Optica Applicata, Vol. XIX, No. 4, 1989

Telecentricity of the interferometric imaging system and its importance in the measurement accuracy

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The influence of the relation between the shape of the detector surface and the position of the exit pupil of an interferometr upon the measurement error is studied. It has been shown that the optical system imaging the fringe pattern does not influence the error if the exit pupil plane is in coincidence with the centre of curvature of the detector surface. For the planar detector the imaging should be telecentric on the image side. The condition to be fulfilled during the interferometr adjustment is proposed.

1. Introduction

In interferometers we can distinguish some main fragments performing different roles. One of them is the optical system imaging the interference fringes on a detector. If the principal rays in a given space of an optical imaging system are parallel to the optical axis then in the considered space we deal with a telecentric system [1]. Using the telecentric system on the image side (the principal rays parallel to the axis in the image space) we may avoid the measurement error related to some longitudinal displacements of the detector. These displacements may change the image sharpness, but they do not influence the image dimensions registered by the detector. By applying the telecentric system to the object and image sides simultaneously, which for afocal systems is possible, we get the system free from the error connected with longitudinal movements of the object and the detector. The traces of the principal rays parallel to the axes in both spaces or even the independency of the afocal system magnification of the object position [2] are the simplest explanations of the phenomena.

Usually, in the interferometric methods the knowledge of the image dimensions is not required, the relative values being sufficient. In these cases, however, there is an additional reason for using the telecentric system on the detector side. In coherent light the imaging by a focusing system occurs, in general, between the object and image spheres [3], and not between the object and image planes. According to the principle of the reference sphere transformation, if the field distribution of an object is given on a plane, then its image appears on a sphere with the centre at the image focus of the system [3]. Usually the photosensitive surface of the detector is planar and the difference between the shapes of the image sphere and the detector plane is an additional source of error. The problem is to find the requirements which should be fulfilled by the system for the assumed maximum measurement error.

2. Analyses

In order to analyse our problem we have taken the Fizeau interferometer for the flatness measurement, Fig. 1. A point source SR emitting the monochromatic light of wavelength λ is set at the focus of the objective U. In the image space of U there appears the plane wave, which, after reflection from the standard plane S and the tested plane T, generates two plane waves propagating in two slightly different directions. The angular difference is related to a small tilt between the planes T and S. Two reflected plane waves, after the repeated transmission through the objective U and the reflection from the beam splitter B, are focused inside the aperture stop AS of the interferometer. The distribution in this stop has the form of two point images. If the influence of defocusing, the aberrations of the optical system and the wave truncation by the finite dimensions of the objective U or the tested element T, see [4], are neglected the field focussed into two points at AS can be assumed. The image AS' of AS produced in the image space of the aberration-free objective U_i has the relevant two-point field distribution (the points G_1 and G_2, see Fig. 1). The distance z_0 between the image AS' and the detector D planes depends on the system design.

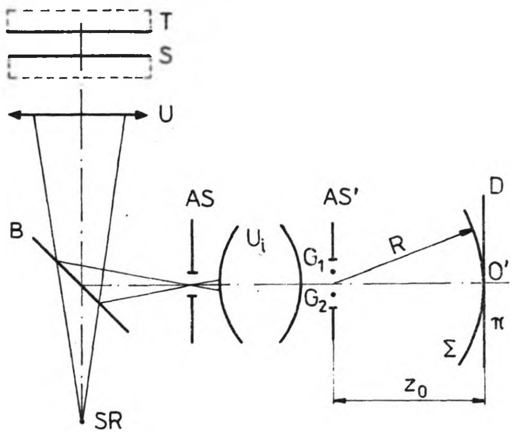

Fig. 1. Optical system of Fizeau interferometer for the flatness measurement

Let us consider the image space of the objective U_i with two light points G_1 and G_2 lying symmetrically with respect to the optical axis OO', Fig. 2 (see, for comparison, Fig. 1). This case concerns the axial tilts of the standard and tested elements in the same meridional plane, but in the opposite directions. The more general case with asymmetrical tilts or the tilts in different planes may be studied, but besides the different positions of the fringe pattern on the image surface it does not introduce anything new from the point of view of the error analysis.

