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Variable wavelength interferometry VI. Some useful modifications of the VAWI-2 technique

MAKSYMILIAN PLUTA

Central Optical Laboratory, ul. Kamionkowska 18, 03-805 Warsaw, Poland.

Alternative approach to the fringe-field variable wavelength method, presented earlier and referred to as the VAWI–2 technique, is now modified in its processing stage. The modification permits us to remove some time-consuming operations. In particular the plots $b(m)$, i.e., the interfringe spacing versus current interference order, can be ignored.

1. Introduction

Two previous papers [1] and [2] of this series dealt with a variable wavelength interferometric procedure, the VAWI–2 technique, which led to the final results via some time-consuming operations. In particular, two graphs $b(m)$ were plotted: one for the empty interference field and the other for the interference image of an object under study. In some instances, two object images produced by a double-refracting interference system were analysed; the graph $b(m)$ for the empty interference field was then needless, but also two graphs $b(m)$ were constructed for the two images of the object under study. Now, another procedure is proposed which does not requires the plots mentioned above.

The VAWI–2 technique uses two parallel pointer lines, L_1 and L_2, in the image plane of an interference system. The zero-order fringe of the empty interference field is adjusted to the coincidence with one pointer line (L_1), and high-order fringes are consecutively brought into coincidence with the other pointer line (L_2) when the wavelength of monochromatic light is continuously varied. This operation is referred to as pointing or testing the empty interference field. Next, the centre of the zero-order fringe of the empty interference field is still kept at the position of the pointer line L_1, while the centres of high-order fringes displaced by an object under study are consecutively brought into coincidence with the pointer line L_2. This operation is referred to as pointing the interference image of the examined object.

2. Testing the empty interference field

It is assumed that a fringe interference field is produced by two plane wavefronts inclined to each other at a small angle ε, and that the distribution of bright and dark interference fringes is identical to the one which is observed in the image plane of a double-refracting interference system with crossed polars, i.e., dark interference fringes occur along straight lines where the optical path differences Δ between the interfering wavefronts are equal to 0, $\pm\lambda$, $\pm 2\lambda$, ...; consequently, bright interference fringes occupy the positions where $\Delta = \pm\lambda/2, \pm 3\lambda/2, \pm 5\lambda/2, \ldots$. If the light wavelength λ is varied, the interfringe spacing (b) also varies according to the relation

$$b = \lambda/\varepsilon. \tag{1}$$

In the VAWI–2 technique, the fringe interference pattern is projected on to a gauging graticule consisting of two pointer lines L_1 and L_2 and placed in the front focal plane of an interferometer ocular. The pointer lines are mutually parallel and separated from each other by a distance d which is as long as possible, e.g., $d = 10b$ or even more. One of these lines L_1 is brought into coincidence with the centre of the zero-order fringe I_0 of the empty interference field (Fig. 1), while the

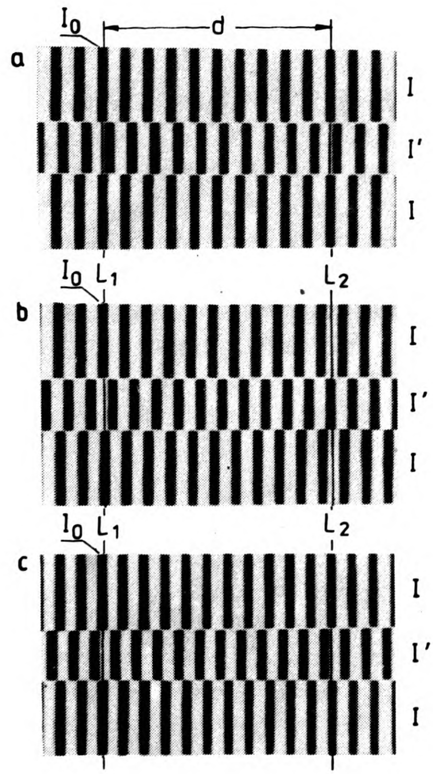

Fig. 1. Testing (pointing) the empty interference field

consecutive hight-order fringes I are brought into coincidence with the other pointer line L_2 when the wavelength of monochromatic light is varied. Starting from long wavelengths permits us to select initially such a first clearly visible red wavelength λ_1 for which one of the high-order dark fringes I becomes coincident with the pointer line L_2 (Fig. 1**a**). The distance d is now covered by m_1 interfringe spacings b_1, i.e.,

