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Multiple matched filter: performance with spherical wave illumination

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The performance of a coherently recorded multiple matched filter under spherical beam illumination is considered. The signals in the target employed to record the filter have different sizes in order to avoid overlappings in the detection process. To do this, a symmetric correlator illuminated by non-parallel light is used; the detection of a fixed character in the input is obtained only when the input has been moved to that position in which the Fourier transform has the suitable scale. Furthermore, under specific conditions, the output plane location varies linearly with respect to the ratio between the size of the signal in the target and that in the input. Experimental results are shown.

1. Introduction

Spatial filtering in the Fourier plane, and specifically the matched filtering technique [1], has been successfully employed by several authors for character recognition. The inconvenience of the usual methods for recording the multiple matched filters [2]–[5] is that, if the separation between symbols when the filter is recorded is not adequate to fit the entire input between them, there is overlapping in the exit plane with consequent problems in the results.

Ferreira et al. [6] proposed a new multiple filter in order to avoid this inconvenience. The filter consists of a hologram of a target in which each character is rotated at a certain angle with regard to an axis perpendicular to the plane where they are located and which passes through the center of the character. The recognition is only obtained when the filter has been suitably rotated.

A different approach to the problem is presented in [7]. The filter is incoherently recorded placing the different targets which the filter is matched to, at different distances from the lens. Now, the corresponding correlations are obtained in different planes depending on all the distances involved in the recording and filtering processes. Thus overlapping is avoided. Moreover, when the input is located in the front focal plane of the first Fourier transformer of the recognition system, the position of the correlation plane varies linearly with respect to the distance between the target and the lens; that is, a simple movement is achieved.

A new situation arises when the characters to be detected have different sizes from those in the target. The usual situation corresponds to characters of the same size in the input (for instance, letters in a text). In a previous paper [8], the results achieved in [6] are improved by means of a variable scale Fourier transform system, using a target with characters of different sizes to build the multiple filter. In such a way, the system is sensitive to the size relation between the characters to be recognized and the characters to which the filter is matched. The variable scale is obtained using a parallel beam illuminating a convergent thin lens; the input is placed behind the lens and the correlation is achieved only when the input is located at the adequate distance from the lens. The location of the output plane depends on the position on the input and is determined by the coupling of the quadratic phase factors introduced by the filter and by a second lens. So, there is no overlapping. However, an hyperbolic dependence between the final correlation plane and the size of the signal to be detected was found.

The goal of this paper is to obtain a greater scale with the use of spherical wave illumination and, at the same time, to extend the working range to positions of the input in front of the lens acting as the first transformer. That is, firstly, it completes the article of [8]. Secondly, movements as simple as possible would be useful in the possible application to character detection in an automatic recognition system. A linear relation between the position of the cross-correlation plane and the size ratio between characters in the target and the input is achieved.

In the following section the basic theory is developed, showing the construction of the multiple filter and the calculation of the scale-dependent location of the correlation-plane. In Sect. 3 experimental results with a multiple matched filter able to detect three different characters are shown.

2. Basic theory

Let us suppose that we want to record a filter matched to n characters, of different sizes. To do this, the Fourier hologram of a transparency, illuminated by nonparallel light, with the n transparent characters on a dark background is recorded. We take as size origin the height of the smallest character. Let $M_i (i = 1, 2, \ldots, n)$ be the ratio of the height of each character to that of the smallest one, which is represented for instance by $i = 1$.

$$1 = M_1 < M_2 < \ldots < M_n.$$

The expression representing these characters can be written as follows:

$$G(x_0, y_0) = \sum_i g_i\left(\frac{x_0}{M_i}, \frac{y_0}{M_i}\right) \otimes \delta(x_0 + a_i, y_0), \tag{1}$$

a_i being the separation of the character i from the origin of coordinates and $\otimes$ representing a convolution.

