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# Elimination of systematic errors occurring in interference dilatometry

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In this paper the systematic errors appearing in measuring methods based on examination of transmission of the systems, in which the optical path of the light wave changes due to parametric change of interference conditions. The derived formulae allow the correction of errors made at determining the position of transmission extremes of the system under test. In the experimental part the influence of the change of both average value of the system transmission and interference fringe amplitude on the accuracy of the thermal dilatability determination in brass are presented when using an interference method.

## 1. Introduction

The thermal coefficient of linear expansion $\alpha(t)$ is one of the thermophysical parameters exploited most frequently in engineering practice. Its knowledge is inevitable in many fields of technology, both in civil engineering and while designing thermal devices of all kinds, in particular, engines, turbines and nuclear reactors. Especially, the design optimization of these devices requires that the thermal properties of the applied materials be known.

Various types of dilatometers (like quartz [1], [2], capacity [3], inductance and interference dilatometers [4], [5]–[8]) are used to measure the thermal coefficient of expansion. Even in the case of pure metals the results obtained by the researchers using different methods differ from one another by 2 to 5 per cent. The Committee on Data for Science and Technology of the International Council of Scientific Unions (CODATA) has worked on unification of the above results by publishing the recommended values of thermal coefficients for the muster materials which should be applied to scaling different types of dilatometers [9].

Interference measuring methods of thermal expansion coefficient for solids may be divided into two basic groups. In one of them the absolute value of the distance $d(t_k)$ between the parallel surfaces is measured (Fabry–Pérot interferometer) in sequent temperatures $t_k$ (Fig. 1**a**), while in the second group one determines only the increment $\Delta d(t)$ of the distance between these surfaces caused by continuous temperature changes (Fig. 1**b**). The layer thickness $d(t_k)$ is determined by the analysis of the interference spectrum of transmission $T(\nu, t_k)$ appearing due to continuous change in wave number $\nu$. Interference of the light wave entering the system with the wave repeatedly reflected from internal surfaces causes the appearance of the extremes of transmission – interference fringes. The layer thickness is calculated

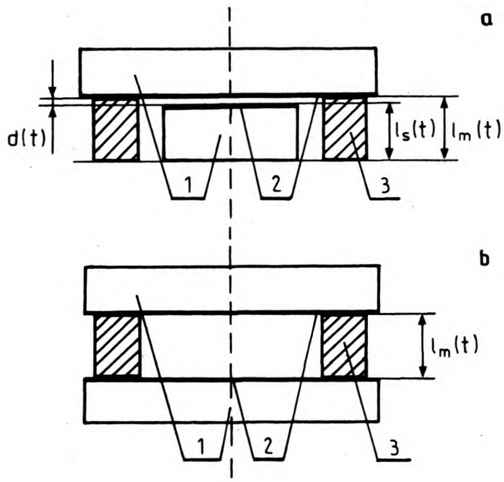


Fig. 1. Ideological scheme of the interference dilatometers for measuring the coefficient of thermal expansion for solids: 1 — glass plate, 2 — reflection layer, 3 — material under test

from the relation [10]

$$d(t_k) = \frac{X(\nu, t_k)}{2n_p \nu} \tag{1}$$

where $X(\nu, t_k)$ is an absolute value of the interference order, while $n_p$ is an refractive index for air or vacuum.

The absolute measurement of $d(t_k)$ is possible for the layers of thickness up to $\sim 100\,\mu m$, this restriction being due to the coherence length of the light beam used in standard spectrophotometers (in visual range). From this restriction it follows that the coefficient of expansion $\alpha(t_k)$ must be determined in an indirect way

$$\alpha(t_k) = \frac{1}{l_{m0}} \frac{\partial l_m(t_k)}{\partial t_k} \tag{2}$$

where $l_m(t_k)$ is a thickness of the ring having plane-parallel front surfaces and made of the examined material (Fig. 1**a**). We measure the distance $d(t_k)$ and calculate the value $l_m(t_k)$ from the relation

$$l_m(t_k) = d(t_k) + l_s(t_k),$$

in which $l_s(t_k)$ denotes the thickness of the glass plate located inside the measurement ring. This plate is made of glass of known coefficient of thermal expansion $\alpha_s(t_k)$. Taking account of the above relation we obtain

$$\alpha(t_k) = \frac{1}{l_{m0}} \frac{\partial d(t_k)}{\partial t_k} + \alpha_s(t_k). \tag{3}$$

By absolute measurement of the thickness of a thin plane-parallel plane $d(t_k)$ the differential coefficent of expansion $\alpha_r(t_k)$ may be determined

$$\alpha_r(t_k) = = \frac{1}{l_{m0}} \frac{\partial d(t_k)}{\partial t_k}, \quad \alpha_r(t_k) = \alpha(t_k) - \alpha_s(t_k). \tag{4}$$

In another variant of the interference method (Fig. 1**b**) the thickness of the examined ring is determined directly

$$l_m(t) = l_{m_0} + \Delta d(t). \tag{5}$$

The value of the thickness increment $\Delta d(t)$ is determined from the analysis of the temperature dependence of the system transmission $T(\nu_0, t)$ recorded for the light of constant wave number $\nu_0$

$$\Delta d(t) = \frac{X(\nu_0, t)}{2n_p \nu_0}. \tag{6}$$

Realization of this variant requires (in the interference methods) a monochromatic light source of coherence length greater than the ring thickness $l_m(t)$, which does not exceed several millimetres. Due to high accessibility of the gas lasers, characterized by much greater coherence length, this variant of the interference method is commonly used. Taking account of the relations (5), (6) and (2) a quasi-continuous dependence of the coefficient of thermal expansion $\alpha(t)$ may be determined for the examined material.

