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Semi-automatic measurement of the refractive index profile of GRIN optical fibre by differential shearing interferometry*

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The semi-automatic measurement of the refractive index profile of gradient-index (GRIN) optical fibre by means of transverse differential interferometry using BIOLAR PI microinterferometer is presented. The accurate procedure for interferometric data interpretation and reduction, based on a strict, close-form relationship between the fringe shift and the deflection function is used. The cladding-immersion index mismatch is considered.

1. Introduction

In transverse interferometric refractive index profiling methods the investigated axially symmetric phase object is illuminated transversally to its axis. The phase shift of a probing beam is detected as fringe shift and the refractive index profile is calculated from the fringe shift distribution [1].

The application of transverse interferometric method was limited by the lack of a strict theoretical approach enabling an accurate reconstruction of the refractive index profile of a gradient-index object. The known and applied three approximate methods, were more or less apparently based on the same assumption that test objects are weakly refracting media [2]–[4]. The exact solution has been given quite recently [5]. It has been shown by computer simulation of measurement that by using this solution the error of reconstruction can be significantly reduced [6].

In the present paper the measurement of the refractive index profile of GRIN optical fibre, using transverse differential shearing microinterferometry, is described. The above-mentioned accurate approach is used to interferometric data interpretation and their reduction.

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2. Theory in brief

Let us consider an axially symmetric phase object (fibre or rod) characterized by the refractive index distribution $n(r)$ being a function of radial distance r only. The object is immersed in a liquid of refractive index close to that of the object edge.

A beam of parallel rays, coming from a monochromatic light source, iliuminates the object perpendicularly to its axis (Fig. 1). Due to the presence of refractive index gradients and/or refractive index steps, almost all the rays are deviated from their initial direction while passing through the object. Let the resulting inclination of these rays be characterized by the set of angles, each being strictly related to the

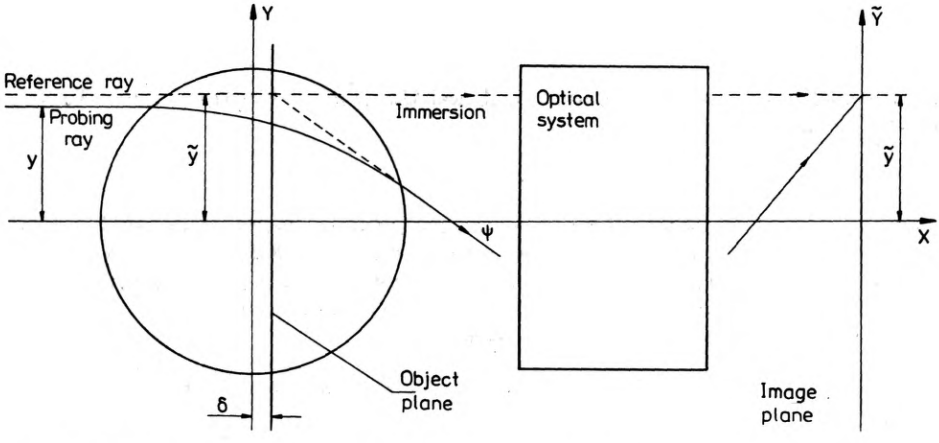

Fig. 1. Configuration scheme of the optical set-up which forms the interference image of the light refracting a cylindrically symmetric object. The object plane of the lens system is assumed to be displaced by the distance from the axis plane of the investigated object

initial coordinate y of a given ray. We refer to this characteristic as the deflection function $\psi(y)$. The deflection function is sufficient to complete the reconstruction of the examined axisymmetric refractive index distribution by means of the parametric relations [5]

$$n = \exp\left[\frac{1}{\pi}\int_u^1 \frac{\psi(y)dy}{(y^2-n^2)^{1/2}}\right], \quad 0 \leqslant u \leqslant 1,$$

and

$$r = u/n. \tag{1}$$

The above relations can be derived from the generalized Snell law, under assumption that $n(r)r$ is strictly increasing function of r. Discontinuities of the refractive index distribution $n(r)$ in the form of steps upward are allowed. For simplicity, the radius of the object as well as the refractive index of the immersion have been assumed to be unity. It means that all the geometrical quantities are

normalized to unity at the edge of the object (becoming dimensionless), and that the index of the object is measured relatively to the surrounding medium.

In the interferometric method the probing beam is superimposed with the reference beam, and the phase shift caused by the examined object is detected as fringe shift $R(\tilde{y})$. The deflection ψ can then be specified directly from the shape of the deviated fringes, due to the strict relation [5]

$$\psi = -\arcsin\left(\frac{\lambda}{D}\frac{dR(\tilde{y})}{d\tilde{y}}\right) \tag{2}$$

where λ is the light wavelength in free space, D – the interfringe spacing related to this wavelength.

