deg/m]
CORE MISALIGNMENT IN LOOP	:	0.00 [deg]
DEGREE OF POLARIZATION	:	1.00
AZIMUTH OF SOP	:	0.00 [deg]
ELLIPTICITY OF SOP.	:	30.00 [deg]
FIBER BEAT LENGTH	:	1.20 [mm]
HALF-WIDTH OF SOURCE	:	1.66 [THz]
POLARIZATION DISPERSION	:	3.60 [ns/km]
FIBER h-PARAMETER	:	4.0E-0005 [1/m]
PHASE PERTURB. ON 1ST END	:	15.00 [deg]
PHASE PERTURB. ON 2ND END	:	18.00 [deg]
PZT TUBE RADIUS	:	30.00 [mm]
MODULATOR WINDING PITCH	:	1.00 [mm]
NUMBER OF TURNS ON MODUL.	:	30.00
EXTINCTION RATIO	:	30.00 [dB]
TRANSFORMATOR LOOP 1	:	0.00 [deg]
TRANSFORMATOR LOOP 2	:	0.00 [deg]
COUPLER LENGTH	:	66.76 [mm]
INTERFIBER COUPLING CONST	:	2.0E-0004 [1/um]
1ST FIBER COUPL.MISALIGNM	:	0.00 [deg]
2ND FIBER COUPL.MISALIGNM	:	5.00 [deg]
TWISTED COUPLER MISALIGNM	:	0.00 [deg]
ELEMENTS MISALIGNMENT	:	0 0 0 0 0 [deg]

Fig. 2. List of the included 36 variables

structural parameters and fiber properties. In simulation we have used the schemes of Chen [12] for coupler and of Burns [5] for high-birefringence loop. Other schemes are described in paper [13] using elastoopic type of fiber deformation.

Introduction of structural parameters and fiber properties as global variables to numerical program (Fig. 2) allowed a simple modification of ELER. In this way, the investigation of the influence of the mentioned above variables on gyro operation (Eq. (8)) is possible. The number of variables included in this program is 36.

4. Simulation results

The values of phase bias-offset Φ_0 (Eq. (8)), intensity scale factor B (Eq. (8)), scale factor K (Eq. (1)) and gyro sensitivity limited by the shot noise [14], were calculated for Sagnac phase changing from $-90°$ to $90°$. The results of gyro simulation for the parameters such as in Fig. 2 are presented in Fig. 3.

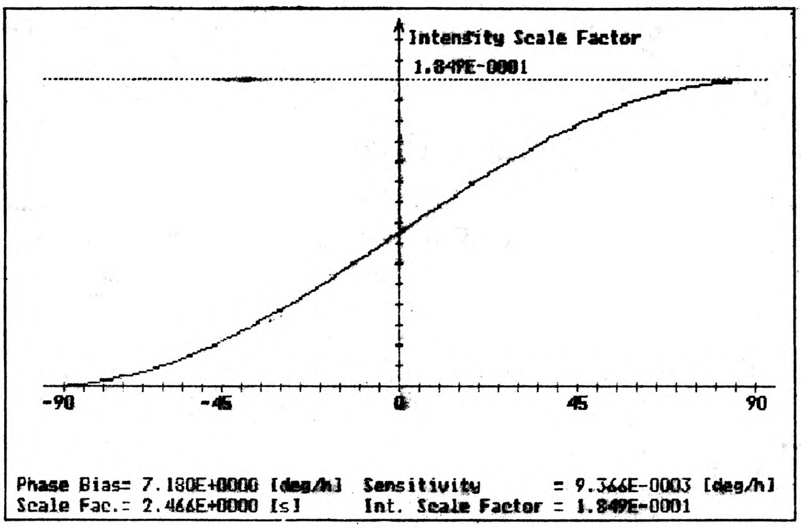

Fig. 3. Simulation of gyro operation

The exemplified field results of simulation with different parameter variations are shown in Figs. 4–7. Figures 4 and 5 describe the operation of unit with high-birefringence loop, and Figs. 6, 7 of that with usual fiber loop.