The errors, which appear when both the amplitude of the interference fringes $A(\nu_0, t)$ and the average value of the temperature dependence $T(\nu_0, t)$ are changing, are shortcomings of this method.

The first variant of the interference method (measurement of the absolute value of the thickness $d(t_k)$ under stabilized temperatures $t_k$) is much more difficult as far as realization and elaboration of the experimental data are concerned; it is, however, much less sensitive to the said errors.

In this paper a detailed analysis of the influence of the change in average transmission $T_s(\nu_0, t)$ on the derivative $\partial l_m(t)/\partial t$ is presented.

## 2. Systematic errors caused by the changes in average value of the transmission and in amplitude of the interference fringes [11]

The transmission spectrum of the parallel layer at the temperature $t_k$ and the temperature dependence of the transmission for the wave number $\nu_0$ are described by the relations

$$T(\nu, t_k) = T_s(\nu, t_k) + A(\nu, t_k)\cos\delta(\nu, t_k), \tag{7}$$

$$T(\nu_0, t) = T_s(\nu_0, t) + A(\nu_0, t)\cos\delta(\nu_0, t) \tag{8}$$

where:

$$\delta(\nu, t_k) = 2\pi X(\nu, t_k) = 4\pi d(t_k) n_p \nu, \tag{9}$$

$$\delta(\nu_0, t) = 2\pi \Delta X(\nu_0, t) = 4\pi \Delta d(t) n_p \nu_0. \tag{10}$$

On the assumption that $T_s$ = idem and $A$ = idem, thus under idealized conditions, the layer transmission reaches the extremal values when $X(\nu, t_k)$ and $X(\nu_0, t)$ are either the entire or half numbers. In reality, $T_s$ = var and $A$ = var (Figs. 3–5), for the reasons which will be described in the further part of this work. The changes both in the average values of transmission $T_s$ and in amplitude $A$ of the interference fringes have some influence on the position of the transmission extremes $T(\nu, t_k)$ and $T(\nu_0, t)$.

In both variants of the interference method the positions of maxima and minima of transmission are determined from the real relations $T_r(\nu, t_k)$ and $T_r(\nu_0, t)$ in order to determine the plane-parallel layer thickness $d(t_k)$ or its incrememt $\Delta d(t)$. In the first case we obtain the sequent wave numbers $\nu_j$, while in the other one the temperatures $t_j$ which correspond to the extremes of the transmissions $X_r(\nu_j, t_k)$ and $X_r(\nu_0, t_j)$. There arises a question, how high would be the orders of interference $X_i(\nu_j, t_k)$ and $X_i(\nu_0, t_j)$, if the measurements were carried out under idealized conditions, i.e., at a constant average transmittivity $T_s$ and constant amplitude of the intererence fringes $A$? The variable $z$ used in further considerations will denote the wave number $\nu$ in the formula (7) and the temperature $t$ in formula (8).

In order to answer the said question, it is necessary to determine a correction $\varrho(z)$ satisfying the relation

$$X_i(z) = X_r(z)+\varrho(z), \tag{11}$$

and consisting of two terms

$$\varrho(z) = \varrho_T(z)+\varrho_A(z) \tag{12}$$

where $\varrho_T(z)$ is a correction connected with the change in average value of transmission $T_s(z)$, while $\varrho_A(z)$ follows from the change in amplitude $A(z)$.

The transmission of the layer is described by the relation

$$T(z) = T_s(z)+A(z)\cos\delta_i(z), \tag{13}$$

in which

$$\delta_i(z) = 2\pi X_i(z) = 2\pi\big(X_r(z)+\varrho(z)\big) = \delta_r(z)+2\pi\varrho(z). \tag{14}$$

Extreme values of the transmission $T(z)$ will be reached when

$$T'(z) = T_s'(z)-A(z)\delta_i'(z)\sin\delta_i(z)+A'(z)\cos\delta_i(z) = 0. \tag{15}$$

By dividing the terms of this equation by

$$B(z) = \sqrt{\big(A(z)\delta_i'(z)\big)^2+\big(A'(z)\big)^2} \tag{16}$$

we obtain

$$\sin\delta_i(z)\frac{A(z)\delta_i'(z)}{B(z)}-\cos\delta_i(z)\frac{A'(z)}{B(z)} = \frac{T_s'(z)}{B(z)}. \tag{17}$$