At first let the image surface be spherical (the image sphere Σ, see Fig. 2) and let P_Σ be a point of the sphere Σ. The distances between the points P_Σ and G_i ($i = 1, 2$) according to the notation given in Fig. 2 can be found from the following vectorial

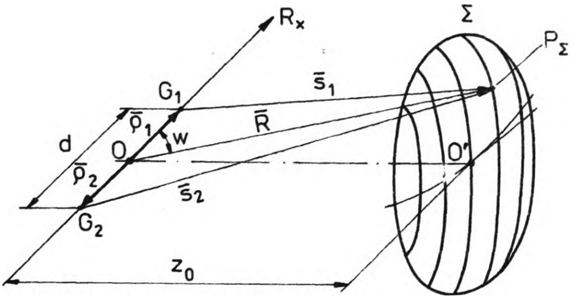

Fig. 2. Image space of an interferometer with the spherical detector surface Σ

relation

$$\mathbf{s}_i = \boldsymbol{R} - \varrho_i, \quad i = 1, 2 \tag{1}$$

where $\boldsymbol{R}$ is the position vector of the point P_Σ defined with respect to the centre O of the sphere Σ. The modulus $R = z_0$ of the vector $\boldsymbol{R}$ is the radius of curvature of the sphere Σ. ϱ — the position vector of the points G_i. Because of symmetry we have $\varrho_2 = -\varrho_1$.

Squarring both sides of Eq. (1), and because $s = \sqrt{s^2}$, we can write

$$s_i = R\sqrt{1+\varepsilon_i} \tag{2}$$

where

$$\varepsilon_i = (\varrho_i/R)^2 - \frac{2\varrho_i}{R}\varrho_1^0 \boldsymbol{R}^0 \tag{3}$$

and $\boldsymbol{R}^0$ is the versor of $\boldsymbol{R}$ ($\boldsymbol{R} = \boldsymbol{R}^0 R$ and $|\boldsymbol{R}^0| = 1$).

The distances ϱ_i ($i = 1, 2$) are considerably smaller than the radius R, it means that $\varepsilon_i \ll 1$, and we can write $\sqrt{1+\varepsilon_i} \approx 1+0.5\varepsilon_i$. The interference order at the point P_Σ will be given by the relation

$$N = \frac{s_2 - s_1}{\lambda} = d\frac{\varrho_1^0 \boldsymbol{R}^0}{\lambda},$$

and finally

$$N = \frac{d}{\lambda}\cos w \tag{4}$$

where $d = |\varrho_2 - \varrho_1| = 2\varrho_1$ is the distance G_1G_2, w denotes the angle between the direction of the vector $\boldsymbol{R}$ and the segment G_1G_2, see Fig. 2.

According to Eq. (4) the interference order N for a given distance d is constant for a constant angle w independently of the distance R of the point P_Σ. The above shadow principle allows us to formulate the following statement: if the form of an interferometric fringe on an image surface is known, then the form of the same fringe on another surface can be found from the geometrical shadow as seen from the point O (see Fig. 2). The above conclusions are valid for the distance d, small with respect to R, and true for the arbitrary shape of the image surface. The zero order fringe lies always in the meridional section ($w = \pi/2$).

Since in the case of the spherical surface, the distance R is constant for the whole surface Σ, the distance of the fringe of the order N from the meridional section is $R_{x,N} = z_0 \cos w$, and in view of Eq. (4) we have

$$R_{x,N} = \frac{RN\lambda}{d}. \tag{5}$$

It means that all the fringes on the sphere have the form of the equidistant lines lying in the section perpendicular to the segment G_1G_2 (Fig. 3), which is in full agreement with the theory of fringes generated by two planar surfaces of the standard S and the tested element T (Fig. 1). The relation (5) is valid for each sphere with the centre at O, Fig. 2, since for the monochromatic illumination the fringes are nonlocalized, and for the planar standard, and the planar tested element the fringe pattern remains the same for different object planes. By the object plane we mean the plane in the space with the standard and the tested element.

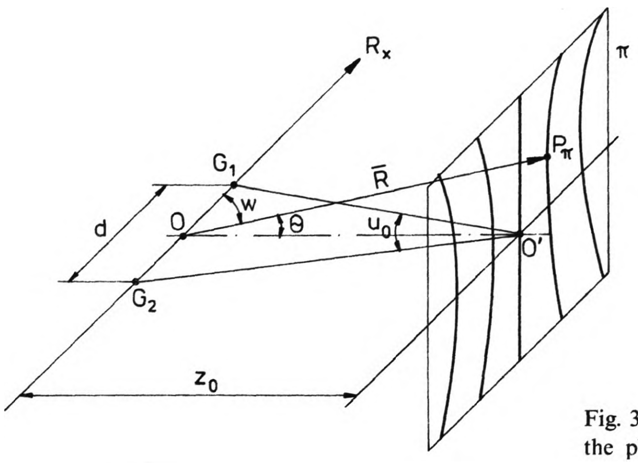

Fig. 3. Image space of an interferometer with the planar detector surface π

Now let the detector surface have the form of the plane π (Fig. 3). The distance R to the arbitrary point P_π of the plane being given by $R = z_0/\cos\theta$ (the angle θ is measured between the direction of the vector $\boldsymbol{R}$ and the optical axis), the distance $R_{x,N}$ of the fringe with the interference order N from the meridional section can be found from the relation

$$R_{x,N} = \frac{z_0 N\lambda}{d}\frac{1}{\cos\theta}. \tag{6}$$

On account of Equations (5) and (6) one can easily conclude, that the fringes with the order different from zero when compared to the case of the spherical detector surface are displaced, being curvilinear (which is well known [5]) and nonequidistant. The conclusion is based on the shadow principle related to Eq. (4). The fringes

