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Coma aberration in off-axis beam optical systems

E. Cojocaru, V. Draganescu, N. Herisanu

National Center of Physics, P.O. Box MG–6, Bucharest, Romania.

In many laser applications optical systems are employed to focus off-axis beams that only partially fill the system aperture. In this paper analytical expressions that give the position of the diffraction focus, normalized peak intensity, and tolerance conditions for uniform and Gaussian off-axis beams in an optical system suffering from primary coma aberration are obtained on the basis of diffraction theory. The results are useful for designing focusing systems for use with laser beams.

1. Introduction

Diffraction patterns of Gaussian beams in aberrated optical systems have been evaluated generally to improve the system performance in the case when the center of the incident beam coincides with the optical axis and the beam is distributed over the whole aperture of the system [1]–[4]. In many laser applications the beam center does not always coincide with the optical axis and the beam only partially fills the system aperture. In such cases the effect of aberrations of the optical system on off-axis beams becomes important and should be investigated. Several studies [5]–[8] have already been carried out. The purpose of this paper is to extend further the previous studies. We derive analytical expressions for uniform and Gaussian off-axis beams which give the intensity degradation near the focus due to primary coma aberration. From these expressions we obtain the position of the focus, the normalized peak intensity and tolerance conditions on the off-axis displacement and aberration parameters.

2. Basic formulae

The aberration function of a focusing system which suffers from primary coma aberration [5], [7] is given by

$$\Phi_C(\varrho, \theta) = C_1 \varrho^3 \cos\theta \tag{1}$$

where C_1 is the coefficient of primary coma aberration and (ϱ, θ) are the polar coordinates whose origin $(0, 0)$ is on the optical axis in the center of aperture of radius a.

In order to discuss the aberration of an off-axis beam, we change, as in [5], the polar coordinates (ϱ, θ) into the polar coordinates (r, φ) the origin of which (ϱ_0, θ_0) is situated at the center of the incident beam (Fig. 1), ϱ and ϱ_0 taking values from

0 to 1. One obtains as in [7]

$$\begin{aligned}\Phi_C(r,\varphi) = {} & C_1\varrho_0^3\cos\theta_0 + (C_1\varrho_0\cos\theta_0)r^2 + [C_1\varrho_0^2(1+2\cos^2\theta_0)] \\ & \times r\cos\varphi + (2C_1\varrho_0^2\sin\theta_0\cos\theta_0)r\sin\varphi + C_1r^3\cos\varphi + (2C_1\varrho_0\cos\theta_0)r^2\cos^2\varphi \\ & + (2C_1\varrho_0\sin\theta_0)r^2\sin\varphi\cos\varphi.\end{aligned} \tag{2}$$

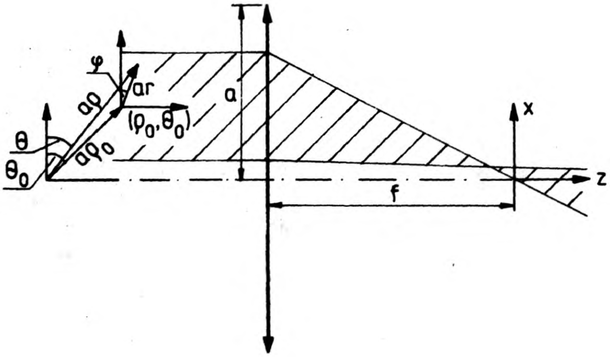

Fig. 1. Off-axis beam optical system

Let (x, y, z) be the Cartesian coordinates in the image field with origin at the focus of the optical system, the z axis along the optical axis, and the x axis parallel to the $\theta = 0$ azimuth at the pupil plane. Let us introduce, as in [1], [8], the normalized variables u and v defined by

$$u = k(a/f)^2 z, \qquad v = k(a/f)(x^2+y^2)^{1/2} \tag{3}$$

where $k = 2\pi/\lambda$ is the wave number, λ is the wavelength of the incident beam and f is the focal length. An arbitrary observation point in the image field can now be specified by using the coordinates (u, v, ψ) where ψ is the azimuth at the plane (x, y), the azimuth $\psi = 0$ being parallel to both the axis x and the azimuth $\theta = 0$ at the pupil plane of the optical system.

