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Focusing element of axial refractive index gradient*

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The problem of spherical aberration correction has been discussed for the case of light focused by a single spherical surface when applying the material of axial refractive index gradient. An analytic form of the formula for the refractive index distribution, which has been derived, assures a complete correction of the spherical aberration for the objects at infinity. The results of calculation of the spherical aberration are reported for the media of small refractive index gradient described by an exponential function.

1. Introduction

In the last period of time, a significant attention has been paid to the role of gradient media in optical instruments. The considerations concern usually the axially symmetric distribution of indices. In a number of works, the focusing properties of the optical elements produced of material with axial gradient have been also considered. As a rule, the theory of third order aberrations is exploited [1]. This approach was proposed to design such elements as: gradient collimating lenses [2], corrections Schmidt plate [3], and telescope objective [4].

In the present paper, the possibilities of a complete correction of the spherical aberration by introducing the material of axial gradient of the refractive index distribution have been analysed. The results concern a single spherical refracting surface under assumption that the object is positioned at infinity.

2. Mathematical considerations

In this part, the focusing properties of a spherical surface of the radius R are considered under the assumption that the object space medium is characterized by a heterogeneous distribution of the refractive index along the optical axis. The coordinate system (Fig. 1) is oriented in such a way that its origin is identical with the centre of the spherical surface, while the axis OZ is identical with the optical axis. The refractive index distribution is described by the function $n = n(z)$.

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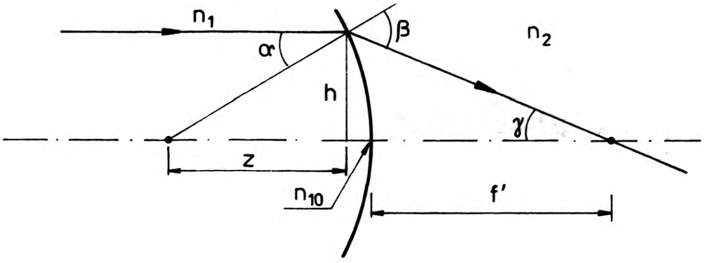


Fig. 1. Single refracting surface (notations in the text)

We assume that the surface is convex and that $n_2 < n_{10}$ (where n_{10} is the refractive index value in object space at the sphere and the optical axis intersection point).

The rays travelling parallelly to the optical axis do not change their direction in such a heterogeneous space, since they are parallel to direction of the gradient n. First, when hitting the surface they suffer from refraction, while the angle of refraction depends on the local value of $n(z)$ on the refracting surface at the respective ray-surface interaction point. By using the notation from Fig. 1, we form the conditions for stigmatic focusing of the beam. The paraxial focal length is defined by the formula

$$f' = R\frac{n_2}{n_{10}-n_2}. \tag{1}$$

In this formula, all the quantities are nonnegative. We start with the scalar law of refraction

$$n_1(z)\sin(\alpha) = n_2\sin(\beta). \tag{2}$$

From Figure 1 the following relations may be easily found:

$$\sin(\alpha) = h/R,$$

$$\cos(\alpha) = z/R,$$

$$\beta = \gamma+\alpha, \text{ while } \beta < \pi/2.$$

The condition for the ray transition through the focus leads to relations:

$$\sin(\gamma) = \frac{h}{\sqrt{(f'+R-z)^2+h^2}},$$

$$\cos(\gamma) = \frac{f'+R-z}{\sqrt{(f'+R-z)^2+h^2}}.$$

From the trigonometric formula for the sinus of a sum of angles we get

$$\sin(\beta) = \frac{h(f'+R)}{\sqrt{(f'+R-z)^2+h^2}}. \tag{3}$$

After substituting (3) to the refraction law (2), we obtain

$$n_1(z) = \frac{f'+R}{\sqrt{(f'+R-z)^2+R^2-z^2}}. \tag{4}$$

Taking additionally (1) into account, we get

$$n_1(z) = \frac{n_{10}n_2}{\sqrt{n_{10}^2-2n_{10}(n_{10}-n_2)z/R+(n_{10}-n_2)^2}}. \tag{5}$$

As may be seen, the distribution of the refractive index depends on the coordinate z normed with respect to the radius R, i.e., z/R.