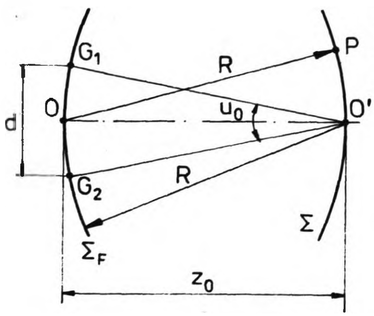

Fig. 4. Image space of an interferometer consistent with the principle of the reference sphere transformation

on the plane π are the geometrical shadow of the fringes on the sphere Σ as seen from the point O (see Figs. 3 and 4). Since the fringes on the sphere have the form of equidistant lines lying in the section perpendicular to the segment G_1G_2, the fringes on the plane are hyperbolic and cannot be equidistant. The difference in the forms of fringes on different detector surfaces disappears if the point O lies in infinity (the telecentric case).

The measurement error introduced by the planar detector used in the case of a finite position of the exit pupil of an interferometer is, according to Eqs. (5) and (6), described by the following expression

$$\Delta N = N\left(\frac{1}{\cos\theta}-1\right). \tag{7}$$

If the imaging system is telecentric on the image side, then $\theta = 0$, and the measurement error disappears. In this case, according to (6), the fringe distance $R_{x,N}$ measured from the zero order fringe is given as

$$R_{x,N} = \frac{N\lambda}{u_0} \tag{8}$$

where $u_0 = d/z_0$ is the angular dimension of the segment G_1G_2 as seen from the centre of the fringe image (see Fig. 3).

According to Equation (7) the measurement error ΔN decreases with the decrease of the interferometric order N. In particular, this means that the error under consideration disappears also for the fringeless observation field and that the detector shape has no significance. The conclusion is evident, because in this case the points G_1 and G_2 (Figs. 2 and 3) are coincident ($d = 0$).

On the basis of the previous consideration it is possible to find the adjustment precision allowing us to obtain the telecentric imaging. The angle θ is small, thus $1/\cos\theta \approx 1+0.5\theta^2$. The maximum angle $\theta_{\max} = 0.5\Phi/z_0$, where the quantity Φ represents the image diameter on the detector plane. Then, according to (7), it may be written

$$\frac{z_0}{\Phi} \geqslant \frac{1}{2}\sqrt{\frac{N}{2\Delta N}}. \tag{9}$$

The relation (9) must be fulfilled when the measurement error introduced by the inaccuracy of the telecentricity adjustment does not exceed the value ΔN. This error is related to the fringe order N in the whole observation field.

For $N = 10$ and $\Delta N = 0.001$ we must have $z_0 \geqslant 36\Phi$. This means that for the case under consideration and the planar surface of the detector, its distance z_0 from the exit pupil of an interferometer (from the stop image AS', see Fig. 1) must be over 36 times greater than the diameter of observation field. This condition is not difficult to fulfil if the optical system is designed as relecentric on the image side.

3. Remarks

Strictly speaking the point sources G_1 and G_2 in Fig. 2 do not lie an the plane, but on a sphere Σ_F with the centre at O' as it is shown in Fig. 4. Moreover, the angle w should be measured from the point O at which the optical axis intersects the sphere sphere Σ_F. This can be proved by taking into account the next terms of the power expansion of the square root in Eq. (2). Such as approach will be in full agreement with the principle of the reference sphere transformation presented in [3]. Since in our case the angle u_0 is very small, the difference between the exact solution obtained with the sphere Σ_F and result of our analysis is negligible.

We have analysed the telecentricity problem on the basis of the exemplary interferometer, i.e., Fizeau interferometer, for the flatness measurement. Our conclusions are however valid for every interferometer in which the fringe pattern is monitored by a detector.

The error described in the paper is systematic, therefore it may be compensated by a numerical method. In this case, however, a constant magnification of the optical imaging system is required. Because of a possible focusing error occurring during the interferometer adjustment the telecentric imaging is desired also in this case.

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Received June 23, 1989

Телецентричность системы изображения в интерферометре и ее значение для точности измерения

Анализировано влияние зависимости между формой поверхности детектора и положением выходного зрачка интерферометра на погрешность измерения. Отмечено, что система составля-

ющая изображение полос не влияет на погрешность измерения, если плоскость выходного зрачка совпадает с центром кривизны поверхности детектора. Дла плоского детектора изображение должно быть телецентрическим в пространстве изображения. Сформулировано условие для юстировки интерферометра.