The relative intensity distribution near the focus of the aberrated optical system [7], [8] is given by

$$I(u,v,\psi) = \left| \int_0^{2\pi}\int_0^{\delta} P(r,\varphi) \times \exp\{i[k\Phi_C(r,\varphi) - vr\cos(\varphi-\psi) - (1/2)u(r^2+\varrho_0^2+2\varrho_0 r\cos\varphi)]\}\, r\,dr\,d\varphi \right|^2 \tag{4}$$

where $P(r, \varphi)$ is the amplitude distribution of the incident beam at the pupil. This relation is a little different from the similar relation used in [7].

It is convenient to redefine [3], [8] the beam aberration function

$$\Phi(r,\varphi) = \Phi_C(r,\varphi) - vr\cos(\varphi-\psi) - (1/2)u(r^2+\varrho_0^2+2\varrho_0 r\cos\varphi), \tag{5}$$

and to introduce the normalized intensity i which expresses the intensity I as a fraction of the intensity I_0 which would be obtained at the focus if no aberration was present

$$i = \frac{I}{I_0} = \frac{\left|\int\limits_0^{2\pi}\int\limits_0^{\delta} P(r,\varphi)\exp[ik\Phi(r,\varphi)]\, r\, dr\, d\varphi\right|^2}{\left|\int\limits_0^{2\pi}\int\limits_0^{\delta} P(r,\varphi)\, r\, dr\, d\varphi\right|^2}. \tag{6}$$

When the beam aberrations are small the following approximate formula can be used

$$i = 1-(2\pi/\lambda)^2 E = 1-(2\pi/\lambda)^2 [\overline{\Phi^2}-(\overline{\Phi})^2] \tag{7}$$

where $\overline{\Phi^n}$ is defined by

$$\overline{\Phi^n} = \int\limits_0^{2\pi}\int\limits_0^{\delta} P(r,\varphi)\Phi^n(r,\varphi)\, r\, dr\, d\varphi \Big/ \int\limits_0^{2\pi}\int\limits_0^{\delta} P(r,\varphi)\, r\, dr\, d\varphi. \tag{8}$$

The upper limit δ of the integration with respect to variable r is set as in [8] assuming that there is no truncation of the beam due to the aperture: in the case of uniform beams δ can be set to the beam radius b which is normalized by the radius of pupil a and in the case of Gaussian beams δ can set to ∞.

3. Shift of diffraction focus

3.1. Uniform off-axis beam

When the off-axis incident beam has a uniform amplitude distribution, $P(r, \varphi) = 1$ and we deduce when the observation point is at $\psi = 0$

$$\begin{aligned} E = {} & (b^2/8)C_1^2[b^4+4b^2\varrho_0^2(1+2\cos^2\theta_0)+2\varrho_0^4(1+8\cos^2\theta_0)]+(1/48)(b^2/k^2) \\ & \times[(b^2+12\varrho_0^2)u^2+12v^2+24\varrho_0\cos\theta_0\, uv]-(1/6)C_1(b^2/k) \\ & \times\{3\varrho_0\cos\theta_0(b^2+3\varrho_0^2)u+[2b^2+3\varrho_0^2(1+2\cos^2\theta_0)]v\}. \end{aligned} \tag{9}$$