It may be easily noticed that

$$\frac{A(z)\delta_i'(z)}{B(z)} = \cos\varphi(z), \quad \text{and} \quad \frac{A'(z)}{B(z)} = \sin\varphi(z) \tag{18}$$

[where $\varphi(z) = 2\pi\varrho(z) = 2\pi(\varrho_T(z)+\varrho_A(z))$], then we obtain

$$\sin\delta_i(z)\cos\varphi(z) - \cos\delta_i(z)\sin\varphi(z) = \frac{T_s'(z)}{B(z)}. \tag{19}$$

The left hand side of the relation (19) may be expressed by the difference $\delta_i(z)-\varphi(z)$

$$\sin(\delta_i(z)-\varphi(z)) = \frac{T_s'(z)}{B(z)}, \tag{20}$$

which means that

$$\delta_i(z)-\varphi(z) = \arcsin\frac{T'(z)}{B(z)} + k2\pi, \tag{21}$$

$k = 1, 2, 3\ldots.$

$$\delta_i(z)-\varphi(z) = \left(\pi - \arcsin\frac{T'(z)}{B(z)}\right) + k2\pi, \tag{22}$$

By transforming the above relations and taking account of the relations (14) and (18) we obtain:

*for maxima*

$$\mathrm{X}_i(z) = \frac{1}{2\pi}\arcsin\frac{T'(z)}{B(z)} + \frac{1}{2\pi}\arcsin\frac{A'(z)}{B(z)} + k, \tag{23}$$

*for minima*

$$X_i(z) = -\frac{1}{2\pi}\arcsin\frac{T_s'(z)}{B(z)} + \frac{1}{2\pi}\arcsin\frac{A'(z)}{B(z)} + \frac{2k+1}{2}. \tag{24}$$

The sought corrections are:

$$\varrho_T(z) = \pm\frac{1}{2\pi}\arcsin\frac{T_s'(z)}{B(z)} = \frac{1}{2\pi}\arcsin\times\frac{T_s'(z)}{\sqrt{(A(z)\delta_i'(z))^2+(A'(z))^2}}, \tag{25}$$

$$\varrho_A(z) = \frac{1}{2\pi}\arcsin\frac{A'(z)}{B(z)} = \frac{1}{2\pi}\arcsin\times\frac{A'(z)}{\sqrt{(A(z)\delta_i'(z))^2+(A'(z))^2}}. \tag{26}$$

Relations (25) and (26) may be simplified by taking advantage of the approximation $\sin x \simeq x$ and truncating the respective power series representation of the denominator after the first term. Assuming additionally that $\delta_r(z) \simeq \delta_i(z)$ we obtain

$$\varrho_r(z) = \pm\frac{1}{4\pi^2 X_r'(z)}\frac{T_s'(z)}{A(z)} = \pm\frac{1}{4\pi^2 A(z)}\frac{\partial T_s(z)}{\partial X_r(z)} \tag{27}$$

("+" – for maxima, "–" – for minima),

$$\varrho_A(z) = \frac{1}{4\pi^2 X_r(z)} \frac{A'(z)}{A(z)} = \frac{1}{4\pi^2 A(z)} \frac{\partial A(z)}{\partial X_r(z)}. \tag{28}$$

From the analysis of the above relations it follows that:

i) The change in the average value of transmission $T_s(z)$ causes the shift of transmission extremes, the maxima being shifted consistently with the sign of the derivative $T_s'(z)$, and the minima – in the opposite direction. This means that when the average value of transmission $T_s(z)$ increases the maxima appear for $z = z_i + \Delta z$ (therefore $X_{r\max}(z) < X_i(z)$), while the minima occur for $z = z_i - \Delta z$ (therefore $X_{r\min}(z) > X_i(z)$).

ii) Due to the change in amplitude of interference fringes their position is changed in accordance with the sign of the derivative $A'(z)$. This concerns both the maxima and minima of transmission (for $A'(z) > 0$, $z = z_i + \Delta z$ and thus $X_r(z) < X_i(z)$).

iii) The difference $\varrho(z) = X_i(z) - X_r(z)$ is proportional to the derivatives $\partial T(z)/\partial X_r(z)$ and $\partial A(z)/\partial X_r(z)$ and inversely proportional to the amplitude $A(z)$.

## 3. Testing measurements

The carried out experiment was aimed at verification of the formulae (25) and (26) and at the assessment of effects of the changes of interference fringe amplitude $A(z)$ and average transmission $T_s(z)$ on the value of the thermal expansion coefficient $\alpha(t)$.

The measurements have been performed using a specially designed cuvette adjusted to the cooperation with the UV VIS spectrophotometer. The cuvette has been equipped with the temperature stabilizing system and with a sensitive system gradient-controlling and recording the temperature changes in time. The structure of the applied interference dilatometer shown in Fig. 2 consists of a brass yoke, four brass pulling screws and two pushing screws.