For the result shown in Figure 4, the variable parameter is the axis birefringent alignment between the coupler and fiber end in ccw direction (matrix $\boldsymbol{R}_5$). As it is seen, the misalignment introduces into gyro operation a significant phase bias-offset and changes in the intensity scale factor. This phase bias-offset may be reduced by decreasing the degree of source polarization, as shown in Fig. 5. These results are in a good agreement with the Burns calculation for SLD and high-birefringence fiber.

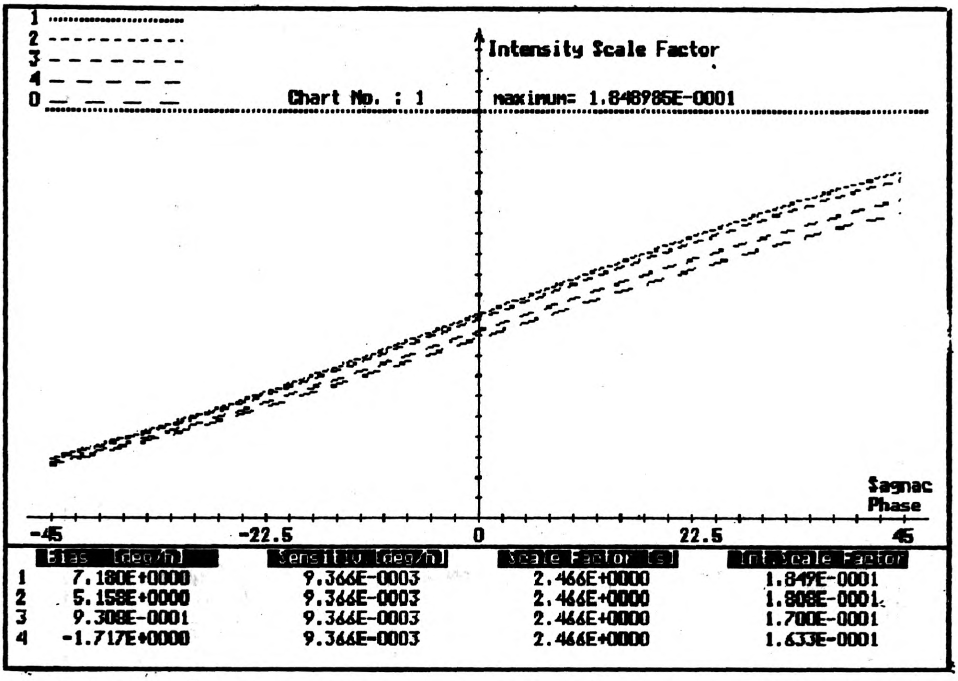

Fig. 4. Simulation for gyro operation with axis birefringent alignment between the coupler and fiber end in ccw direction as the variable parameter (the angle θ_5 in matrix $\boldsymbol{R}_5$): 1 – $\theta_5 = 0$ deg, 2 – $\theta_5 = 3$ deg, 3 – $\theta_5 = 9$ deg, 4 – $\theta_5 = 12$ deg