Taking the derivates of E with respect to u and v and setting those equal to zero, we obtain the position of maximum intensity, that is of the diffraction focus

$$u = 4kC_1\varrho_0\cos\theta_0(b^2+6\varrho_0^2\sin^2\theta_0)/(b^2+12\varrho_0^2\sin^2\theta_0), \tag{10}$$

$$\begin{aligned} v &= kC_1[(2/3)b^2+\varrho_0^2(1+2\cos^2\theta_0)]-u\varrho_0\cos\theta_0 \\ &= (kC_1/3)[b^2(2b^2-3\varrho_0^2\cos^2\theta_0)+9\varrho_0^2\sin^2\theta_0(3b^2+4\varrho_0^2)]/(b^2+12\varrho_0^2\sin^2\theta_0). \end{aligned} \tag{11}$$

Equations (10) and (11) give the shift of the diffraction focus for a uniform off-axis beam due to the primary coma aberration. When $\theta_0 = 0$, one obtains in Cartesian coordinates at $y = 0$

$$z_F = 4(f/a)^2C_1\varrho_0, \quad x_F = (f/a)C_1[(2/3)b^2-\varrho_0^2]. \tag{12}$$

The position of maximum intensity when $y = 0$ and $z = 0$ is

$$x_M\Big|_{\substack{y=0\\z=0}} = (f/a)C_1[(2/3)b^2+\varrho_0^2(1+2\cos^2\theta_0)]. \tag{13}$$

3.2. Gaussian off-axis beam

When the amplitude distribution of the off-axis beam is Gaussian, $P(r, \varphi)$ is given by

$$P(r, \varphi) = \exp[-(a/\varepsilon)^2 r^2] = \exp[-(r^2/\omega^2)] \tag{14}$$

where ε and ω are the actual and normalized spot radii of the Gaussian beam, respectively. We obtain for azimuth $\psi = 0$

$$\begin{aligned} E = {} & (C_1\omega^2/2)[6\omega^4 + 2\varrho_0^2\omega^2(3+8\cos^2\theta_0) + \varrho_0^4(1+8\cos^2\theta_0)] + (1/4)(\omega^2/k^2) \\ & \times [(\omega^2+2\varrho_0^2)u^2 + 2v^2 + 4\varrho_0\cos\theta_0 uv] - (C_1\omega^2/k)\{\varrho_0\cos\theta_0(4\omega^2+3\varrho_0^2)u \\ & + [2\omega^2+\varrho_0^2(1+2\cos^2\theta_0)]v\}. \end{aligned} \tag{15}$$

Taking the derivatives of E with respect to u and v and setting those equal to zero, we obtain the position of the diffraction focus

$$u = 4kC_1\varrho_0\cos\theta_0(\omega^2+\varrho_0^2\sin^2\theta_0)/(\omega^2+2\varrho_0^2\sin^2\theta_0), \tag{16}$$

$$\begin{aligned} v &= kC_1[2\omega^2+\varrho_0^2(1+2\cos^2\theta_0)] - \varrho_0\cos\theta_0 u \\ &= kC_1[\omega^2(2\omega^2-\varrho_0^2)+2\varrho_0^2\sin^2\theta_0(\varrho_0^2+3\omega^2)]/(\omega^2+2\varrho_0^2\sin^2\theta_0). \end{aligned} \tag{17}$$

Equations (16) and (17) give the shift of diffraction focus for an off-axis Gaussian beam due to primary coma aberration.

In Figure 2 the positions of diffraction focus (x_F, z_F) near the diffraction focus are given for various values of ϱ_0 from 0 to 0.6 when $\theta_0 = 45°$, $f/a = 1$, $C_1 = 5\lambda$ and $\omega = 0.2$, 0.4.