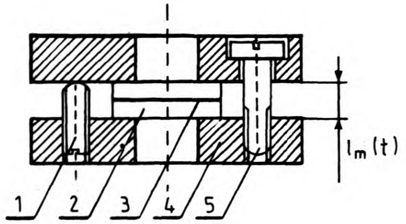


Fig. 2. Construction of the interference dilatometer: 1 – regulation screw, 2 – glass plate, 3 – spacer, 4 – front plate of the dilatometer, 5 – regulation screw

The plane-parallel plates have been made of BK–7 glass. An optimal (with reference to the coherence length of the light beam in the spectrophotometer) thickness of the air slit $d_0$ ranging within 10–20 μm was obtained by using the distance spacers made of teflon folie. The interferometer has been adjusted with the help of the regulating screws, so that the internal surfaces of the glass plates creating the air slit be parallel. The parallelism of the slit was controlled by the sodium lamp. The measuring setup was described in a detailed way in papers [11], [12].

### 3.1. Experimental verification of the correction formulae $\varrho_T(z)$ and $\varrho_A(z)$

The amplitude of interference fringes and the average value of transmission depend essentially on the properties of the reflecting layers deposited on the internal surfaces of the glass plates (Fig. 1). The interference spectrum of transmission $T(\nu)$ has been recorded by using the plates with both metal and semiconductor layers. In Figure 3

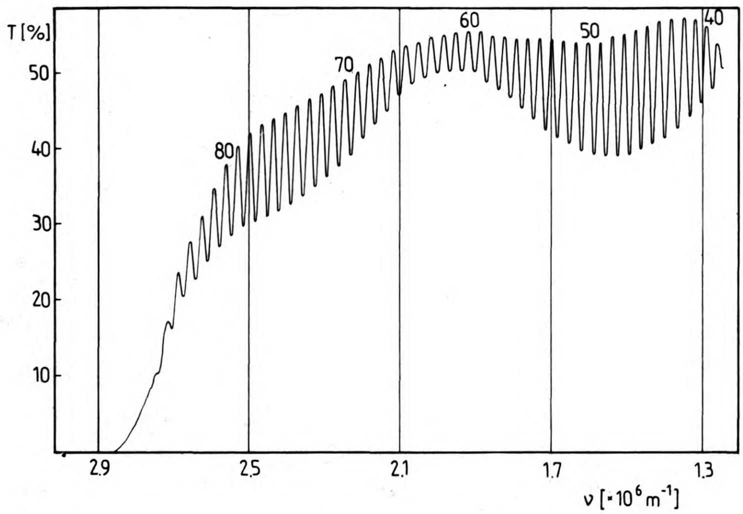


Fig. 3. Interference spectrum of transmission for a plane-parallel air layer of thickness $d = 15.59\,\mu m$ created between the plates of BK–7 glass with the reflection layer $SnO_2$

the transmission spectrum of the slit of thickness $d = 15.59\ \mu m$ is shown. The glass plates were covered with the $SnO_2$ layers. The amplitude $A(\nu)$ of interference fringes changes taking its maximum values for wave numbers $\nu = 1.4 \times 10^6 m^{-1}$ and $\nu = 2.4 \times 10^6 m^{-1}$, and its minimum values for $\nu = 2.0 \times 10^6 m^{-1}$. By depositing a thin layer of silver on the glass plates an almost constant amplitude of the interference fringes was achieved (Fig. 4).

In view of the obvious fact that the layer thickness $d(\nu_j)$ calculated on the base of interference spectrum of transmission recorded at a steady temperature, should be the same for all the interference fringes $X_r(\nu_j)$, the formulae (25) and (26) have been verified by comparing the results $d_p(\nu_j)$ and $d(\nu_j)$ obtained respectively with and without the correction terms $\varrho_A(\nu)$ and $\varrho_T(\nu)$ being taken into account

$$d_p(\nu_j) = \frac{X_r(\nu_j) + \varrho_A(\nu_j) + \varrho_r(\nu_j)}{2n_p \nu_j}; \quad d(\nu_j) = \frac{X_r(\nu_j)}{2n_p \nu_j}.$$

In the first case a steady thickness of the layer $d_p(\nu_j) =$ idem has been obtained with a random error within the limits 0.1%. In the other case $d(\nu_j) =$ var, since apart from the random error there appeared also the systematic errors which were especially

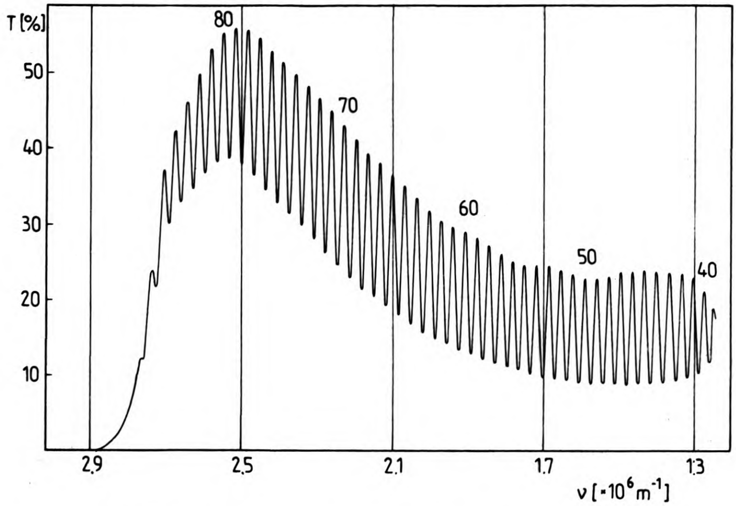


Fig. 4. Interference spectrum of transmission for plane-parallel air layer of thickness $d = 15.66\,\mu m$ created between the plate of BK–7 glass with silver layer