The system for recording the symmetric filter is shown in Fig. 1. A point source, located at a distance $d_1 = 2f$ in front of a spherical lens L_1 of focal length f, illuminates the 2-D target placed at a distance z from the source. The lens

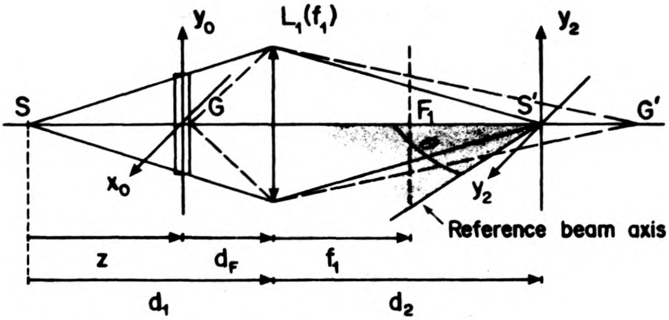

Fig. 1. Arrangement for the filter recording

L_1 performs a non-exact Fourier transform in the image plane of the source, showing a quadratic factor which represents a spherical wave converging to (or diverging from) the image point S' through L_1 of the axial point of the entrance signal [9]. Therefore, if the Fourier transform of $G(x_0, y_0)$ is denoted by $\tilde{G}(x_2, y_2)$, in the image plane of the point source, we have

$$A_2(x_2, y_2) \propto \exp\left\{j\frac{k}{2d_2}[1-d_F d_1/zd_2](x_2^2+y_2^2)\right\}\tilde{G}(x_2, y_2) \tag{2}$$

with

$$\tilde{G}(x_2, y_2) = \sum M_i^2 \tilde{g}_i(M_i x_2, M_i y_2)\exp\{j2\pi(a_i d_1/\lambda z d_2)x_2\}. \tag{3}$$

The scale is given by

$$\frac{d_1}{\lambda z d_2}. \tag{4}$$

Taking into account the reference beam, the resulting amplitude on the plate is

$$A_p(x_2, y_2) \propto \exp\left\{j\frac{k}{2d_2}\left[1-\frac{d_F d_1}{zd_2}\right](x_2^2+y_2^2)\right\}\tilde{G}(x_2, y_2)+R_0 e^{-j2\pi\alpha x_2} \tag{5}$$

with

$$\alpha = \frac{\sin\theta}{\lambda}.$$

Assuming that we are working in the linear region of the t–E curve, the amplitude transmission of the developed plate is

$$t_2(x_2, y_2) \propto R_0^2+|\tilde{G}(x_2, y_2)|^2+R_0\exp\{j2\pi\alpha x_2\}\exp\left\{j\frac{k}{2d_2}\left[1-\frac{d_F d_1}{zd_2}\right](x_2^2+y_2^2)\right\}$$
$$\times\tilde{G}(x_2, y_2)+R_0\exp\{-j2\pi\alpha x_2\}\exp\left\{-j\frac{k}{2d_2}\left[1-\frac{d_F d_1}{zd_2}\right](x_2^2+y_2^2)\right\}\tilde{G}^*(x_2, y_2). \tag{6}$$

The fourth term of Eq. (6) contains the filter matched to the characters to be recognized in a text. If we take into account Eq. (3), this term can be written as

$$t_{24}(x_2, y_2) \propto R_0 \exp\{-j2\pi\alpha x_2\} \exp\left\{-j\frac{k}{2d_2}\left[1-\frac{d_F d_1}{zd_2}\right](x_2^2+y_2^2)\right\}$$

$$\times\left[\sum_i M_i^2 \tilde{g}_i^*(M_i x_2, M_i y_2) \exp\left\{-j2\pi\frac{a_i d_1}{\lambda z d_2} x_2\right\}\right]. \quad (7)$$

To carry out the detection of one character, the hologram that contains the multiple matched filter, represented by Eq. (7), is introduced in the Fourier plane of the system shown in Fig. 1, the reference wave is removed and a simple spherical lens, which transforms back to the space domain, is added at a distance d_R behind the Fourier plane (Fig. 2).

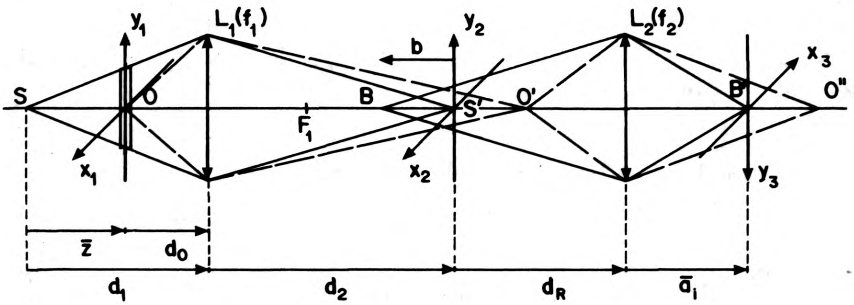

Fig. 2. Coherent filtering setup for character recognition

The input is placed at a distance $\bar{z}$ from the point source. The cross-correlation is obtained at a distance a_i from L_2. This is the output plane, where the detector is placed.

