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Correction procedures of the immersion mismatching in interferometric determination of refractive index profile. Part III. Correction of the interference order for radial shearing case*

W. Kowalik, B. Dubik

Institute of Physics, Technical University of Wrocław, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

In the method presented the orders of interference are subject to direct correction procedure, which renders possibility of immediate usage of the algorithm employed for the case of perfect immersion-to-coat matching. The errors of the correction method have been analysed. The method of mismatching measurement for the refractive indices of the immersion liquid and the coat, respectively, has been proposed and accuracy of this measurement determined.

1. Introduction

This work is third of the cycle [1]–[3] devoted to the problem of the accuracy, to which the refractive index distribution in preforms and light waveguides may be determined in the case of mismatching of the immersion refractive index to that of the measured object (preform, light waveguide) coat.

In the mathematical models used to describe the wavefront reconstruction and calculation of the refractive index distribution in the core of the measured object, the following assumptions are usually taken: the wavefront is a continuous function, the examined object itself and the refractive index distribution within it are of cylindric symmetry, and the refractive index of the immersion liquid and the coat are usually equal to each other [4]. Any deviations from these assumptions are sources of errors.

In the present work, only the problem of immersion mismatching to the object coat will be considered. When using only single-component immersion liquid, the refractive index of which being usually different from that of the core, assures the uniformity of the immersion which facilitates the measurements, especially those which are performed frequently and when the measurement results must be quickly obtained, for example, the examination of the preforms produced in the mass scale in

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the industry. The method proposed of the interference order correction renders possibility of making quick calculations and therefore may be applied to the said cases, if the demand on accuracy is not too high.

2. Radial shearing interferogram ($n_p \neq n_i$). Correction of the interference orders

A plane wavefront passing through the examined object perpendicularly to the axis of symmetry of the latter suffers from deformations within the respective regions of the core, the coat and the immersion liquid in which the measured object is suspended. The emerging wavefronts (Fig. 1) may be referred either to the wavefront

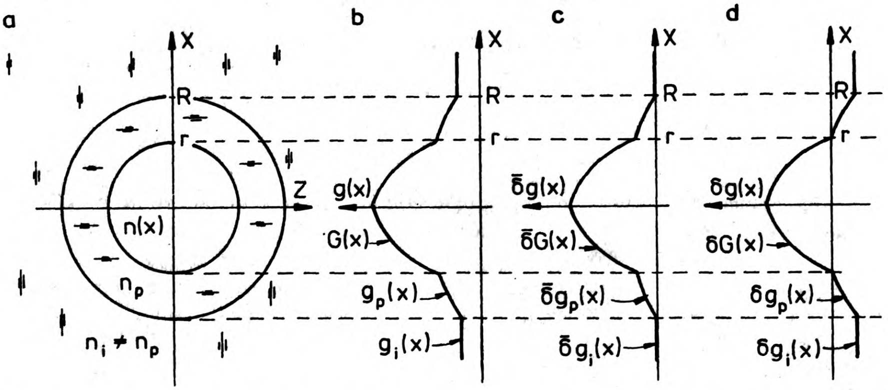

Fig. 1. Wavefronts after having passed the object under test: **a** – object, **b** – wavefront, **c** – wavefront referred to the immersion, **d** – wavefront referred to the clad

emerging from the coat (Fig. 1**d**) or to that emerging from the immersion (Fig. 1**c**). Consequently, the following relative wavefronts may be obtained:

– *in the core of the object* ($0 \leqslant |x| \leqslant r$)

$$\delta G(x) = \delta g(x) + \delta G_k(x), \quad \bar{\delta} G_k(x) = \bar{\delta} g(x) + \bar{\delta} G_k(x); \tag{1}$$

– *in the coat of the object* ($r < |x| \leqslant R$)

$$\delta g_p(x) = 2\delta n_i \sqrt{R^2 - x^2} - \sqrt{R^2 - r^2}, \quad \bar{\delta} g_p(x) = 2\delta n_i \sqrt{R^2 - x^2}; \tag{2}$$