When $\theta_0 = 0$ one obtains in Cartesian coordinates at $y = 0$

$$z_F = 4(f/a)^2 C_1\varrho_0, \qquad x_F = (f/a)C_1(2\omega^2-\varrho_0^2). \tag{18}$$

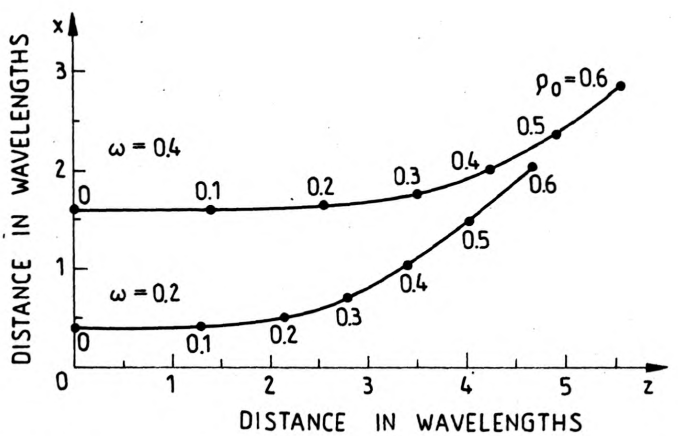

Fig. 2. Positions of diffraction focus (x_F, z_F) for Gaussian beams of normalized spot radii $\omega = 0.2$ and 0.4 for various values of ϱ_0 from 0 to 0.6 for $\theta_0 = 45°$, $f/a = 1$, $C_1 = 5\lambda$

When $y = 0$ and $x = 0$ the position of maximum intensity is

$$x_M \Big|_{\substack{y=0 \\ z=0}} = (f/a)\, C_1 \left[2\omega^2 + \varrho_0^2 (1 + 2\cos^2\theta_0)\right]. \tag{19}$$

It should be noted that these results are different from those given in [7] for the shift of diffraction focus of an off-axis Gaussian beam due to primary coma aberration.

4. Normalized peak intensity

Taking the maximum value of the normalized intensity i defined by Eq. (7) we evaluate the normalized peak intensity at diffraction focus, which is often called the Strehl intensity I_S.

4.1. Uniform off-axis beam

Substituting the conditions (10) and (11) into Eqs. (9) and (7) we obtain the Strehl intensity

$$\begin{aligned} I_S = 1 - (\pi^2/18)(C_1 b^3/\lambda)^2 &[1 + 12(\varrho_0/b)^2 + 24(\varrho_0/b)^2 \sin^2\theta_0 \\ &+ 360(\varrho_0/b)^4 \sin^2\theta_0 + 72(\varrho_0/b)^4 \sin^4\theta_0 + 1728(\varrho_0/b)^6 \sin^4\theta_0 \\ &+ 864(\varrho_0/b)^6 \sin^4\theta_0 \cos^2\theta_0]/[1 + 12(\varrho_0/b)^2 \sin^2\theta_0]^2. \end{aligned} \tag{20}$$

It depends on θ_0, the off-axis displacement parameter ϱ_0/b, and the aberration parameter $C_1 b^3/\lambda$. When $\theta_0 = 0$ one obtains

$$I_S = 1 - (\pi^2/18)(C_1 b^3/\lambda)^2 [1 + 12(\varrho_0/b)^2]. \tag{21}$$

When $\varrho_0 = 0$ the Strehl intensity is given by

$$I_S = 1 - (\pi^2/18)(C_1 b^3/\lambda)^2. \tag{22}$$

4.2. Gaussian off-axis beam

Substituting the conditions (16) and (17) into Eqs. (15) and (7) we obtain the Strehl intensity

$$\begin{aligned} I_S = 1 - 4\pi^2 (C_1 \omega^3/\lambda)^2 &[1 + (\varrho_0/\omega)^2 + 4(\varrho_0/\omega)^2 \sin^2\theta_0 \\ &+ 6(\varrho_0/\omega)^4 \sin^2\theta_0 + 2(\varrho_0/\omega)^4 \sin^4\theta_0 + 4(\varrho_0/\omega)^6 \sin^4\theta_0 \\ &+ 4(\varrho_0/\omega)^6 \sin^4\theta_0 \cos^2\theta_0]/[1 + 2(\varrho_0/\omega)^2 \sin^2\theta_0]^2. \end{aligned} \tag{23}$$