Let us note at this moment that fringe shift distribution is read in the image plane. Without loss of generality we can establish that the transverse magnification of optical system is equal to unity. However, due to refraction and possible defocusing δ of the optical system (see Fig. 1), the coordinate $\tilde{y}$ in the image plane will be, in general, different from the initial coordinate y. According to the results of analysis given in [7], they are connected by the relation

$$\tilde{y} = y\sec\psi - \delta\tan\psi. \tag{3}$$

Equations (2)–(3) provide the exact determination of the ray deflection function $\psi(y)$ in a set of data points in which the differential of the fringe shift can be calculated numerically or measured directly (differential interferometry).

3. Measuring technique

The main part of the measuring system is the polarizing interference microscope BIOLAR PI [8], [9], developed in the Central Laboratory of Optics by Maksymilian Pluta and manufactured by PZO (Polish Optical Works) since 1975.

The general working principle of the microscope is as follows. A plane light wave, polarized linearly by the polarizer P, leaves the condenser C (see Fig. 2). When passing through an object under examination O, the wave is subject to a phase shift corresponding to the optical path difference occurring in the object. The distorted wavefront enters the objective Ob, and is split by the system of two Wollaston prisms (W_1 and W_2) into two wavefronts polarized at right angles. When passing through the analyser A, both wavefronts interfere with each other and make visible the transparent object in form of two images laterally duplicated.

The shearing system used in the microscope is capable of giving uniform and fringe interference fields with both the great and differential image duplication. The most characteristic feature of this system is a combination of two simultaneously acting Wollaston prisms. One of then (W_1), located above the objective Ob at a constant distance is rotatable round the objective axis. The rotation enables the amount and the direction of image duplication to be changed. The other prims (W_2) is placed in the microscope tube and can be moved in two, parallel (p) and transverse

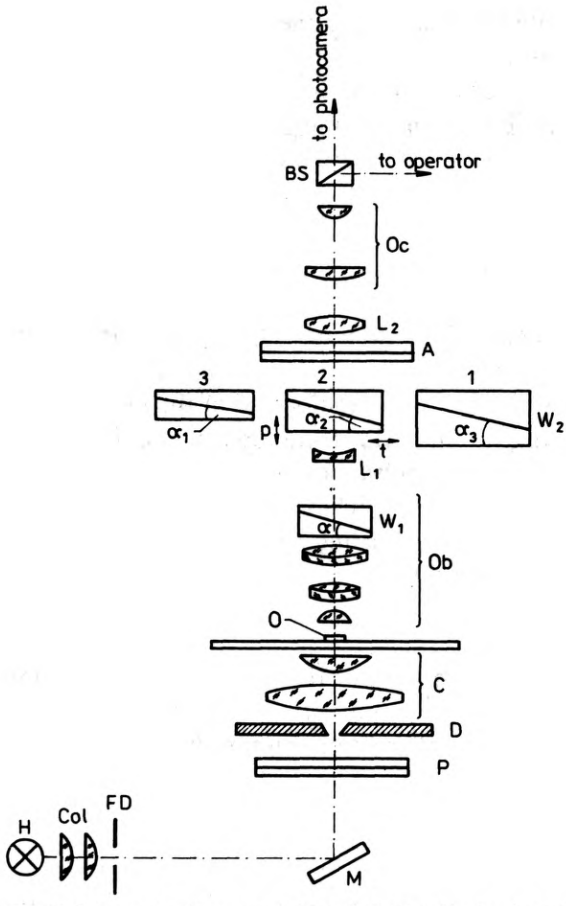

Fig. 2. Schematic diagram of the BIOLAR PI microinterferometer. H – tungsten, halogen lamp 12 V/100 W, Col – collector, FD – field diaphragm, M – mirror, P – polarizer, D – slit diaphragm, C – substage condenser, O – object under study, Ob – objective PI, W_1 – objective birefringent prism, W_2 – birefringent prism localized in the microscope tube, A – analyzer crossed with the polarizer P, L_1 and L_2 – auxiliary lenses, Oc – photographic (or projection) ocular, BS – beam splitter

(t), directions, to the objective axis. The transverse translation for shifting the phase is between sheared light waves which influences the measuring of the interfringe spacing.

The BIOLAR PI microscope is equipped with three tube prisms with different wedge angles (α_1, α_2, and α_3). Two of them (numbered 1 and 3) are used for observation in uniform field interference with great ($d_2 = 6.5$ μm for obj. magn. 10×), and small ($d_2 = 1.84$ μm for obj. magn. 10×) image duplications. By using tube prism no. 2 one obtains fringe field interference with great image duplication ($d_2 = 32$ μm for obj. magn. 10×).

