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Calculation of the reflection coefficient with a system non-absorbing layers

MARCELA TUTUNARU, L. TĂTAR

Craiova University, Bucharest, Romania.

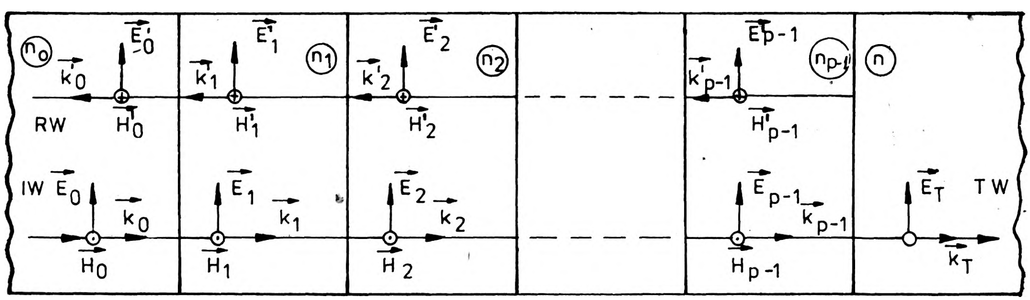
I. N. MIHĂILESCU

Central Institute of Physics, Bucharest, Romania.

An expression is derived for the reflectivity of the electromagnetic radiation on a multilayer system with the help of the transfer matrix.

The multilayer films obtained with very thin transparent coatings deposited on different supports are often used in laser optics interference filters, antihals layers, etc. The adequate selection of both the nature and the thickness of these layers may result in the formation of system such that the fraction of reflected light in a certain wavelength range is minimal.

The reflectivity of such a multilayer transparent system gives direct information about the nature and the geometry of multilayers. Further, based on the so-called transfer matrix [1–3], a calculation of the reflectivity of the electromagnetic radiation on such a multilayer system is presented.



Propagation of the electromagnetic radiation through and out of the multilayer system

Let us consider the case of a normal incidence of the electromagnetic radiation on a multilayer (see the Figure) consisting of $(p-1)$ layers of $h_1, h_2, \ldots, h_{p-1}$ thickness with the corresponding refraction indexes of $n_1, n_2, \ldots, n_{p-1}$. The support medium is supposed to have a refraction index n, the propagation medium of the incident TM wave being the air with refraction index n_0.

The limit conditions with the components of electric and magnetic fields can be written separately at each border [4] (for notations see Fig.) as:

$$\begin{cases} \left.\begin{aligned} &E_0 + E_0' = E_1 + E_1' \\ &n_0(E_0 - E_0') = n_1(E_1 - E_1') \end{aligned}\right\} \text{border } (0,1) \\ \left.\begin{aligned} &E_1 e^{ik_1h_1} + E_1' e^{-ik_1h_1} = E_2 + E_2' \\ &n_1(E_1 e^{ik_1h_1} + E_1' e^{-ik_1h_1}) = n_2(E_2 - E_2') \end{aligned}\right\} \text{border } (1,2), \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \left.\begin{aligned} &E_{p-1} e^{ik_{p-1}h_{p-1}} + E_{p-1}' e^{-ik_{p-1}h_{p-1}} = E_T \\ &n_{p-1}(E_{p-1} e^{ik_{p-1}h_{p-1}} - E_{p-1}' e^{-ik_{p-1}h_{p-1}}) = nE_T \end{aligned}\right\} \text{border } (p-1,p). \end{cases} \tag{1}$$

Let us introduce the following notations:

$$\begin{aligned} \vec{A}_0 &= \begin{pmatrix} E_0 + E_0' \\ n_0(E_0 - E_0') \end{pmatrix}, \\ \vec{A}_1 &= \begin{pmatrix} E_1 + E_1' \\ n_1(E_1 - E_1') \end{pmatrix}, \\ &\cdots\cdots\cdots\cdots \\ \vec{A}_{p-1} &= \begin{pmatrix} E_{p-1} + E_{p-1}' \\ n_{p-1}(E_{p-1} - E_{p-1}') \end{pmatrix}, \\ \vec{A}_p &= \begin{pmatrix} E_T \\ nE_T \end{pmatrix}. \end{aligned} \tag{2}$$

Let us write the transfer matrices $\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_{p-1}$ of the first, second and $(p-1)$ layers

$$\begin{aligned} \hat{M}_1 &= \begin{pmatrix} \cos k_1 h_1 & -\frac{i}{n_1} \sin k_1 h_1 \\ -in_1 \sin k_1 h_1 & \cos k_1 h_1 \end{pmatrix}, \\ \hat{M}_2 &= \begin{pmatrix} \cos k_2 h_2 & -\frac{i}{n_2} \sin k_2 h_2 \\ -in_2 \sin k_2 h_2 & \cos k_2 h_2 \end{pmatrix}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \hat{M}_{p-1} &= \begin{pmatrix} \cos k_{p-1} h_{p-1} & -\frac{i}{n_{p-1}} \sin k_{p-1} h_{p-1} \\ -in_{p-1} \sin k_{p-1} h_{p-1} & \cos k_{p-1} h_{p-1} \end{pmatrix}. \end{aligned} \tag{3}$$