Another form of this relation may be given for the distribution depending on the normed depth ($G = 1-z/R$) measured from the surface vertex

$$n_1(G) = \frac{n_{10}}{\sqrt{1-\frac{2n_{10}(n_{10}-n_2)}{n_2^2}G}}. \tag{6}$$

The subsequent simplification of this formula may be achieved after introducing the relative refractive index

$$n_r(z) = n_1(z)/n_2, \qquad n_{r0} = n_{10}/n_2.$$

Now we have

$$n_r(G) = \frac{n_{r0}}{\sqrt{1+2n_{r0}(n_{r0}-1)G}}. \tag{7}$$

In Figure 2, a few curves illustrating the determined relation of the relative refractive index to the normed depth G are shown for several parameters n_{r0}.

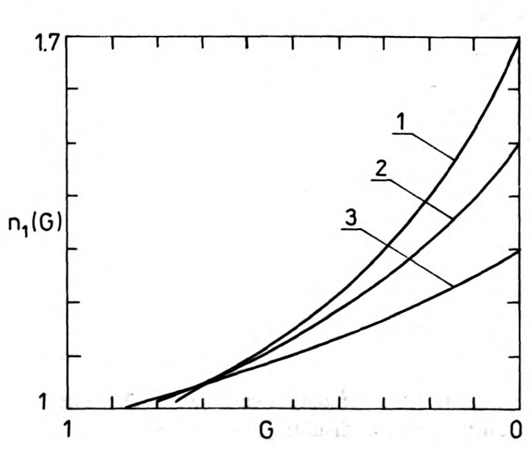


Fig. 2. Dependence $n_1(G)$ for a medium correcting spherical aberration for: $n_{10} = 1.7$ (1), 1.5 (2) and 1.3 (3)

3. Correction of the spherical aberration for glasses of small refractive index gradient

The distribution of refractive index (7) determined theoretically assures the complete correction of the spherical aberrations for the beam of rays parallel to the axis. The correctness of the obtained distribution has been checked with the help of the geometrical transitions of the ray. The numerical results showed a stigmatic concentration of all the rays passing through the limiting surface.

It may be easily noticed that the determined distribution is realizable only within a limited region of depth G (for G equal nearly to 1 the relative refractive index tends to 1; in the case of $n_2 = 1$ it would mean smooth transition of the medium material from glass to gaseous state which is practically unrealizable).

Also, for practical reasons not the whole semisphere is used to produce a plane-convex lens but a part of it around its vertex. For the lenses of relative aperture $2h_{max}/f'$, the complete correction may be achieved if the above changeability of the refractive index is assured up to the depth

$$G \in (0,\ 1-\sqrt{1-(h_{max}/R)^2}).$$

It appears that even for the significant values of the relative aperture the change of the refractive index within this range is not high (Fig. 3).

For small depths, the change of the refractive index determined in (7) becomes almost linear. The exponential distributions of relatively small total change of the refractive index may be technologically realized by using the ion diffusion method. The correction effects, appearing when applying such glasses, seem to be very interesting.

Below, the numerical results of the spherical aberration correction are presented for the glass of axial gradient. It has been assumed that the refractive index in these

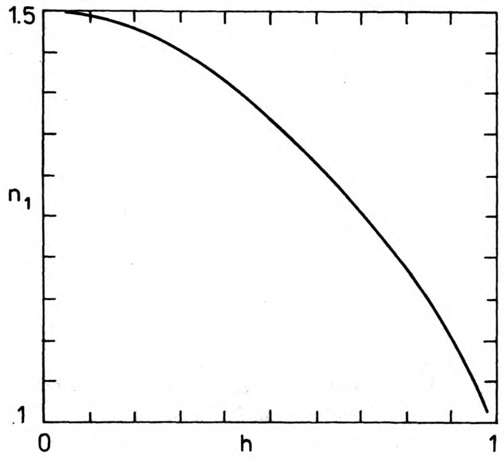
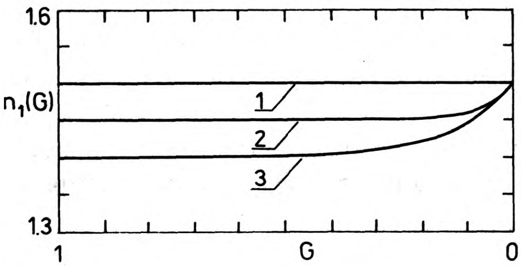


Fig. 3. Value of refractive index at the spherical surface vs the distance from the axis ($n_{1_0} = 1.5$)

glasses is described by the formula

$$n_1(z) = n_{10} - \Delta n\left[\exp\left(-n_{10}\frac{n_{10}-n_2}{\Delta n\, n_2^2}G\right)\right], \tag{8}$$