great at the borders of the measurement range, where the amplitude of the interference fringes decreased rapidly. In these regions the deviation from the average value of the thickness $\bar{d}(\nu_j)$ amounts to several per cent.

The above analysis has been carried out for five different reflecting layers. It has been stated that the application of the approximate formulae (27) and (28) to calculation of the corrections $\varrho_A(\nu)$ and $\varrho_T(\nu)$ is justified.

### 3.2. Measurement of the coefficient of thermal expansion $\alpha(t)$

The measurements of the coefficient of linear expansion $\alpha(t)$ for brass have been performed with the help of the dilatometer shown in Fig. 2. The plates of BK–7 glass of thickness 2.77 mm covered with a thin semitransparent silver layer have been applied. The coefficient of thermal expansion $\alpha_s(t)$ for BK–7 glass in the 0–500°C temperature range is a linear function of temperature and may be described by the relation

$$\alpha_s(t) = \alpha_{s1} + \alpha_{s2} t, \tag{29}$$

in which $\alpha_{s1} = 7.26 \times 10^{-6}$ 1/°C and $\alpha_{s2} = 7.37 \times 10^{-9}$ 1/(°C)$^2$ [13].

When realizing the first variant of the interference method the transmission spectrum $T(\nu, t_k)$ of the layer has been recorded for twenty temperatures within 0–100°C range. The thickness of the air slit $d(t_k)$ has been calculated taking account of all the interference fringes $X_r(\nu_j, t_k)$ occurring in the transmission spectrum of the layer in the wave number range $\nu = 1.4\text{–}2.5 \times 10^6$ m$^{-1}$, thus from the range in which their amplitude is constant (Fig. 4). Taking account of the relation (3) the coefficient of linear expansion $\alpha(t)$ of the brass has been calculated. In the above temperature range the linear relation has been obtained. It may be described by the coefficient

$\alpha_1 = 17.70 \times 10^{-6}$ 1/°C and $\alpha_2 = 11.2 \times 10^{-9}$ 1/(°C)$^2$. The obtained results are consistent with the literature data for the brass [14].

When realizing the second variant of the interference method, the measurements of transmission $T(\nu_0, t)$ were made at a continuous change of temperature within the range 0–100°C for the light of wave number $\nu = 1.7 \times 10^6$ m$^{-1}$ (Fig. 5).

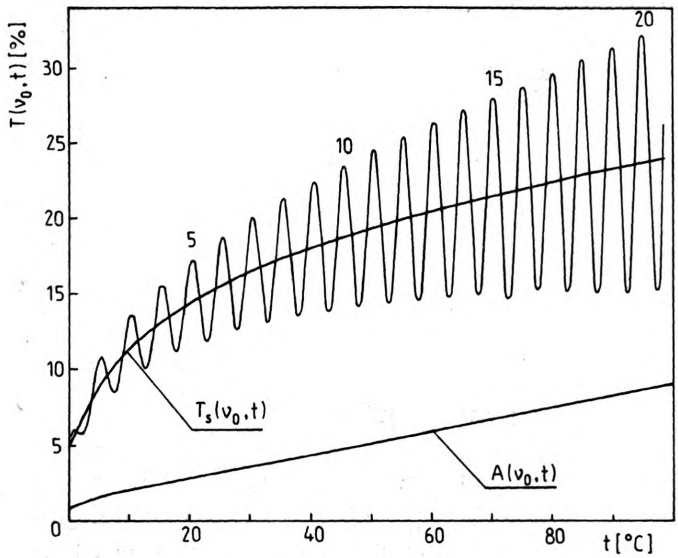


Fig. 5. Temperature changes of transmission $T(\nu_0, t)$ of the brass dilatometer recorded for the light of wave number $\nu_0 = 1.7 \times 10^6$ m$^{-1}$ ($l_{m_0} = 5.575$ mm)

The derivative $p(\nu_0, \bar{t})$ has been calculated by determining the temperature $t_j$ corresponding to the sequent extremes of transmission

$$p(\nu_0, \bar{t}) = \frac{\partial X(\nu_0, \bar{t})}{\partial t} = \frac{X(\nu_0, t_{j+1}) - X(\nu_0, t_j)}{t_{j+1} - t_j}; \quad \bar{t} = \frac{1}{2}(t_{j+1} - t_j). \tag{30}$$