– *in the immersion liquid* ($|x| \geqslant R$)

$$\delta g_i(x) = -2\delta n_i \sqrt{R^2 - r^2}, \quad \bar{\delta} g_i(x) = 0 \tag{3}$$

where $\delta G_k(x)$, $\bar{\delta} G_k(x)$ are the correcting wavefronts:

$$\delta G_k(x) = 2\delta n_i [\sqrt{R^2 - x^2} - \sqrt{R^2 - r^2}], \quad \bar{\delta} G_k(x) = 2\delta n_i \sqrt{R^2 - x^2}. \tag{4}$$

Here δn_i denotes the difference of the refractive indices of the coat n_p and the immersion liquid n_i

$$\delta n_i = n_p - n_i, \tag{5}$$

while $\delta g(x)$ denotes the wavefront (referred to the coat) emerging from the core of the object in the case when $n_i = n_p$

$$\delta g(x) = 2\int_0^{z_r} \delta n(x) dz \tag{6}$$

where $\delta n(x)$ is the sought distribution of the refractive index in the object core (referred to the coat).

$$\delta n(x) = n(x) - n_p \tag{6a}$$

and

$$z_r = \sqrt{r^2 - x^2}. \tag{6b}$$

All the above wavefronts (within the three regions distinguished above), after having passed the radial shearing element, interfere with the radially magnified wavefronts [5], [6]. The optical path difference between these wavefronts is recorded on the interferogram in the form of interference fringes.

The subject of the further analysis is reduced to the interference fringes from the region of the object core, because of the information about the wavefront $\delta g(x)$ recorded in this region, which is necessary to calculate the distribution of the refractive index in the core of the object $\delta n(x)$. In the region $-r_2 < x < r_2$ (Fig. 2), either the wavefronts $\delta G_1(x)$ and $\delta G_2(x)$ (in the coordinate system as shown in Fig. 1**d**) or the wavefronts $\bar{\delta} G_1(x)$ and $\bar{\delta} G_2(x)$ (in the coordinate system shown in Fig. 1**c**)

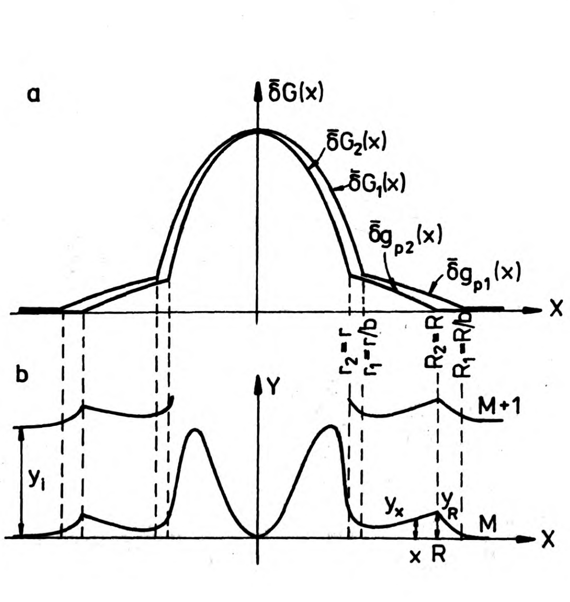

Fig. 2. Interfering wavefronts (**a**) and the result of interference of radial shearing type (**b**)

interfere respectively with each other. The optical path differences between those wavefronts amount to:

$$\delta Z(x) = \delta G(bx) - \delta G(x), \quad \bar{\delta} Z(x) = \bar{\delta} G(bx) - \bar{\delta} G(x). \tag{7}$$

Taking account of (1) and the relation between the optical path difference and the order of interference, it may be easily shown that the orders of interference are described by the expressions:

$$\delta M(x) = \delta m(x) + \delta M_k(x), \quad \bar{\delta} M(x) = \bar{\delta} m(x) + \bar{\delta} M_k(x) \tag{8}$$

where $\delta m(x)$ and $\bar{\delta} m(x)$ are the orders of interference of the fringes recorded on interferograms in the case when $n_i = n_p$

$$\delta m(x) = (1/\lambda)\big(\delta g(bx) - \delta g(x)\big), \quad \bar{\delta} m(x) = (1/\lambda)\big(\bar{\delta} g(bx) - \bar{\delta} g(x)\big). \tag{9}$$