$$d = m_1 b_1 \tag{2}$$

where m_1 is referred to as the initial interference order at the pointer line L_2. The optical path difference Δ_1 at this line is given by

$$\Delta_1 = d\varepsilon_1 = m_1 \lambda_1 . \tag{3}$$

Next, let the light wavelength be continuously decreased from λ_1 to $\lambda_2, \lambda_3, \lambda_4, \ldots$ for which the pointer line L_2 becomes consecutively coincident with the bright (Fig. 1**b**), dark (Fig. 1**c**), ... fringes I whose interference orders are higher by $q_s = 0.5, 1, 1.5, 2, \ldots$ with respect to m_1. These consecutive coincidences may be expressed as

$$\Delta_s = d\varepsilon_s = (m_1 + q_s)\lambda_s \tag{4}$$

where $s = 2, 3, 4, \ldots$. This equation expresses the optical path differences $\Delta_2, \Delta_3, \Delta_4, \ldots$ between the interfering wavefronts for the wavelengths $\lambda_2, \lambda_3, \lambda_4, \ldots$ at the pointer line L_2.

From Eqs. (3) and (4) it follows that

$$m_1 = q_s \frac{\lambda_s}{\varepsilon_{s1}\lambda_1 - \lambda_s} \tag{5}$$

where

$$\varepsilon_{s1} = \frac{\varepsilon_s}{\varepsilon_1}. \tag{6}$$

According to relation (1), the wavelengths λ_1 and λ_s may be expressed as $\lambda_1 = b_1 \varepsilon_1$ and $\lambda_s = b_s \varepsilon_s$. Equation (5) can therefore be rewritten as

$$m_1 = q_s \frac{b_s}{b_1 - b_s}, \tag{7}$$

or as

$$(m_1 + g_s) b_s = m_1 b_1 . \tag{8}$$

The latter equation shows that the product $(m_1 + q_s) b_s$ is a constant and wavelength independent parameter across the spectrum. This parameter can exactly be determined by the mean value

$$\frac{\sum_{s=1}^{s=N} (m_1 + q_s) b_s}{N} = \overline{m_s b_s} = C \tag{9}$$

where N is the total number of interference fringe coincidences with the pointer line L_2. It is important to note that the only quantities which are directly measured are interfringe spacings b_s, while the interference order increments q_s are observed, and the initial interference order m_1 is calculated from Eq. (7) or even visually recorded. The interfringe spacings can be determined extremely accurately as the distance $l = 20b_s$, $40b_s$ or even $100b_s$ is measured rather than a single interfringe spacing b_s.

3. Pointing the interference image of an object under study

Let us now consider the interference fringes I' (Fig. 2) displaced by a plate-like transparent object transilluminated normally by a parallel beam of monochromatic light. The pointer line L_1 is still coincident, as before (Fig. 1), with the centre of the