Taking account of the fact that for the neighbouring maxima and minima of transmission

$$X(\nu_0, t_{j+1}) - X(\nu_0, t_j) = 1/2 \tag{31}$$

we obtain

$$p(\nu_0, \bar{t}) = \frac{1/2}{(t_{j+1} - t_j)}. \tag{32}$$

The values of the derivative $p(\nu_0, \bar{t})$ calculated for the sequent temperatures $\bar{t}$ are of oscillatory character caused by the changes of the mean value of the transmission $T_s(\nu_0, t)$, Fig. 5. The derivative $\partial T_s(\nu_0, t)/\partial t$ is positive and therefore the maxima transmission are shifted by $\Delta_T t_i$ toward the higher temperatures, while the minima are displaced by the same value but in the opposite direction. A consequence of this fact is the following relation:

$$\frac{1}{2[(t_{j+1}+\Delta_T t_{j+1})-(t_j-\Delta_T t_j)]} < \frac{1}{2[(t_{j+2}-\Delta_T t_{j+2})-(t_{j+1}+\Delta_T t_{j+1})]}. \tag{33}$$

This inequality is satisfied worse and worse with the increasing amplitude of the interference fringes and the decreasing derivative $\partial T_s(\nu_0, t)/\partial t$ (Figs. 5 and 6).

The derivative $p(\nu_0, t)$ has been calculated once more taking account of the correction $\varrho_T(t_j)$ in the relation (30)

$$p_T(\nu_0, \bar{t}) = \frac{[X(\nu_0, t_{j+1})+\varrho_T(t_{j+1})]-[X(\nu_0, t_j)+\varrho_T(t_j)]}{t_{j+1}-t_j}$$
$$= \frac{1/2+[\varrho_T(t_{j+1})-\varrho_T(t_j)]}{t_{j+1}-t_j}. \tag{34}$$

It may be also written in the form

$$p_T(\nu_0, \bar{t}) = \frac{1/2}{t_{j+1}-t_j}+\frac{\varrho_T(t_{j+1})-\varrho_T(t_j)}{t_{j+1}-t_j} = p(\nu_0, \bar{t})+\Delta p_T(\bar{t}). \tag{35}$$

The oscillatory character of the changes of the derivative $p(\nu_0, \bar{t})$ follows from the fact that the factor $\Delta p_T(\bar{t})$ correcting its value takes sequentially positive and negative signs. This, in turn, is due to the fact that for the maxima the correction $\varrho_T(t_j)$ is added to the number of the interference fringe $X_r(\nu_0, t_j)$, while for the minima it is substrated. In Figure 6 the corrected values of the derivative calculated from the relation (34) are denoted by squares. Under the circumstances when the average transmission $T_s(\nu_0, t)$ does not change too fast (as compared to the amplitude of the interference fringes) it may be assumed that $|\varrho_T(t_{j+1})| \simeq |\varrho_T(t_j)|$ which means that the neighbouring extremes (maxima and minima) of transmission are shifted in the

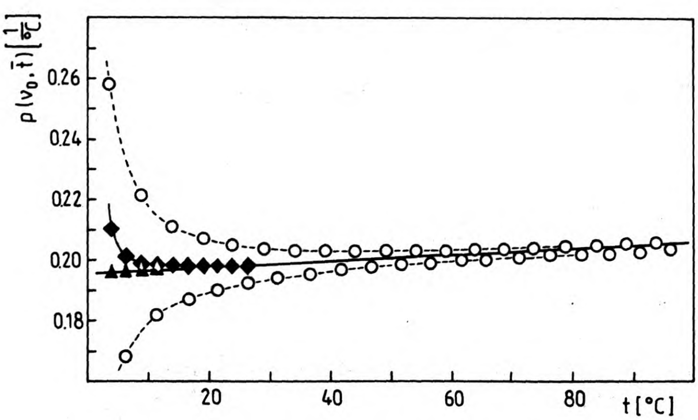


Fig. 6. Derivative of the interference order with respect to temperature $p(\nu_0, \bar{t}) = \partial X(\nu_0, \bar{t})/\partial t$ (○ – $p(\nu_0, \bar{t})$, ◆ – $p_T(\nu_0, \bar{t}) = p(\nu_0, \bar{t})+\Delta p_T(\bar{t})$, ▲ – $p_{T,A}(\nu_0, \bar{t}) = p_T(\nu_0, \bar{t})+\Delta p_A(\nu_0, \bar{t})$)