For simplicity, a signal with only the characters to which the filter is matched is chosen, but with a distance c_i from the origin. Let H be the ratio of the height of each character to be recognized to that of the smallest one to which the filter is matched, i.e., the size of the characters to be recognized is the same. Then, the signal introduced can be written as

$$O(x_1, y_1) = \sum_i g_i\left(\frac{x_1}{H}, \frac{y_1}{H}\right) \otimes \delta(x_1 + c_i, y_1). \quad (8)$$

To achieve the filtering we put the input between the point source S, and the lens L_1, at a distance d_0 in front of this lens.

In the Fourier plane (x_2, y_2) the amplitude distribution is

$$U(x_2, y_2) \propto \exp\left\{\frac{k}{j2d_2}\left[1-\frac{d_0 d_1}{\bar{z} d_2}\right](x_2^2+y_2^2)\right\} \tilde{O}(x_2, y_2)\, t_2(x_2, y_2) \quad (9)$$

with

$$\tilde{O}(x_2, y_2) = H^2 \sum_i \tilde{g}_i(Hx_2, Hy_2) \exp\left\{ j2\pi \frac{c_i d_1}{\lambda \bar{z} d_2} x_2 \right\}. \tag{10}$$

The scale is now given by

$$\frac{d_1}{\lambda \bar{z} d_2}. \tag{11}$$

Taking into account the term t_{24}, given by Eq. (7), which concerns us, we have

$$U_{24}(x_2, y_2) \propto R_0 \exp\left\{ j \frac{kd_1}{2d_2^2} \left[\frac{d_F}{z} - \frac{d_0}{\bar{z}} \right] (x_2^2 + y_2^2) \right\} \cdot \tilde{O}(x_2, y_2)$$

$$\times \left[\sum_i M_i^2 \tilde{g}_i^* (M_i x_2, M_i y_2) \exp\left\{ -j2\pi \frac{a_i d_1}{\lambda z d_2} x_2 \right\} \right] \exp\{-j2\pi\alpha x_2\} \tag{12}$$

The recognition of the character represented by g_i is achieved only when the spatial frequencies match each other, so

$$\frac{Hd_1}{\lambda \bar{z} d_2} = \frac{M_i d_1}{\lambda z d_2}, \qquad \bar{z} = \frac{H}{M_i} z. \tag{13}$$

Taking into account that $\bar{z} \leqslant d_1$, Eq. (13) implies that the condition

$$H \leqslant \frac{d_1 M_i}{z} \tag{14}$$

must be fulfilled by the characters to be detected. So, characters of any size are detected if d_1 is great enough, and we can get the recognition of the character g_i when $\bar{z}$ fulfills the Eq. (13).

The quadratic term of the Eq. (12) provides the output plane (x_3, y_3), when the lens L_2 has taken the Fourier transform of $U_{24}(x_2, y_2)$. With the condition of Eq. (13), the distance $\bar{a}_i$ between the lens L_2 and the output plane is given by

$$\bar{a}_i = f_2 \frac{Hd_2^2 z + Hd_1 d_F d_R - M_i d_0 d d_R}{Hd_2^2 z + d_1 (Hd_F - M_i d_0)(d_R - f_2)}. \tag{15}$$

If it is assumed that $d_R = f_2$, and taking into account that

$$d_F = d_1 - z,$$

$$d_0 = d_1 - \bar{z} = d_1 - \frac{Hz}{M_i} \tag{16}$$

we obtain

$$\bar{a}_i = - \frac{d_1^2 f_2^2}{d_2^2 z} \cdot \frac{M_i}{H} + f_2 \left[1 + \frac{d_1^2 f_2}{d_2^2 z} \right], \tag{17}$$

that is, there is a linear dependence between the position of the correlation plane and the ratio between the size of the character in the filter and that of the input, with the only condition that the distance between the Fourier plane and the lens L_2 be the same as its focal length, f_2.