It depends on θ_0, the off-axis displacement parameter ϱ_0/ω, and the aberration parameter $C_1 \omega^3/\lambda$. When $\theta_0 = 0$ one obtains

$$I_S = 1 - 4\pi^2 (C_1 \omega^3/\lambda)^2 [1 + (\varrho_0/\omega)^2]. \tag{24}$$

When $\varrho_0 = 0$ the Strehl intensity is given by

$$I_S = 1 - 4\pi^2 (C_1 \omega^3/\lambda)^2. \tag{25}$$

5. Tolerance conditions

Following the Marechal criterion [1], [8] by virtue of which an optical system is regarded to be well-corrected when the Strehl intensity degradation is < 20%, that is $I_S \geqslant 0.8$, we derive the tolerance conditions on the aberration and off-axis displacement parameters.

5.1. Uniform off-axis beam

We obtain the tolerance condition

$$C_1 b^3/\lambda \leqslant (\sqrt{3.6}/\pi)\,[1+12(\varrho_0/b)^2\sin^2\theta_0]\,[1+12(1+2\sin^2\theta_0)(\varrho_0/b)^2 + 72\sin^2\theta_0(5+\sin^2\theta_0)(\varrho_0/b)^4 + 864\sin^4\theta_0(2+\cos^2\theta_0)(\varrho_0/b)^6]^{-1/2}. \tag{26}$$

In the case of $b = 1$ and $\varrho_0 = 0$ one obtains the tolerance condition

$$C_1 \leqslant (\sqrt{3.6}/\pi)\lambda. \tag{27}$$

For the off-axis displacement parameter ϱ_0/b and the aberration parameter $C_1 b^3/\lambda$ the tolerance region is shown in Fig. 3**a** for uniform beams when $\theta_0 = 45°$. The values of the Strehl intensity I_S as a function of the aberration parameter $C_1 b^3/\lambda$ in the tolerance region are plotted in Fig. 4**a** for $\varrho_0/b = 0, 1, 2, 3, 4$ and $\theta_0 = 45°$.

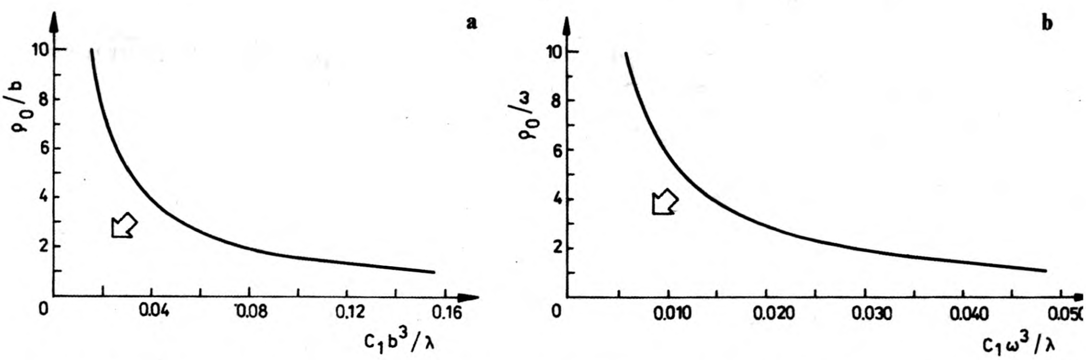

Fig. 3. Tolerance region for uniform beams (**a**) for parameters ϱ_0/b and $C_1 b^3/\lambda$ and for Gaussian beams (**b**) for parameters ϱ_0/ω and $C_1\omega^3/\lambda$