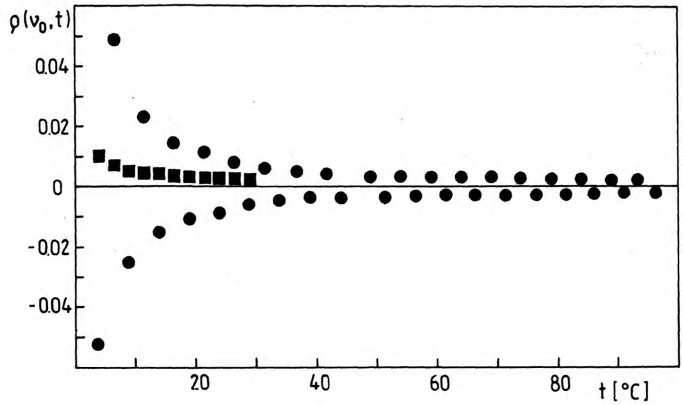


Fig. 7. Corrections $\varrho_T(t_j)$ and $\varrho_A(t_j)$ calculated for sequent extremes of transmission (● $-\varrho_T(t_j)$, ■ $-\varrho_A(t_j)$)

opposite directions by almost the same value (Fig. 7). In this situation the elimination of the influence of the slope of the transmission $T_s(\nu_0, t)$ is reduced to averaging the derivative $p(\nu_0, \bar{t})$, calculated for the sequent points $t = (t_{j+1} + t_j)/2$, i.e., for $p_T(\nu_0, \bar{t}) = \bar{p}(\nu_0, \bar{t})$.

Based on the preliminarily corrected value of the derivative $p_T(\nu_0, \bar{t}) = p(\nu_0, \bar{t}) + \Delta p_T(\bar{t})$ the coefficient of thermal expansion $\alpha(t)$ for brass (denoted in Fig. 8 by black squares) has been calculated. In the 0–15°C temperature range the value of this coefficient decreases with the increase of temperature which is an unexpected effect for this alloy and proves the occurrence of some systematic errors. In the above range the amplitude of the interference fringes $A(t)$ increases rapidly, causing shift of the transmission extremes toward the higher temperatures. This shift is proportional to the quotient $A'(t_j)/A(t_j)$ defining the relative change of the amplitude $A(t_j)$ caused by a unit increment of the temperature.

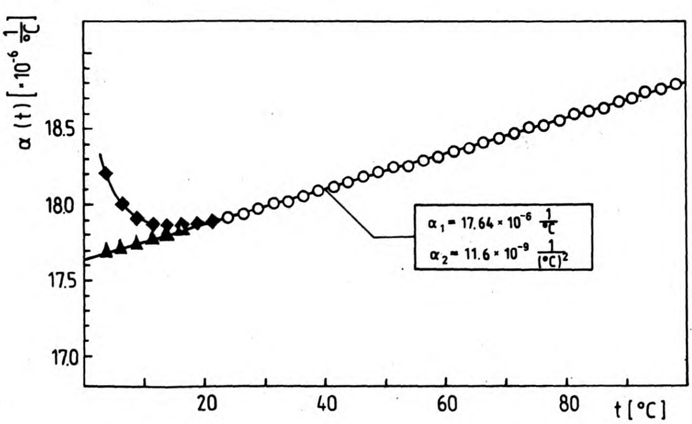


Fig. 8. Coefficient of thermal expansion $\alpha(t)$: ◆—calculated for $p_T(\nu_0, t)$, ▲—calculated for $p_{T,A}(\nu_0, t)$

In Figure 7 a correction $\varrho_A(t_j)$ is calculated from the relation (28). It has the same sign for all the maxima and minima of transmission. The complete correction of the derivative $p(\nu_0, \bar{t})$ consists in eliminating the influence of the changes of both the average values of transmission $T_s(\nu_0, t_j)$ and the amplitude $A(\nu_0, t_j)$:

$$p_{T,A}(\nu_0, \bar{t}) = p(\nu_0, \bar{t}) + \Delta p_T(\bar{t}) + \Delta p_A(t),$$
$$\Delta p_A(\bar{t}) = \frac{\varrho_A(t_{j+1}) - \varrho_A(t_j)}{t_{j+1} - t_j}. \tag{36}$$

The corrected derivative $p_{T,A}(\nu_0, \bar{t})$ is illustrated by black triangles in Fig. 6. The coefficient of thermal expansion $\alpha(t)$ for brass calculated on the base of $p_{T,A}(\nu_0, \bar{t})$ changes linearly within the whole examined temperature range and the coefficients $\alpha_{m1}$ and $\alpha_{m2}$ have the values correspondingly equal to $17.64 \times 10^{-6}$ 1/°C and $11.60 \times 10^{-9}$ $1/(°C)^2$.

The values of the coefficient of thermal expansion are consistent (the divergences not exceeding 0.5%). This consistency provides the proof that the influence of the change of both the average transmission $T_s(\nu_0, t_j)$ and amplitude $A(\nu_0, t_j)$ of the interference fringes on the derivative $p(\nu_0, \bar{t})$ and, consequently, on the value of the coefficient of thermal expansion $\alpha(t)$ may be eliminated efficiently by taking account of the corrections $\varrho_T(\nu_0, t_j)$ and $\varrho_A(\nu_0, t_j)$.