On the other hand, $\delta M_k(x)$ is a correcting factor for the interference orders $\delta M(x)$ and $\bar{\delta} M(x)$ recorder on the interferogram in the case when $n_p \neq n_i$. This correcting factor is identical in both the reference systems (Figs. 1**e**, 1**d**):

$$\delta M_k(x) = \frac{2}{\lambda} \delta n_i \left[\sqrt{R^2 - (bx)^2} - \sqrt{R^2 - x^2}\right]. \tag{10}$$

Since the function $\delta M_k(x)$ correcting the interference order is proportional to both δn_i and R, the normalized form of its course has been shown in Fig. 3. This course has been given for three parameters b of the radial shearing. In order to find the correcting factor for the interference orders (10), it is necessary to know the value of the refractive index mismatching δn_i. This value may be determined by making an additional measurement. The corresponding difference operator introduced to distinguish the additional measurement from the basic one has been denoted by $\bar{\delta}$.

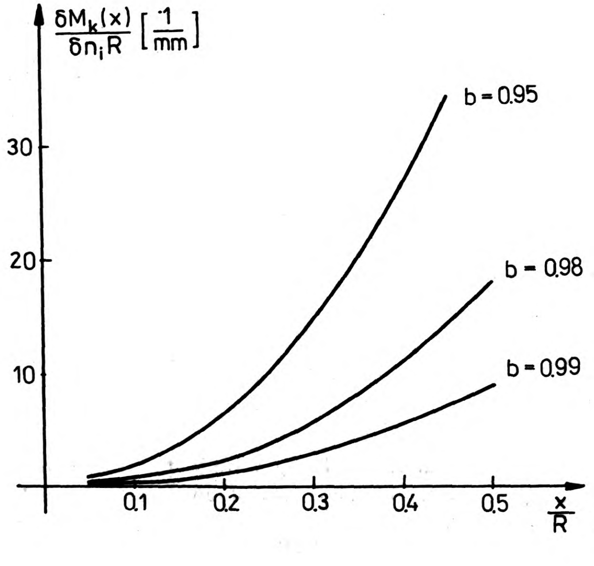

Fig. 3. Run of the function correcting the interference order $\delta M_k(x)$

The additional measurement was carried out in the same measuring setup, which was used for the basic measurement. In a particular case, δn_i may be determined from the same interferogram that was used for calculation of $\delta n(x)$. In general, the method of measuring δn_i and the measuring setup are quite arbitrary, however, the most convenient way is to measure δn_i and $\delta n(x)$ by using the same setup.

3. Calculation of the difference δn_i between the refractive indices in the immersion liquid and the coat of the phase object on the base of an additonal interference measurement

Below, we present the method of measurement of δn_i in the same setup in which the refractive index distribution $\delta n(x)$ in the object core was determined, i.e., in the radial shearing interferometer [7]. In this measurement the immersion liquid has been assumed as a reference level (in contrast to the basic measurement where the object coat was accepted as a reference). The information recorded on the radial shearing interferogram in the region of the phase object coat have been exploited to calculate δn_i. Analogically as it was case in Fig. 2**a** the optical path difference between the interfering wavefronts in this region ($r_2 \leqslant |x| \leqslant R_2$) is equal to

$$\bar{\delta} Z_p(x) = \bar{\delta} g_{p1}(x) - \bar{\delta} g_{p2}(x). \tag{11}$$

Taking account of the above results, the definition of the radial shearing and the relation between the optical difference and the interference order, it has been obtained

$$\delta n_i = \frac{\lambda}{2} \bar{\delta} M_p(x) / [\sqrt{R^2 - (\bar{b}x)^2} - \sqrt{R^2 - x^2}]. \tag{12}$$