For the measurement of the refractive index profile of an optical fibre the fringe field interference is used. The investigated fibre is oriented perpendicularly to the

direction of interference fringes. The image duplication is changed by rotation of the objective prism (W_1).

By adjusting the objective prism at angles 90° and 270° relatively to the tube prism one obtains the maximum available duplication in the direction perpendicular to the fibre axis. The resultant image duplication, related to the object plane, is then equal to $\sqrt{d_1^2+d_2^2}$, where d_1 and d_2 are the duplications produced by the prism W_1 and W_2, respectively. After rotation of the prism W_1 the light interference in the image plane of the microscope disappears. In order to restore an effective interference in this plane, the slit of the condenser diaphragm should be oriented perpendicularly to the direction of resultant image duplication.

If the diameter of the investigated fibre is smaller then the duplication is d_1, and separated images of the fibre are obtained (total shearing method) [10]. The shift of the fringe $R(\tilde{y})$ is then measured in one of the images and the derivative of the shift $dR(\tilde{y})/d\tilde{y}$ with respect to the image coordinate $\tilde{y}$ is calculated numerically.

Fibres of a greater diameter can be examined by the differential shearing method to which the present paper is devoted. In this method a small duplication d is produced in the direction perpendicular to the fibre axis. In BIOLAR PI microscope such a duplication can be realized while rotating the objective prism by a small angle from the subtraction position. As previously, the slit must be oriented perpendicularly to the direction of the resultant image duplication.

It is assumed that the refractive index profile is invariable along the fibre axis at the distance equal to axial component of the duplication and that the examined fibre is isotropic. If these conditions are fulfilled the obtained fringe shift $\Delta R(\tilde{y})$ and the optical path difference $P(\tilde{y})$ occurring in the object are related by

$$\Delta R(\tilde{y}) = \frac{D}{\lambda}\left[P\left(\tilde{y}+\frac{d}{2}\right)-P\left(\tilde{y}-\frac{d}{2}\right)\right]. \tag{5}$$

Simultaneously, the fringe shift $R(\tilde{y})$ is related to $P(\tilde{y})$ by

$$R(\tilde{y}) = \frac{D}{\lambda}P(\tilde{y}). \tag{6}$$

Substituting Equation (6) into (5), and assuming that the duplication distance d is small enough, the following relation between the fringe shift $\Delta R(\tilde{y})$ and the derivative $dR(\tilde{y})/d\tilde{y}$ can be obtained

$$\frac{dR(\tilde{y})}{d\tilde{y}} \simeq \frac{R(\tilde{y}+(d/2))-R(\tilde{y}-(d/2))}{d} = \frac{\Delta R(\tilde{y})}{d}. \tag{7}$$

For the axisymmetric, isotropic fibre the fringe shift at the centre of coordinate axis is equal to zero ($\Delta R(0) = 0$). In the area of the width d near the fibre edge a fragment of ordinary fringe is observed (see Fig. 3).

The fringe shift is read from the magnified photo of the obtained interference pattern. To this end the densitometric technique can be used [11]. In order to reduce the measurement time, the fringe shape can be read directly by a hand digitizer and the refractive profile is calculated numerically with the aid of the Eqs. (1)–(3).

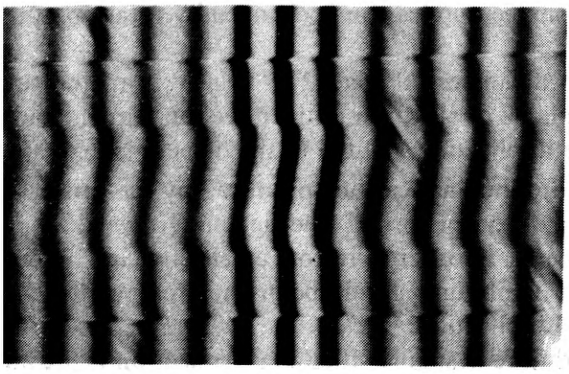

Fig. 3. Differential shearing interference pattern of the gradient-index fibre

4. Results

The gradient-index fibre prepared by the MCVD method in the Chemical Physics Department of the UMCS in Lublin was examined. The differential shearing interference pattern of the examined sample is shown in Fig. 3. The duplication (d) in the direction perpendicular to the fibre axis is equal to 2.5 μm, the fibre diameter is equal to 112.5 μm.

There are two possible configurations in which the refractive index profile can be completely reconstructed from the deflection function with the aid of Eq. (1):

1. The immersion refractive index is equal to cladding index of refraction.
2. The immersion refractive index is higher then cladding index of refraction.