The results obtained in the first variant of the interference method – based on measuring the thickness of the air slit $d(t_k)$ at the stabilized temperature $t_k$ – have been treated as a kind of muster. The thickness $d(t_k)$ has been calculated by taking account of the interference fringes from this range of the transmission spectrum in which their amplitude is constant ($\nu = 1.4$–$2.5 \times 10^6$ m$^{-1}$, Fig. 5) which means that $\varrho_A(\nu, t_k) = 0$. By using the method of linear regression we have eliminated the influence of the change of the average transmission $T_s(\nu, t_k)$ on the derivative $\partial X(\nu, t_k)/\partial \nu$ being proportional to the layer thickness $d(t_k)$. This influence was small and only in the wave numbers $\nu = 2.1$–$2.5 \times 10^6$ m$^{-1}$ the shift of maxima and minima of transmission in the opposite direction was slightly greater than the accuracy of the read-outs.

The first variant of the interference method is very tedious and much more difficult in the experimental realization and for this reason is less commonly used. The realization of the measurements in the second variant of the method is much faster and simpler. Its wide application is also due to better susceptibility to automation of both the measurement and data processing. The accuracy may be much higher when taking account of the corrections $\varrho_A(\nu_0, t)$ and $\varrho_T(\nu_0, t)$.

## 4. Recapitulation

In the first part of this work, the influence of the changes in the average values of both transmission $T_s(z)$ and amplitude $A(z)$ of interference fringes on the positions of the transmission extremes due to continuous change of the optical path are described. The formulae derived for the corrections $\varrho_A(z)$ and $\varrho_T(z)$ allow us to correct the systematic errors. This correction is indispensable since even small errors in the read-outs of the position of the interference fringes $X(z)$ have a significant influence on the measurement accuracy of the physical magnitudes depending on the derivative $\partial X(z)/\partial z$.

The second part of the work is devoted to an experimental verification of the above formulae. A dilatometer which has been designed enabled us to realize two variants of the interference method for determining the coefficient of thermal expansion for metals. In the first variant of the method the above errors do not appear practically and therefore the results obtained are treated as the master ones in relation to the second variant of the method where they are very essential.

It has been stated that in the case where the change of the average transmission $T_s(z)$ between the neighbouring extremes is small (as compared to the value of the amplitude $A(z)$ of interference fringes) the corrections $\varrho_T(z)$ take practically the same absolute values but their signs are opposite. By applying the respective averaging procedure the influence of the slope of the transmission $T_s(z)$ may be eliminated without taking account of the formula (27). The change of amplitude $A(z)$ of the interference fringes causes some shift of the maxima and minima of transmission in the same direction. This has a significant influence on the value of the derivative $\partial X(z)/\partial z$. The only way to eliminate this error is to take account of the correction $\varrho_A(z)$ defined by formula (28).

When measuring the coefficient of thermal expansion of the solids (method of recording the temperature changes of transmission $T(\nu_0, t)$ for the light of constant wave number $\nu_0$) the effect of changes in the amplitude $A(\nu_0, t)$ of the interference fringes occurs frequently, especially for high temperatures. This is caused by the change of reflection layer properties as well as by the effect of spoiling the parallelism of the glass plates of the dilatometer due to nonuniform distribution of temperature in the measuring ring. Even for very slow heating of the dilatometer some regions in the temperature characteristics $T(\nu_0, t)$ may appear in which the amplitude $A(\nu_0, t)$ may first fall down rapidly to increase quickly thereafter. In this situation, the derivative $\partial X(\nu_0, t)/\partial t$ increases initially and then decreases, while the coefficient of thermal expansion $\alpha(t)$ changes similarly as it is the case for ferromagnetics at the vicinity of the temperature $T_C$ (Curie temperature) of transition to the paramagnetic state.

The errors caused by the change of amplitude of the interference fringes change the value of derivative $\partial X(z)/\partial z$ by few to several per cent, thus deforming the shape of characteristics $f(z)$ depending on its values. The corrections $\varrho_T(z)$ and $\varrho_A(z)$ determined on the base of the shape of the real envelope of transmission $T_r(z)$ may be applied in all the measurements in which the examination of the changes in the optical path of the ligh wave constitutes a basis to determine such physical magnitudes as the coefficient of refraction or the thickness of the plane parallel layers of solids.

The interference methods of dispersion measurements for the indices of refraction of fluids and isotropic layers of solids [15–18] are very sensitive to any inaccuracies in determining the position of the transmission extremes. The highest influence on the improvement of the accuracy of the results has the application of the corrections $\varrho_T(z)$ and $\varrho_A(z)$ in the Caris and Vamfo methods [19–22] enabling the measurements of the thickness of the thin dielectric layers.

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## Элиминация систематических ошибок возникающих в измирительных методах интерференционной дилатометрии

В работе охарактеризованы систематические ошибки, возникающие в измирительных методах основанных на исследовании оптической передачи систем, в которых изменяется оптическая длина пути световой волны в результате параметрического изменения условий интерферометрии. Введены формулы позволяющие исправить ошибки возникающие при определении положения экстремумов оптической передачи исследуемой системы. В исследовательской части работы представлено влияние изменения среднего значения оптической передачи системы и амплитуды интерференционных полос на точность обозначения коэффициента теплового расширения латуни с использованием интерференционного метода измерения.