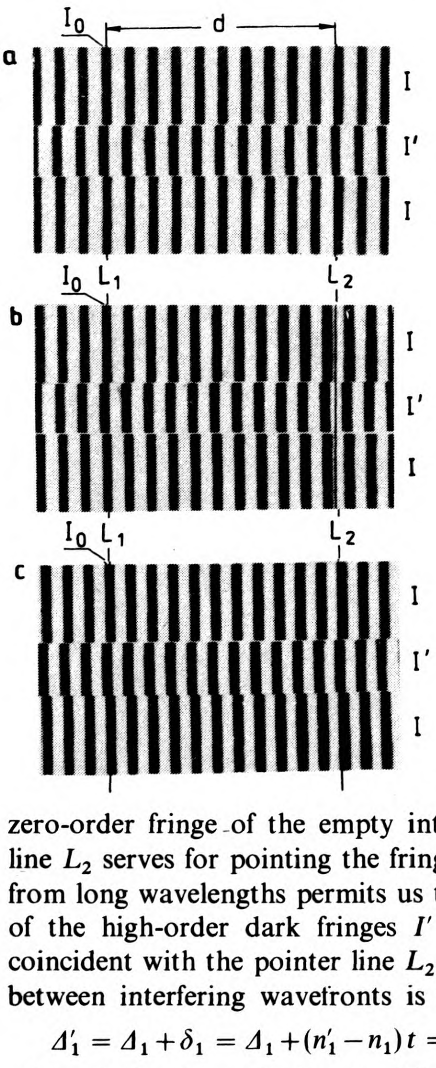

Fig. 2. Pointing the interference image of an object under study

zero-order fringe of the empty interference field. On the other hand, the pointer line L_2 serves for pointing the fringes I' displaced by the examined object. Starting from long wavelengths permits us to select such a red wavelength λ_1 for which one of the high-order dark fringes I' displaced by the object under study becomes coincident with the pointer line L_2 (Fig. 2**a**). At this line, the optical path difference between interfering wavefronts is now given by

$$\Delta'_1 = \Delta_1 + \delta_1 = \Delta_1 + (n'_1 - n_1)t = m_1 \lambda_1 \tag{10}$$

where δ_1 is the optical path difference produced by the examined object, i.e.

$$\delta_1 = (n'_1 - n_1)t. \tag{11}$$

Here n_1 and t are the refractive index and thickness of the object, and n'_1 is the refractive index of a medium which surrounds the object. The symbol Δ_1 denotes the optical path difference associated with the empty interference field at the pointer line L_2. It is selfevident that Δ_1, m_1 and λ_1 are now, in general, other than in Eq. (3).

If the light wavelength is decreased from λ_1 to shorter wavelengths, we can select such of them ($\lambda_2, \lambda_3, \lambda_4, \ldots$) for which consecutive bright (Fig. 2**b**), dark (Fig. 2**c**), bright, ... fringes I' become coincident with the pointer line L_2. These consecutive coincidences may be expressed as

$$\Delta'_s = \Delta_s + \delta_s = \Delta_s + (n'_s - n_s)\, t = (m_1 + q_s)\, \lambda_s \tag{12}$$

where $s = 2, 3, 4, \ldots$ and $q_s = 0.5, 1, 1.5, \ldots$.

By combining Eqs. (10), (12), and (1) we obtain

$$m_1 = q_s \frac{b_s}{N'_{s1}\, \varepsilon_{1s} b_1 - b_s} + d \frac{N'_{s1}\, \varepsilon_{1s} - 1}{N'_{s1}\, \varepsilon_{1s} b_1 - b_s} \tag{13}$$

where

$$\varepsilon_{1s} = \frac{\varepsilon_1}{\varepsilon_s}. \tag{14}$$

Note that according to Eq. (6) the above coefficient $\varepsilon_{1s} = 1/\varepsilon_{s1}$, i.e., $\varepsilon_{s1}\, \varepsilon_{1s} = 1$.

If the Biolar PI microinterferometer is used [1], the term $N'_{s1}\, \varepsilon_{1s}$ is nearly equal to unity for many objects to be studied, and the initial interference order m_1 can be calculated from an approximative formula, which is quite similar to Eq. (7). For the interested reader more details regarding this problem may be found in [1].

4. Determining the optical path difference δ produced by an object under study

In conventional interferometry, the optical path difference δ produced by an object under study is determined from the formula

$$\delta = \frac{c}{b} \lambda \tag{15}$$

where b is the interfringe spacing of the empty interference field, c is the displacement of interference fringes observed in the object image, and λ is the wavelength of light used. The displacement c must be measured between the displaced and undisplaced fringes belonging to the same interference order.

Equation (15) may also be written as

$$\delta = \frac{c}{b} \lambda = (m_1 + q)\, \lambda = m\lambda \tag{16}$$

where m_1 is an integer number of interfringe spacings b by which the object displaces interference fringes, q is fraction of b which, when added to m_1, gives the total amount (m) of interfringe spacings by which the object displaces the fringes. For the VAWI method the number m is referred to as the current interference order, m_1 is called the initial interference order expressed always by an integer number, and q is specified as the increment (or decrement) of the current interference order with respect to m_1. We can therefore write the following relation

$$c/b = m_1 + q = m. \tag{17}$$

When this equation is multiplied by b, we have

$$c = (m_1 + q)\, b = mb. \tag{18}$$

As can readily be seen, Eq. (18) is similar to Eq. (8).