The change in the interference order $\bar{\delta} M_p(x)$ has been measured by using the method of deviation of the fringe from the rectilinearity

$$\bar{\delta} M_p(x) = y_x / y_i \tag{13}$$

where y_i is the interference distance in the region of the immersion liquid (Fig. 2**b**). The measurement was performed at the point where the fringe deformation in the coat region y_x was maximal, thus at $x = R$ (Fig. 2**b**), in order to minimize the relative error

$$\delta n_i = \frac{\lambda}{2R} \bar{\delta} M_p(R) / \sqrt{1 - \bar{b}^2}, \quad \bar{\delta} M_p(R) = y_R / y_i. \tag{14}$$

4. Measurement accuracy for δn_i

Since the measurement accuracy of the refractive index mismatching δn_i in the coat of the examined object and the immersion liquid has an influence on both accuracy with which the wavefront $\delta g(x)$ is corrected [1], and that of the interference order correction $\delta m(x)$, the analysis of the factors influencing this accuracy is very important. The absolute and relative errors of δn_i have been defined as follows:

$$\Delta\delta n_i = \frac{\lambda}{2}\frac{\Delta\bar{\delta}M_p(x)}{R\sqrt{1-\bar{b}^2}}+\delta n_i\left[\frac{\Delta R}{R}\;\frac{\bar{b}\Delta\bar{b}}{\sqrt{1-\bar{b}^2}}\right], \tag{15}$$

$$\frac{\Delta\delta n_i}{\delta n_i} = \frac{\lambda}{2}\frac{\Delta\bar{\delta}M_p(x)}{R\sqrt{1-\bar{b}^2}}\frac{1}{\delta n_i}+\left[\frac{\Delta R}{R}+\frac{\bar{b}\Delta\bar{b}}{\sqrt{1-\bar{b}^2}}\right],$$