To avoid additional reflections at the immersion-cladding boundary, the immersion of refractive index equal to cladding index is usually used. In practice, however, it may be troublesome to prepare such an immersion. So, in the present paper, the second configuration is considered as being more convenient. It should be also noted that similar steps upward the refractive index are observed in the index profiles of fibre and preform rods between deposited cladding and the substrate.

The shape of the fringe is obtained by averaging "borders" of the zero order fringe. The hand digitizer Summagraphics–1201 working under ACAD (Computer-Aided Drafting and Design Program) and the IBM AT computer are used. The sampling interval related to the object plane is equal to $\cong 0.8$ μm, the accuracy R of the fringe shift data reading is estimated as $\delta R \cong D/40$. The normalized fringe shift $(\lambda/D)dR(\tilde{y})/d\tilde{y}$ and the deflection function calculated with the aid of the Eq. (2) are shown in Fig. 4. The defocusing δ is assumed to be equal to zero ($\delta = 0$). The reconstructed profile is shown in Fig. 5. To compute the integral appearing in Eq. (1) the procedure described in [6] is used.

From the detailed error analysis given in [1], [6] it results that, as long as the conditions assumed in Sects. 2 and 3 are fulfilled, the main measurement error is caused by the random uncertainty of the fringe shift data reading. This error ε can be estimated by means of the following formula [1]:

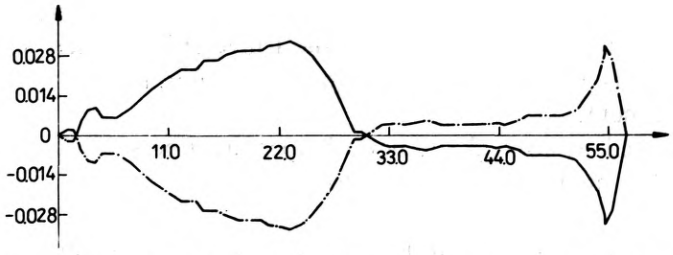

Fig. 4. Normalized fringe shift $(\lambda/D)dR(\tilde{y})/d\tilde{y}$ $(- \cdot -)$, and deflection function $\psi(y)$ (———) (rad)

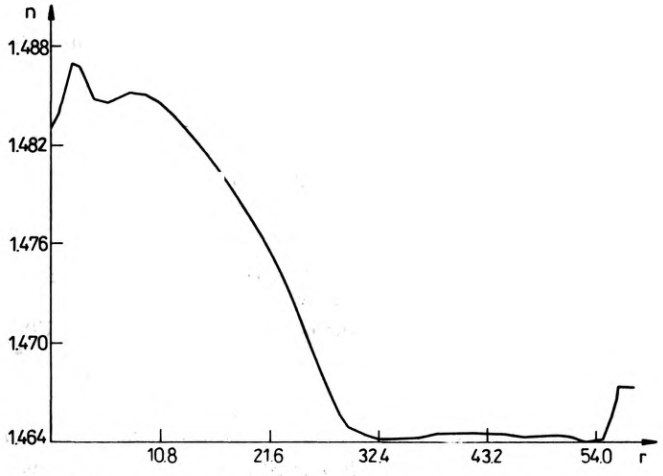

Fig. 5. Refractive index profile reconstructed from the deflection function presented in Fig. 4

$$\frac{\varepsilon}{\Delta n} = K_d \frac{\lambda}{2d\Delta n} \frac{\delta R}{D}$$

where $K_d = 0.19$ is constant number independent of the index profile, Δn is the refractive index difference between the core and the cladding.

Here this error is equal to 2.3% of the maximum refractive index difference. Taking into account other error factors, i.e., numerical evaluation of Eq. (1), optical differentation of the fringe shift, and the possible slight fibre assymmetry and anisotropy, the measurement accuracy is estimated as being equal to 4% of the refractive index difference.

5. Conclusions

The measurement of gradient-index fibre by transverse differential interferometry was presented. The presented interferometric refractive index profiling method has the following advantages:

1. Easy identification of fringe order even for high numerical aperture fibre.
2. Reduced time of computation (the differential of the fringe shift, needed for refractive index reconstruction, is read directly from the interferogram).
3. Strictly theoretical treatment.
4. Application to low as well as to high numerical aperture fibres.
5. Statisfactory accuracy for many practical purposes.

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Полуавтоматическое измерение профиля показателя преломления градиентного оптического волокна методом дифференциальной интерферометрии сдвига

Представлено измерение профиля показателя преломления градиентного оптического волокна методом дифференциальной интерферометрии сдвига с применением микроинтерферометра БИОЛАР ПИ. Реконструкция профиля из интерферометрических данных основана на тесной связи между отклонением полоски и функцией дефлекции. Исследовано несогласование показателя преломления иммерсии с показателем преломления оболочки волокна.