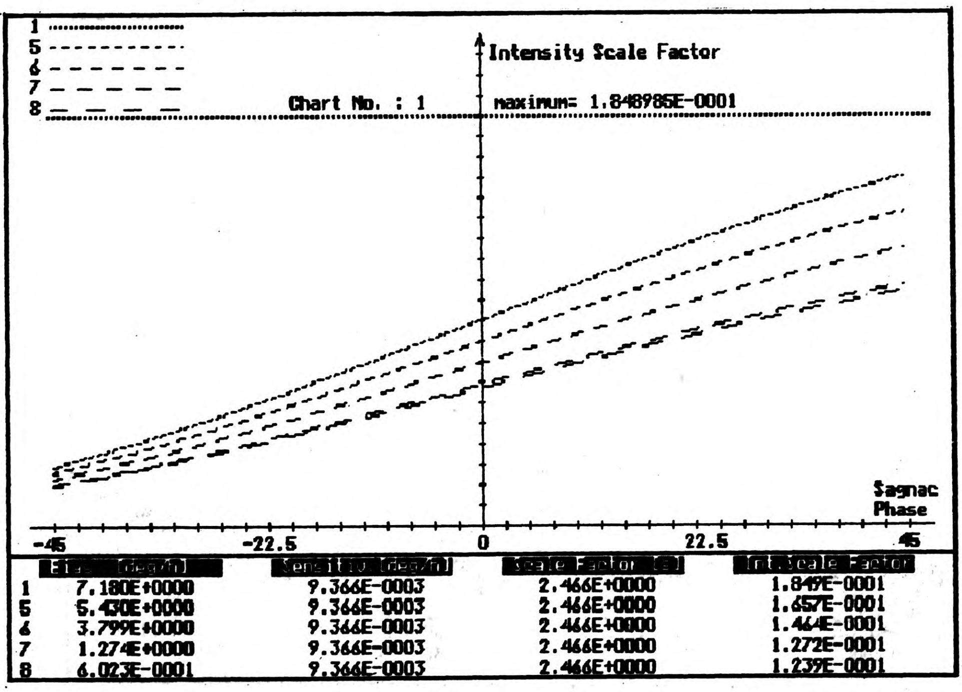

Fig. 5. Simulation for gyro operation with the degree of source polarization (DOP) as the variable parameter: 1 – DOP = 1.0, 5 – DOP = 0.7, 6 – DOP = 0.4, 7 – DOP = 0.1, 8 – DOP = 0.05

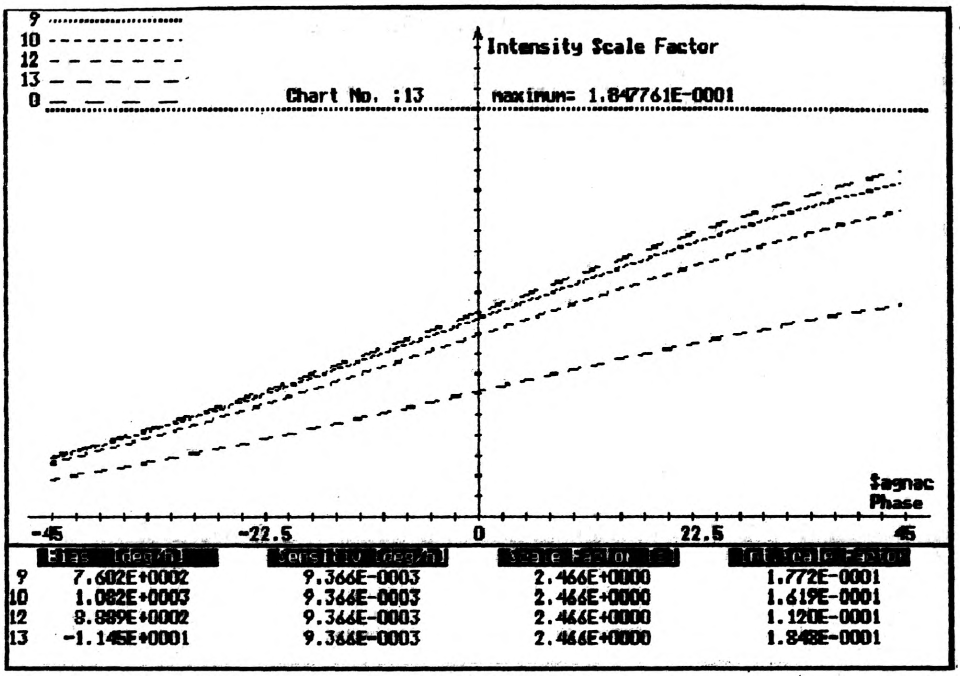

Fig. 6. Simulation for gyro operation with the second transformer loop angle (the angle ψ in matrix CP) as the variable parameter: 9 – $\psi = 0$ deg, 10 – $\psi = 10$ deg, 12 – $\psi = 45$ deg, 13 – $\psi = 90$ deg