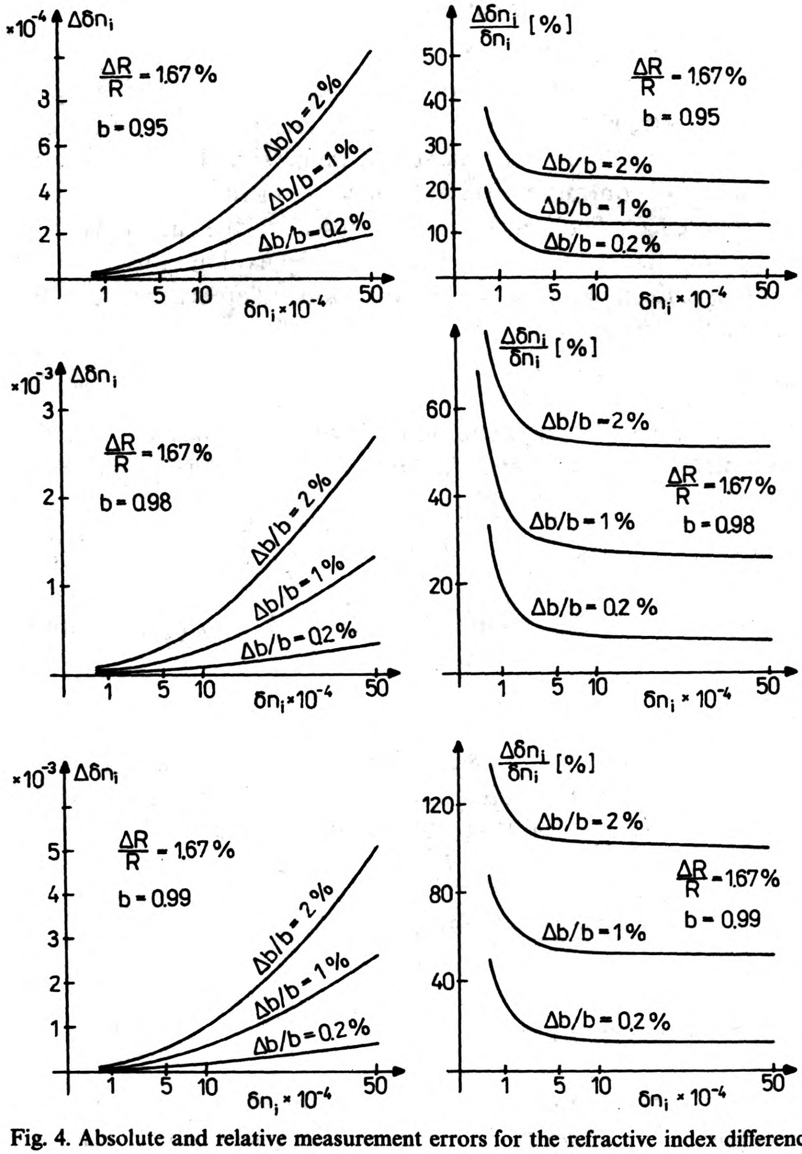

Fig. 4. Absolute and relative measurement errors for the refractive index difference δn_i for an established accuracy of the coat diameter measurement $\Delta R/R = 1.67\%$. Case of the preform $R = 6\times 10^{-3}$ m