5.2. Gaussian off-axis beam

We obtain the tolerance condition

$$C_1\omega^3/\lambda \leqslant (1/(2\sqrt{5}\pi))\,[1+2(\varrho_0/\omega)^2\sin^2\theta_0]\,[1+(1+4\sin^2\theta_0)(\varrho_0/\omega)^2 + 2\sin^2\theta_0(3+\sin^2\theta_0)(\varrho_0/\omega)^4 + 4\sin^4\theta_0(1+\cos^2\theta_0)(\varrho_0/\omega)^6]^{-1/2}. \tag{28}$$

In the case of $\varrho_0 = 0$ one obtains the tolerance condition

$$C_1 \leqslant \lambda\omega^3/(2\sqrt{5}\pi). \tag{29}$$

The tolerance region for the off-axis displacement parameter ϱ_0/ω and the aberration parameter $C_1\omega^3/\lambda$ is shown in Fig. 3**b** for Gaussian beams when $\theta_0 = 45°$. The values of the Strehl intensity I_S as a function of the aberration parameter $C_1\omega^3/\lambda$ in the tolerance region are plotted in Fig. 4**b** for $\varrho_0/\omega = 0, 1, 2, 3, 4$ and $\theta_0 = 45°$. It can be

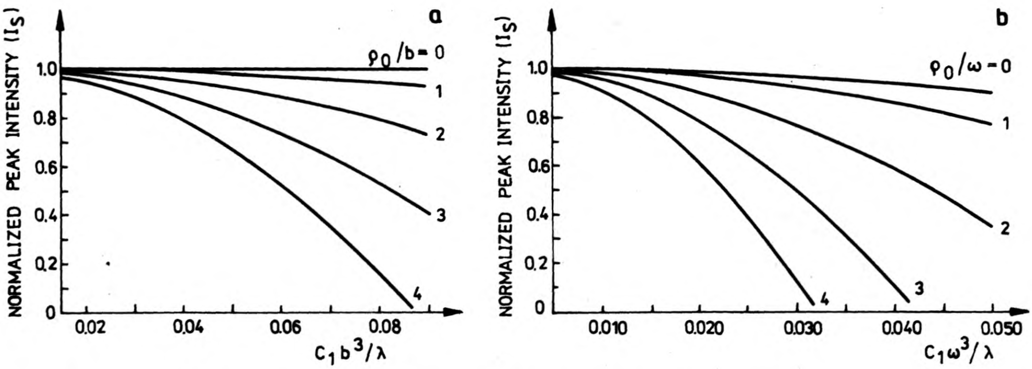

Fig. 4. Normalized peak intensity I_S in the tolerance region for uniform beams (**a**) as a function of the aberration parameter C_1b^3/λ for $\varrho_0/b = 0, 1, 2, 3, 4$, and for Gaussian beams (**b**) as a function of the aberration parameter $C_1\omega^3/\lambda$ for $\varrho_0/\omega = 0, 1, 2, 3, 4$

seen that the dependence of the Strehl intensity on the aberration parameter in the tolerance region is stronger for a Gaussian than for a uniform beam at the same value of the off-axis displacement parameter ϱ_0/b and ϱ_0/ω.

6. Conclusion

In this paper we have presented a study based on the diffraction theory of aberrations of the focusing property of optical systems which suffer from primary coma aberration for off-axis incident beams that only partially fill the system aperture. Analytical expressions are obtained for the position of the diffraction focus, normalized peak intensity (Strehl intensity) and, following the Marechal criterion, the tolerance conditions on the off-axis displacement and aberration parameters for uniform and Gausian beams. These expressions are useful for designing focusing systems for use with laser beams.

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Кома в оптических системах работающих во внеосевом пучке

В многих применениях лазерные оптические системы используются для сжатия внеосевых пучков, которые лишь частично заполняют отверстие системы. На основе теории дифракции в работе получено аналитические выражения, определяющие положение дифракционного фокуса, нормализированного пикового напряжения, условия допуска для внеосевых однородных и гауссовских пучков в оптических системах, которые обременены комой третьего ряда. Результаты полезны при проектировании фокусировочных систем с использованием лазерных пучков.