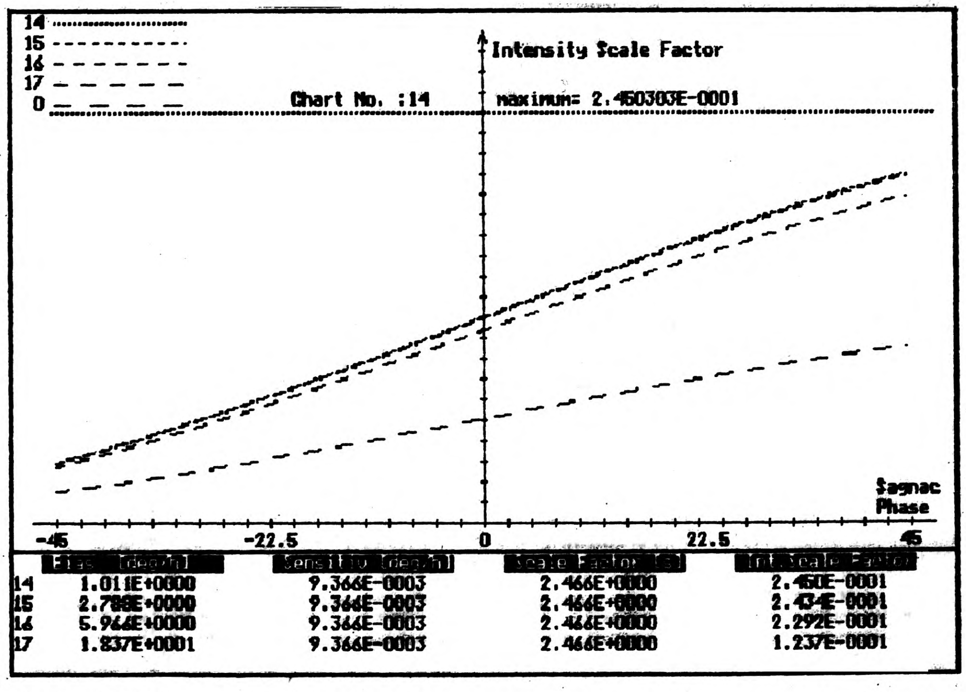

Fig. 7. Simulation of gyro operation with the state of source polarization ellipticity (ε) as the variable parameter: 14 – $\varepsilon = 0$ deg, 15 – $\varepsilon = 5$ deg, 16 – $\varepsilon = 15$ deg, 17 – $\varepsilon = 30$ deg

The possibility of bias-offset minimization by polarization of control unit (matrix ***CP***) is shown in Fig. 6. In this case, the variable parameter is the axis of second quarter wave loop of LE FEVRE transformer [15]. Figure 7 shows the influence of SOP ellipticity on the output gyro parameters. The obtained results are in conformity with the KINTER ones [2].

5. Conclusions

The obtained results show that the Jones method can be successfully used for description of the influence of the element parameters of gyro on its operation. The program presented in this paper can be applied to the use of all-fiber elements for gyro set. This program can be modified so as to take into account other parameters, like the influence of external fields, and so on. This problem will be solved in the near future.

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Действие фибрового гироскопа – численное моделирование

Дано аналитическое описание фибрового гироскопа, базирующее на вычислении Джонса. Это вычисление позволяет однородно описать фибровые одномодные элементы из минимальной конфигурации гиро. Аналитическое моделирование позволяет рассчитать влияние параметров элемента на чувствительность гиро, а также на сдвиг начальной поляризации.