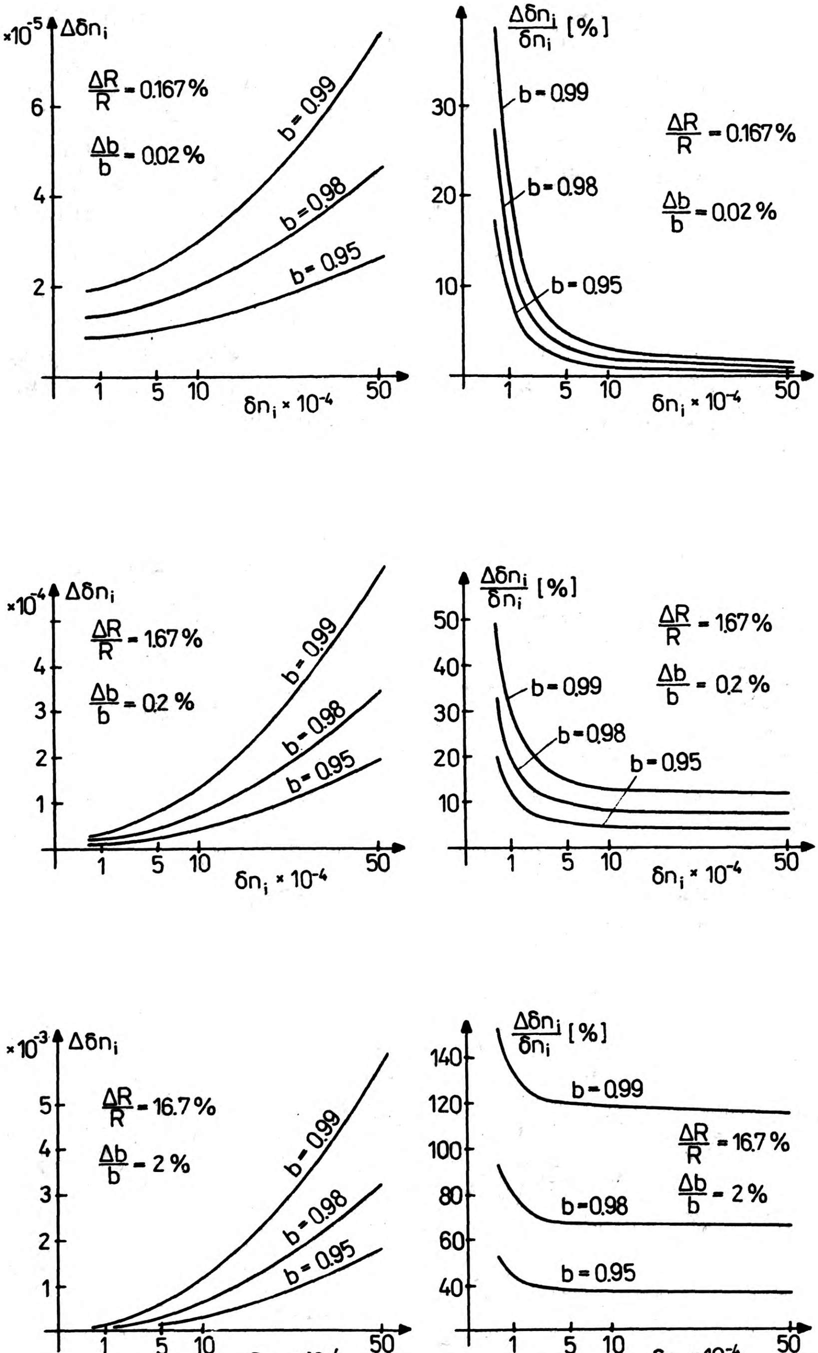

Fig. 5. Absolute and relative measurement errors for the refractive index difference for different accuracies of the coat diameter measurement $\Delta R/R$ and different value of the radial shearing parameter $\Delta b/b$. Case of the preform $R = 6 \times 10^{-3}$

respectively, where $\bar{\delta}M_p(R)$, ΔR and $\Delta\bar{b}$ are the errors of the interference order measurement at the point $x = R$, the object radius R and the radial shearing parameter b. The purpose of the error analysis carried out below is to determine the measuring parameters for which the error of the measurement δn_i is contained within the admissible limits. The absolute errors $\Delta\delta n_i$ grow up nonlinearly while the relative error $\Delta\delta n_i/\delta n_i$ diminish nonlinearly with the increase of the measured mismatching of the refractive index δn_i. The relative errors of the shearing parameter $\Delta b/b$ (Fig. 4) and those of the object geometry $\Delta R/R$ (Figs. 5–7) exert the greatest influence on the measurement accuracies. The greater is $\Delta R/R$ the more nonlinear are the relations for δn_i. In a similar way, the size of the examined object influence the nonlinearity (Figs. 6, 7). From the given relations the conditions may be determined which are necessary to obtain the required measurement accuracies. The given relations take no account of the errors introduced by the zero-order approximation. In Figs. 4–7 the parameter of the radial shearing for the additional measurement has been denoted by $\bar{b}$.