Lens L_2 takes the Fourier transform of $U_2(x_2, y_2)$, in the plane (x_3, y_3). A reflected coordinate system in the exit plane is introduced to avoid the sign change due to the double Fourier transform. From Eq. (12) and considering the filter in the corresponding position to recognize the character represented by $g_K(x_1/H, y_1/H)$, we have

$$\begin{aligned} U_{34}(x_3, y_3) \propto & \left\{ \sum_i M_i^2 \tilde{g}_i^* \left[\frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{x_3}{M_i}, \frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{y_3}{M_i} \right] \right. \\ & \left. \otimes \delta \left[\frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) x_3 - a_i - \lambda z \beta'_1 \alpha, \frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) y_3 \right] \right\} \\ & \otimes \left\{ \sum_j g_j \left[\frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{x_3}{M_K}, \frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{y_3}{M_K} \right] \right. \\ & \left. \otimes \delta \left[\frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) x_3 + \frac{M_K}{H} c_j, \frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) y_3 \right] \right\} \end{aligned} \quad (18)$$

where β'_1 and β'_2 are the lateral magnifications between the plane of the coresponding source and its image plane through the first and second transformers respectively.

So, only when $i = j = k$, is the term

$$\begin{aligned} M_K^2 & \left\{ g_K \left[\frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{x_3}{M_K}, \frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{y_3}{M_K} \right] \right. \\ & * g_k \left[\frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{x_3}{M_K}, \frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) \frac{y_3}{M_K} \right] \\ & \otimes \delta \left[\frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) x_3 + \frac{M_K}{H} c_j - a_i - \lambda z \beta'_1 \alpha, \frac{1}{\beta'_1 \beta'_2} \left(1 - \frac{M_K}{H}\right) y_3 \right] \end{aligned} \quad (19)$$

the autocorrelation (*) of $g_K(x_1/H, y_1/H)$ whereas the other terms are cross-correlations due to the different size of the character in the filter.

Of course, the process can be generalized using other entrance signals and filters matched to more characters, but the idea is always the same.

3. Experimental results

To obtain the filters, a setup to record a Fourier hologram was used, as that of Fig. 1. The recognition was achieved in a two lens filtering system. The experimental setup is shown in Fig. 3. Both focal lengths were $f = 150$ mm and $d_1 = d_2 = 300$ mm. To record the filter the target was placed at distance $z = 115$ mm from the point source.