According to formulae (8) and (9), Eq. (12) may be rewritten as

$$\Delta'_s = C\frac{\lambda_s}{b_s} + \delta_s. \tag{19}$$

On the other hand, the optical path difference Δ'_s can also be expressed as

$$\Delta'_s = (m_1 + q_s)\,\lambda_s = (m_1 + q_s)\, b_s \frac{\lambda_s}{b_s}. \tag{20}$$

By combining Eqs. (19) and (20) we have

$$\delta_s = [(m_1 + q_s)\, b_s - C]\frac{\lambda_s}{b_s} = (C_s - C)\frac{\lambda_s}{b_s} \tag{21}$$

where

$$C_s = (m_1 + q_s)\, b_s. \tag{22}$$

In the VAWI–2 technique, the wavelengths $\lambda_s = \lambda_1, \lambda_2, \lambda_3, \ldots$ are selected for which interference fringes of orders m_1 and $m_1 + q_s$ become consecutively coincident with the pointer line L_2. Equation (15) must therefore be rewritten as

$$\delta_s = \frac{c_s}{b_s}\lambda_s \tag{23}$$

where $s = 1, 2, 3, \ldots$. Comparing Eqs. (21) and (23) leads to the statement that

$$C_s - C = c_s. \tag{24}$$

Here C refers to the empty interference field and is calculated from Eq. (9); this is a wavelength independent constant quantity. On the other hand, C_s refers to the interference image of an object under study and is calculated from Eq. (22). In general, C_s is a quantity dependent on light wavelength, but sometimes this quantity is also a constant parameter. When for a given object $C_s = \text{const}$, a specific kind of interferometry, referred to as the object adapted interferometry, is obtained. This is possible if the term $N'_{s1}\varepsilon_{1s}$ in Eq. (13) is equal to unity; consequently, Eq. (13) reduces to $m_1 = q_s b_s/(b_1 - b_s)$.

Equation (21) shows that the above-described procedure leading to δ_s is quite simple and no graphs $b(m)$ are required which were recommended for the procedure described earlier [1].

It is worthwhile noting in connection with Eqs. (2), (8), and (9) that the quantity C should be equal to the distance d between the pointer lines L_1 and L_2 (Fig. 2) if this distance and the interfringe spacings b_s are measured identically. In practice, however, $C = d$ if the distribution of interference fringes of the empty interference field is ideally symmetrical with respect to the zero-order fringe. It has been stated that some commercially available interferometers suffer from a variable interfringe spacing across the image plane, i.e., the interfringe spacing between interference fringes of higher and higher orders continuously increases on one side of the zero order fringe and decreases on the other side. Such a variable distribution of interference fringes occurs especially in double-refracting microinterferometers which use typical birefringent Wollaston prisms [3]. In this instance the quantity C is not ideally equal to d, and some small discrepancy occurs between these two parameters. This defect can be overcome if the so-called symmetrical Wollaston prism is used. Another remedy is to use the typical Wollaston prism adjusted somewhat obliquely with respect to the optical axis of the double-refracting interferometer. However, the latter way is less favourable than the former.

5. Illustrative examples

In order to illustrate practically the procedure presented here, a layer of photographic emulsion (10E56 Holotest plate), which was measured previously [1] by using the Biolar PI microinterferometer, is taken into consideration. The results are listed in Table 1. Interfringe spacings b_s, which were directly measured, interference order increments q_s, which were observed, initial interference orders m_1, and current interference orders $m = m_1 + q_s$ are rewritten from Table 5 of Ref. [1]. These data are now supplemented by the results of calculation obtained according to the procedure described in Sections 2–4. Among these new results are the products $m_s b_s = (m_1 + q_s) b_s$ and their mean value C for the empty interference field, the products $(m_1 + q_s) b_s = C_s$ for the interference images of the emulsion layer, the displacements $c_s = |C_s - C|$ of interference fringes, the optical path differences δ_s calculated from Eq. (21), and the refractive indices n_s of the photographic emulsion. The indices n_s result from the relation $n_s - 1 = \delta_s/t$, where t is the emulsion thickness ($t = 6.297$ μm, as reported in [1]). The light wavelengths λ_s corresponding to the interfringe spacings b_s were read out from the calibration plot $b(\lambda)$.