(Δn is the maximal change of $n_1(z)$). The function defined above is tangent to the optimal one (6) at the vertex of the sphere. The function $n_1(z)$ for several values of n is shown in Fig. 4. The graphs of the transversal spherical aberration for the above materials are shown in Fig. 5. Analogical results of calculations for slightly modified



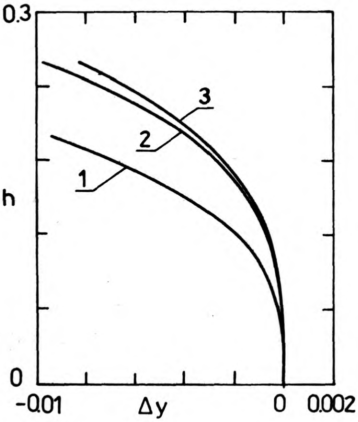


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Fig. 4. Dependence $n_1(G)$ for the uniform (1), and for the gradient: $\Delta n = 0.5$ (2), 0.1 (3) media. $n_{10} = 1.5$

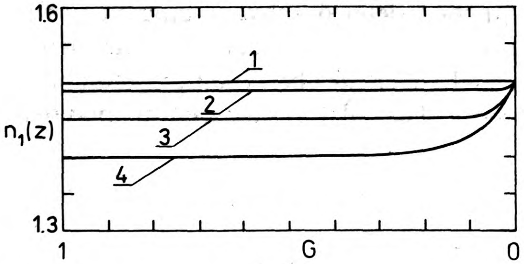
Fig. 5. Transversal spherical aberration for the uniform (1), and the gradient: $\Delta n = 0.5$ (2), 0.1 (3) materials. $n_{10} = 1.5$

distributions, i.e., those of the form

$$n_1(G) = n_{10} - \Delta n\left[\exp\left(-kn_{10}\frac{n_{10}-n_2}{\Delta n\, n_2}G\right)\right], \quad k > 1, \tag{9}$$

are shown in the subsequent figures. In these examples, the function of refractive index distribution appears to be steeper at the spherical surface than the curves corresponding to the optimal distribution. The function $n_1(z)$ for the chosen values Δn and k is shown in Fig. 6. The graphs of the spherical aberration for the above materials are shown in Fig. 7.

The presented results of calculation shown that the spherical aberration correction may be significantly affected also by the changes of refractive index deviating from that determined theoretically. In particular, a distinctly improved correction may be achieved for the distributions described by formula (9). The lens of such a distribution is slightly overcorrected in the central part, but the value of aberration remains small even for significant (1/4) values of the relative aperture (Fig. 7, curve 4).



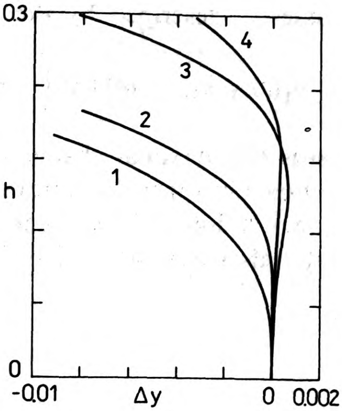


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Fig. 6. Dependence $n_1(G)$ for the uniform (1), and the gradient: $\Delta n = 0.10$, $k = 2$ (2), $\Delta n = 0.05$, $k = 2$ (3), $\Delta n = 0.1$, $k = 1.7$ (4) media. $n_{10} = 1.5$

Fig. 7. Transversal spherical aberration for the uniform (1), and the gradient: $\Delta n = 0.01$, $k = 2$ (2), $\Delta n = 0.05$, $k = 2$ (3), $\Delta n = 0.1$, $k = 1.7$ (4) materials. $n_{10} = 1.5$

It appeared also that a relatively small modification of the refractive index ($\Delta n = 0.01$) made in a thin layer resulted in significant decrease of aberrations (Fig. 6, curve 2, and Fig. 7, curve 2).

4. Concluding remark

The results presented above indicate the applicability of materials with axial gradient to the spherical aberration correction.

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Фокусирующий элемент с аксиальным градиентом коэффициента преломления

Обсужден вопрос коррекции сферической аберрации в случае фокусировки света отдельной сферической поверхностью при применении материала с аксиальным градиентом коэффициента преломления. Выведена аналитическая форма формулы распределения коэффициента преломления, обеспечивающая полную коррекцию сферической аберрации для объекта в бесконечности. Представлены результаты расчетов сферической аберрации для сред с небольшими изменениями коэффициента преломления, описанными экспоненциальной функцией.