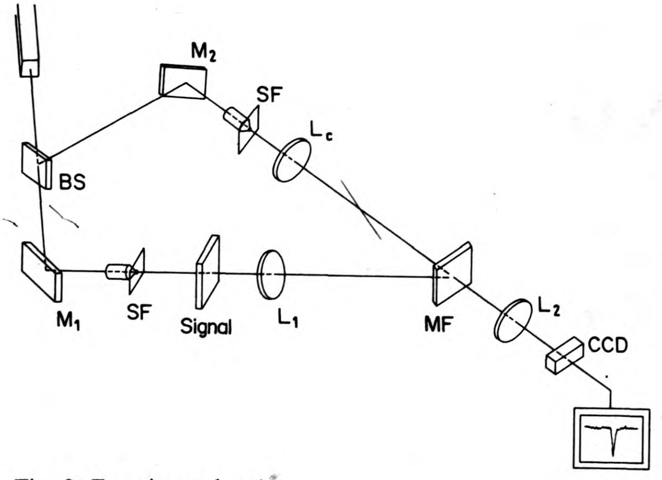

Fig. 3. Experimental setup

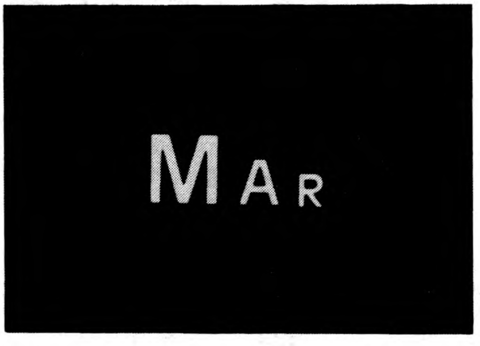

Fig. 4. Target

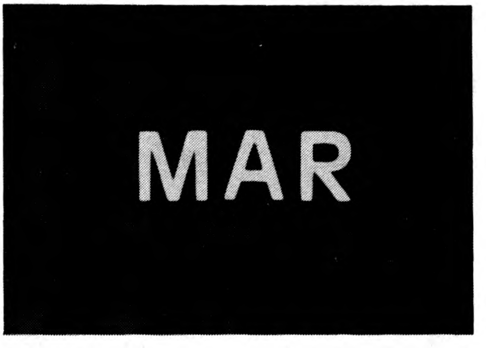

Fig. 5. Input image

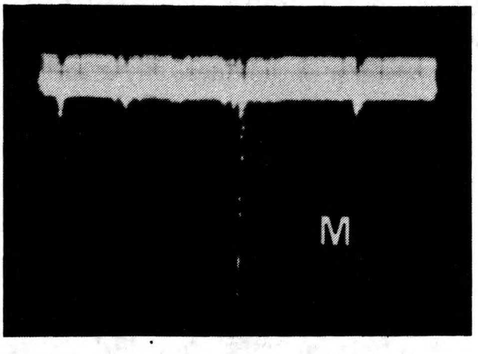

Fig. 6. Detection of character M

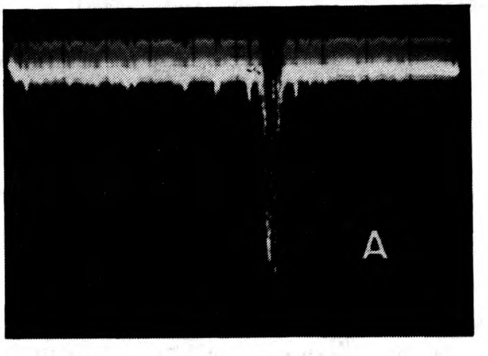

Fig. 7. Detection of character A

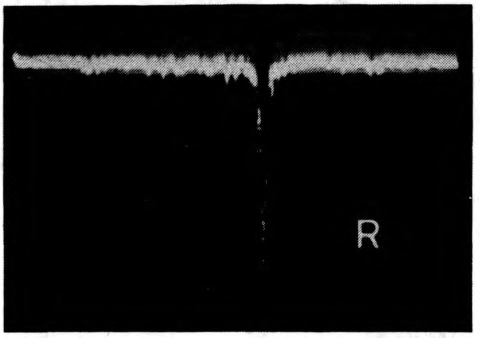

Fig. 8. Detection of character R

In our case, to build the filter, a transparency with the characters M, A, R; the last being the smallest one as can be seen in Fig. 4, was utilized as the entrance signal.

Figure 5 shows the input. The ratio between the size of the characters in the input and those in the filter are given in the Table. This table also contains the position $\bar{z}$ from the point source to the input and the theoretical distances, a_i, from L_2 lens, to the correlation planes. There are significant changes in the location of the correlation planes. So, erroneous detections are reduced.

Variable parameters involved in the recognition of the characters M, A, R

Character	M/H	$\bar{z}$ (mm)	$\bar{a}_i$ (mm)
M	1'01	114'3	295'3
A	0'67	172'5	560'9
R	0'49	234'6	699'0

The detection was performed placing a CCD array coupled to an oscilloscope in the predicted correlation planes. The succesive detection of the characters M, A, R, is shown in Fig. 6, 7 and 8. Note that with each position of the filter only a specific character is determined. Nevertheless, from the experimental point of view, there exists a certain indetermination in the position of the detection plane, due to the dimensions of the converging beam giving rise to the autocorrelation point.

4. Conclusions

We have considered the performance of a multiple matched filter when we use spherical beam illumination either during the construction of the filter or during the detection process. We are dealing with a variable scale Fourier transform system which allows greater scale variations than in the case of plane wave illumination and object placed behind the lens.

The filter is matched to a target containing alphanumerics of different sizes; the detection of these characters, with a fixed size, in an input is obtained only when the input is moved along the axis of the system, to the adequate distance determined by the ratio between sizes of the characters in the target and in the input. In this way, the corresponding Fourier transforms match each other. The position of the correlation plane is a function of the input and the target and it is different for each character in the input, thus avoiding ambiguities in detection, due to the overlappings which are present when other multiple filters are used. Furthermore, under specific conditions, it has been shown that the position of the detection plane varies linearly with the ratio between sizes of the alphanumerics in the target and the input. The experimental results confirm the theoretical predictions. The filter was matched to three characters of different sizes.

Better results can be obtained using a filter like the previous one but with the characters being rotated different angles when recording the filter, as in [8]. We have not considered this type of filters because they add nothing to the performance we were looking for.

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Многократный согласованный фильтр действующий при освещении сферической волной

Обсуждено действование согласованного фильтра когерентно регистрированного при освещении сферической волной. Сигналы в таргете применяемом для регистрации фильтра характеризуются разными размерами, чтобы избечь покрытия во время детекции. С той целью был применен симметрический коррелометр, освещенный непараллельным пучком. Детекция фиксированного символа на входе происходит только тогда, когда вход перемещен к позиции, в которой изображение Фурье имеет соответственную шкалу. Более того, в специальных условиях локализация выходной поверхности изменяется линейно с отношением между размером сигнала в таргете и размером сигнала на входе. Показаны также опытные результаты.