Figure 3 shows the plot $n(\lambda)$ obtained according to the data listed in Table 1. As can readily be seen the plot is identical with that given previously as Fig. 15 in

Table 1. Results of the measurement of the refractive index dispersion $n(\lambda)$ of the photosensitive emulsion of a Holotest 10E56 plate of thickness $t = 6.297$ μm by using the transmitted-light VAWI-2 technique

empty interference field							
q_s	0	1	2	3	4	4.5	5
b_s [μm]	231.825	210.900	194.100	179.500	165.950	160.675	155.615
initial interference order $m_1 = 10$							
$m_s = m_1 + q_s$	10	11	12	13	14	14.5	15
$m_s b_s$ [μm]	2318.25	2319.90	2329.20	2333.50	2323.30	2329.79	2334.23
$C = \overline{m_s b_s} = 2326.8814$ μm							
1-st interference image of the emulsion							
q_s	0	0.5	1	1.5	2		
b_s [μm]	239.433	212.525	189.425	172.40	157.017		
λ_s [nm]	687.7	615.3	554.0	508.8	468.1		
initial interference order $m_1 = 4$							
$m_s = m_1 + q_s$	4	4.5	5	5.5	6		
$C_s = m_s b_s$ [μm]	957.752	956.363	947.125	948.200	942.102		
$c_s = C - C_s$ [μm]	1369.1494	1270.5189	1379.7564	1378.6814	1384.7794		
$\delta_s = c_s \lambda_s / b_s$ [μm]	3.9325	3.9679	4.0353	4.0689	4.1283		
$n = 1 + \delta_s / t$	1.6245	1.6301	1.6408	1.6462	1.6556		
2-nd interference image of the emulsion							
q_s	0	1	2	3	4	5	6
b_s [μm]	230.800	217.275	205.500	194.815	185.075	176.800	168.675
λ_s [nm]	664.6	628.2	596.6	568.1	542.6	520.4	499.0
initial interference order $m_1 = 16$							
$m_s = m_1 + q_s$	16	17	18	19	20	21	22
$C_s = m_s b_s$ [μm]	3692.800	3693.675	3699.000	3701.485	3701.500	3712.800	3710.850
$c_s = C_s - C$ [μm]	1365.9186	1366.7936	1372.1186	1374.6036	1374.6186	1385.9186	1383.9686
$\delta_s = c_s \lambda_s / b_s$ [μm]	3.9332	3.9518	3.9835	4.0085	4.0300	4.0794	4.0943
$n_s = 1 + \delta_s / t$	1.6246	1.6301	1.6326	1.6366	1.6400	1.6478	1.6502

Table 2. Results of the measurement of the thickness (t) of an AZ1350 photoresist film

empty interference field							
q_s	0	1	2	3	4	4.5	5
b_s [μm]	116.735	106.05	97.125	89.725	83.325	80.500	77.650
initial interference order $m_1 = 10$							
$m_s = m_1 + q_s$	10	11	12	13	14	14.5	15
$m_s b_s$ [μm]	1167.350	1166.550	1165.500	1167.075	1166.550	1167.250	1164.750
$C = \overline{m_s b_s} = 1166.432$ μm							
1-st interference image of the film							
q_s	0	1	2	3			
b_s [μm]	111.725	100.650	91.550	84.575			
λ_s [nm]	635.9	577.4	529.9	493.8			
initial interference order $m_1 = 9$							
$m_s = m_1 + q_s$	9	10	11	12			
$C_s = m_s b_s$ [μm]	1005.525	1006.500	1007.050	1014.900			
$c_s = C - C_s$ [μm]	160.907	159.932	159.382	151.532			
$\delta_s = c_s \lambda_s / b_s$ [μm]	0.9158	0.9175	0.9225	0.8846			
$t = \delta_s / 2$ [μm]	0.4579	0.4587	0.4612	0.4423			
2-nd interference image of the film							
q_s	0	1	2	3	4		
b_s [μm]	111.250	102.625	95.075	88.825	83.850		
λ_s [nm]	633.3	587.8	548.3	515.8	483.8		
initial interference order $m_1 = 12$							
$m_s = m_1 + q_s$	12	13	14	15	16		
$C_s = m_s b_s$ [μm]	1335.000	1334.125	1331.050	1332.375	1329.360		
$c_s = C_s - C$ [μm]	168.568	167.693	164.618	165.943	162.928		
$\delta_s = c_s \lambda_s / b_s$ [μm]	0.9596	0.9605	0.9494	0.9636	0.9517		
$t = \delta_s / 2$ [μm]	0.4798	0.4803	0.4747	0.4818	0.4759		
by averaging, $t = 0.4681$ μm							