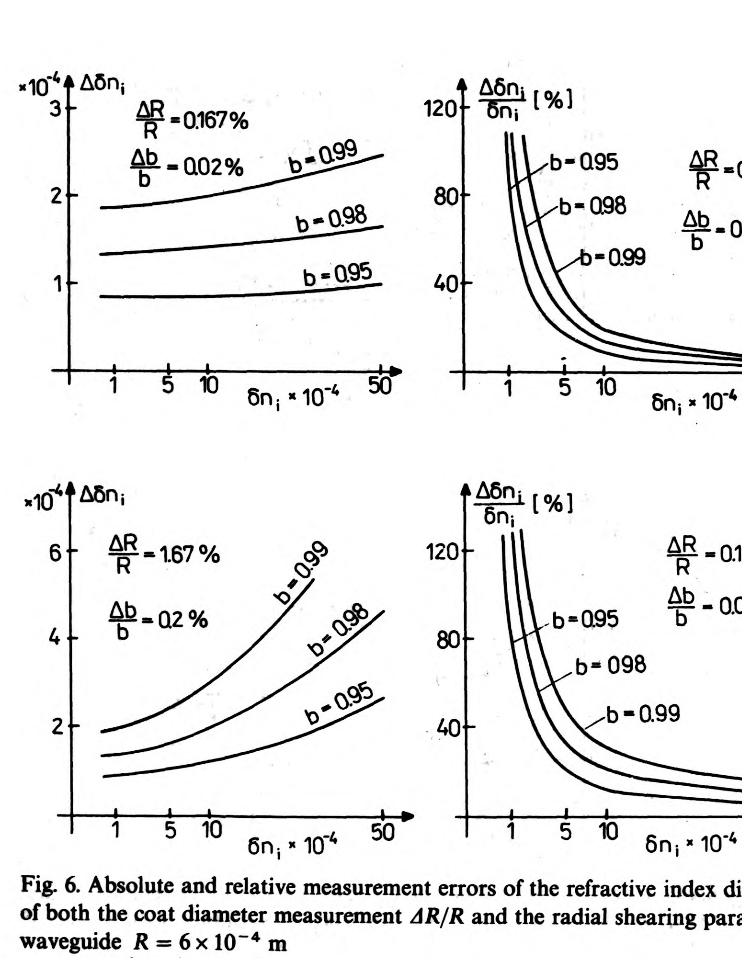

Fig. 6. Absolute and relative measurement errors of the refractive index difference for different accuracies of both the coat diameter measurement $\Delta R/R$ and the radial shearing parameter $\Delta b/b$. Case of thick-core waveguide $R = 6 \times 10^{-4}$ m

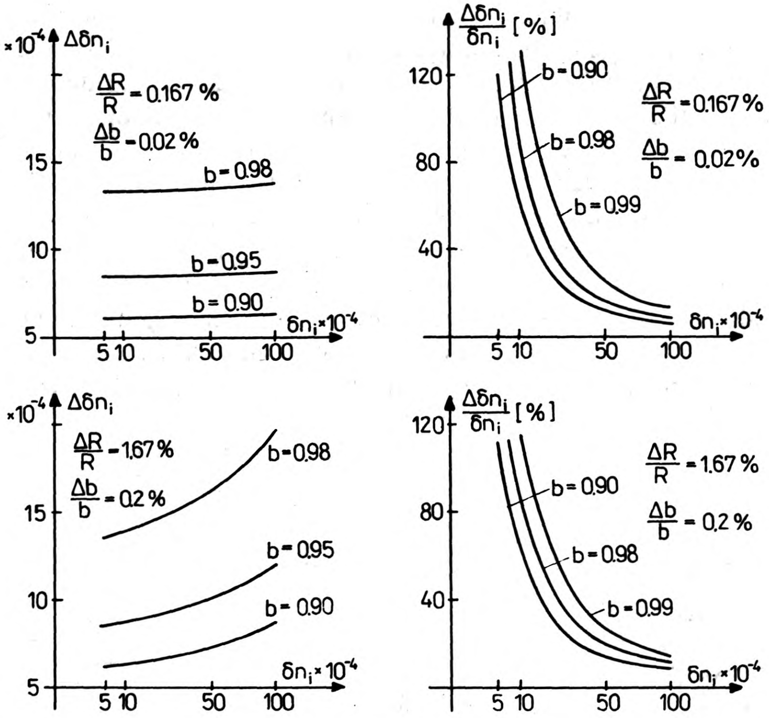

Fig. 7. Absolute and relative measurement errors of the refractive index difference δn_i for different accuracies of both the coat diameter measurement $\Delta R/R$ and the radial shearing parameter $\Delta b/b$. Case of the light waveguide $R = 6\times 10^{-5}$

5. Interference order correction error

The relative error of the correcting factor for the interference order $\delta M_k(x)$ is defined by

$$\frac{\Delta\delta M_k(x)}{\delta M_k(x)} = \frac{\Delta\lambda}{\lambda} + \frac{\Delta\delta n_i}{\delta n_i} + A''\Delta R + B''\Delta x + C''\Delta b \tag{17}$$

where:

$$A'' = \left|\frac{R}{\sqrt{R^2-(bx)^2}-\sqrt{R^2-x^2}}\left[\frac{1}{\sqrt{R^2-(bx)^2}} - \frac{1}{\sqrt{R^2-x^2}}\right]\right|,$$

$$B'' = \left|\frac{x}{\sqrt{R^2-(bx)^2}-\sqrt{R^2-x^2}}\left[\frac{b^2}{\sqrt{R^2-(bx)^2}} - \frac{1}{\sqrt{R^2-x^2}}\right]\right|,$$

$$C'' = \left|\frac{1}{\sqrt{R^2-(bx)^2}-\sqrt{R^2-x^2}}\,\frac{x^2 b}{\sqrt{R^2-(bx)^2}}\right|. \tag{18}$$

Its value is defined for a class of objects characterized by the radial shearing parameter b. As may be seen from (17), the relative error $\Delta\delta M_k(x)/\delta M_k(x)$ depends: on the error due to mismatching of the refractive index of the object coat to that of the immersion $\Delta\delta n_i/\delta n_i$, on the error $\Delta\lambda/\lambda$ of the wavelength determination of the light used to the measurements, as well as on the error of the radial shearing parameter Δb, and that of the distance from the core centre Δx. The value $\Delta\lambda/\lambda$ is negligibly small in most cases and the value $\Delta\delta n_i/\delta n_i$ is estimated in the way shown in the previous section. The value of the error depends in a linear way on both the error of the measurement of the object coat diameter and that of the radial shearing parameter, i.e., if the value of $\Delta R/R$ and $\Delta b/b$ increases by an order magnitude the value of the errors $\Delta\delta M_k(x)/\delta M_k(x)$ increases by an order of magnitude, as well. As it may be seen from Fig. 8, the relative error of the correcting factor $\delta M_k(x)$ decreases

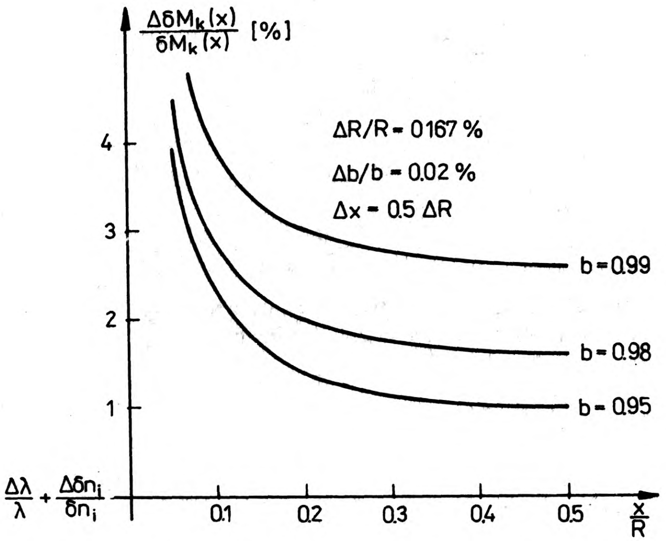

Fig. 8. Relative error of the functions correcting the interference order $\delta M_k(x)$: **a** – object, **b** – wavefront, **c** – wavefront referred to the immersion, **d** – wavefront referred to the clad

with the increase of the distance from the core centre x. Such a dependence has been obtained under the assumption that $\Delta x = 0.5\Delta R$. Under this assumption two coefficients B'' and C'', the value of which decreases with the increase of x (the coefficient A'' increases with the increase of x) decide about the character of the function $\Delta\delta M_k(x)/\delta M_k(x)$.

6. Concluding remarks

The measurement of the profile of the refractive index by using the shearing interference methods is possible both for the cases of matching and mismatching of the immersion refractive index to that of the examined (preform, light waveguide) object. The second case was worth considering since the achievement of perfect matching is hardly possible or requires some tedious manipulations, at least. The measurement method suggested in this work is simple, and is reduced to correcting the interference orders measured on the interferogram. The presented analysis of the measurement accuracy enables to make a choice of the measurement conditions which must be fulfilled in order to achieve the presumed accuracies.

The suggested method of calculation of the refractive index difference in the immersion liquid and the coat of the examined object may be used for the purposes other than that defined above. The accuracies of the refractive index profile may be increased by applying more accurate methods of wavefront calculations, for instance, those reported in [1].

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Процедуры, корректирующие несогласование иммерсии при интерферометрическом определении профиля коэффициента преломления. III. Корректировка порядка интерференции для случая интерференции радиальный shearing

В представленном методе корректировка проводится непосредственно на порядках интерференции, что дает возможность выполнения расчетов по алгоритму, применяемому для примера согласования иммерсии оболочки исследуемого объекта. Проведен анализ погрешностей метода корректировки. Дан метод измерения величины несогласования коэффициентов преломления иммерсионной жидкости и оболочки исследуемого объекта, а также определена степень точности этого измерения.