[1]. However, the previous plot was obtained via graphs $b(m)$ shown in Fig. 13 of the paper [1]; no graphs $b(m)$ or $\lambda(m)$ are now required.

Another example is represented by Table 2, where the results of the measurement of the thickness t of a photoresist film are listed. Directly measured and observed parameters (b_s, q_s, m_1, m_1+q_s) are rewritten from Table 2 of Ref. [1]. These are now supplemented by new data derived according to the procedure

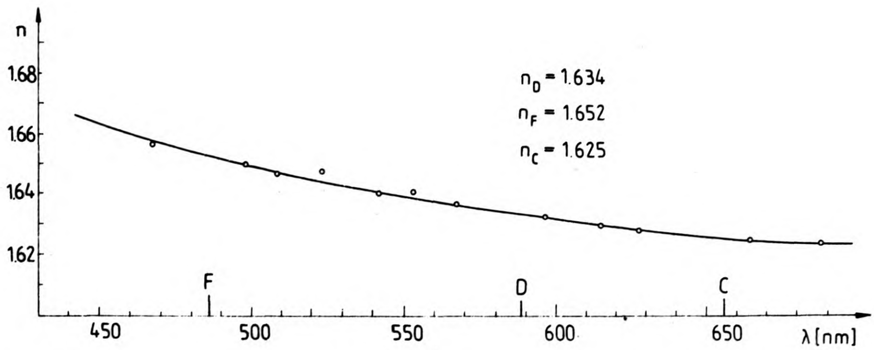

Fig. 3. Graphical representation of the results of measurement obtained for the Holotest 10E56 (n – refractive index of the Holotest emulsion, λ – light wavelength)

described in this article. The thickness obtained here is equal to 0.4681 μm and it differs from the previous result by only 0.0003 μm. This very small discrepancy is unimportant and both results are consistent with each other.

6. Conclusion

The VAWI–2 procedure presented previously in [1] and that described in this paper are equivalent to each other. In its stage of calculation processing, the latter is however less time-consuming and more suitable for computer programming since no graphs $b(m)$ or $\lambda(m)$ are required for determining the final interferometric results such as the optical path difference δ produced by an object under study, the object thickness, and the spectral dispersion of the refractive index. On the other hand, the former procedure and the graphs $b(m)$ permit us to correct some possible inaccuracies due to not always ideal adjustments of interference fringes at positions consecutively coincident with the pointer line L_2 (Fig. 2). Moreover, the graphs $b(m)$ enable the optical path difference δ to be calculated for an arbitrary wavelength λ (see Eq. (21) and Fig. 4 in [1]), while the procedure proposed in this paper permits us to calculate δ for wavelengths λ_s for which the interfringe spacings b_s were measured.

In conclusion, we can state that the two procedures are complementary to each other.

References

[1] PLUTA M., Opt. Appl. **16** (1986), 301–323.
[2] PLUTA M., Opt. Appl. **17** (1987), 47–63.
[3] PLUTA M., J. Microsc. **146** (1987).

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Интерферометрия с плавно-переменной длиной волны VI. Некоторые полезные модификации метода VAWI-2

Описанный раньше подход к полосатой интерферометрии с плавно-переменной длиной волны, т.е. метод VAWI–2, модифицирован теперь на этапе обработки измерительных данных. Модификация позволяет исключить некоторые трудоёмкие операции. Лишними оказываются графики $b(m)$ (b — период интерференционных полос, m — интерференционный порядок).