Applying Euler's relations $e^{\pm ia} = \cos a \pm i \sin a$ we have

$$\begin{aligned} &(E_1 + E_1')\cos k_1 h_1 + i(E_1 - E_1')\sin k_1 h_1 = E_2 + E_2', \\ &n_1(E_1 - E_1')\cos k_1 h_1 + in_1(E_1 + E_1')\sin k_1 h_1 = n_2(E_2 - E_2'). \end{aligned} \tag{4}$$

Solving the system (4) against the components of $\vec{A}_1$ vector we obtain

$$\begin{cases} E_1+E_1' = \begin{vmatrix} E_2+E_2' & \frac{i}{n_1}\sin k_1 h_1 \\ n_2(E_2-E_2') & \cos k_1 h_1 \end{vmatrix} = (\hat{M}_1\vec{A}_2)_1, \\ n_1(E_1-E_1') = \begin{vmatrix} \cos k_1 h_1 & E_2+E_2' \\ in_1 \sin k_1 h_1 & n_2(E_2-E_2') \end{vmatrix} = (\hat{M}_1\vec{A}_2)_2. \end{cases} \tag{5}$$

By $(\hat{M}_1\vec{A}_2)_1$, and $(\hat{M}_1\vec{A}_2)_2$ we denote the components of the vector $(\hat{M}_1\vec{A}_2)$. Accordingly we can write

$$\vec{A}_1 = \hat{M}_1\vec{A}_2. \tag{6}$$

Similarly for the other vectors we get

$$\vec{A}_2 = \hat{M}_2\vec{A}_3, \ldots, \vec{A}_{p-1} = \hat{M}_{p-1}\vec{A}_p. \tag{7}$$

Combining these equations we have

$$\vec{A}_1 = \hat{M}_1\hat{M}_2, \ldots, \hat{M}_{p-1}\vec{A}_p = \hat{M}\begin{pmatrix} E_T \\ nE_T \end{pmatrix}. \tag{8}$$

The vector $\vec{A}_1$ can be written in a simple manner as

$$\vec{A}_1 = \begin{pmatrix} E_0+E_0' \\ n_0(E_0-E_0') \end{pmatrix} = \begin{pmatrix} 1 \\ n_0 \end{pmatrix} E_0 + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix} E_0'. \tag{9}$$

We assume the notations:

$$\frac{E_0'}{E_0} = R, \quad \frac{E_T}{E_0} = T. \tag{10}$$

The reflectivity r and the transmission t of the multilayer system can be further derived as

$$r = |R|^2, \quad t = |T|^2. \tag{11}$$

Using the notation (10) and substituting (9) in (8) we have

$$\begin{pmatrix} 1 \\ n_0 \end{pmatrix} + \begin{pmatrix} 1 \\ -n_0 \end{pmatrix} R = \hat{M}\begin{pmatrix} 1 \\ n \end{pmatrix} T. \tag{12}$$

Multiplying Eq. (12) with the inverse matrix $\hat{M}^{-1}$ we get

$$\hat{M}^{-1}\begin{pmatrix} 1 \\ n_0 \end{pmatrix} + \hat{M}^{-1}\begin{pmatrix} 1 \\ -n_0 \end{pmatrix} R = \begin{pmatrix} 1 \\ n \end{pmatrix} T. \tag{13}$$

By further multiplying Eq. (13) with the line matrix $(n_1 - 1)$ we obtain

$$(n_1-1)\hat{M}^{-1}\begin{pmatrix}1\\ n_0\end{pmatrix}+(n_1-1)\hat{M}^{-1}\begin{pmatrix}1\\ -n_0\end{pmatrix}R=0, \tag{14}$$

where from we get

$$R=-\frac{(n_1-1)\,\hat{M}^{-1}\begin{pmatrix}1\\ n_0\end{pmatrix}}{(n_1-1)\hat{M}^{-1}\begin{pmatrix}1\\ -n_0\end{pmatrix}}. \tag{15}$$

The inverse matrix of the first layer is equal to

$$\hat{M}_1^{-1}=\begin{pmatrix}\cos k_1h_1 & \frac{i}{n_1}\sin k_1h_1\\ in_1\sin k_1h_1 & \cos k_1h_1\end{pmatrix}, \tag{16}$$

while its complex conjugate is

$$\hat{M}^{-1*}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\hat{M}_1^{-1}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{17}$$

With the aid of the Pauli matrix

$$\sigma_3=\begin{pmatrix}1 & 0\\ 1 & -1\end{pmatrix} \tag{18}$$

the Eq. (18) can be written as

$$\hat{M}_1^{-1*}=\sigma_3\,\hat{M}_1^{-1}\sigma_3. \tag{19}$$

Then the inverse transfer matrix and its conjugate are given by

$$\begin{aligned}\hat{M}^{-1}&=\hat{M}_{p-1}^{-1}\cdots\cdots\ \hat{M}_1^{-1},\\ \hat{M}^{-1*}&=\sigma_3\hat{M}_1^{-1}\sigma_3.\end{aligned} \tag{20}$$

Finally, the complex conjugate reflection coefficient is

$$R^*=\frac{(n_1-1)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\hat{M}^{-1}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ n_0\end{pmatrix}}{(n_1-1)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\hat{M}^{-1}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ -n_0\end{pmatrix}}. \tag{21}$$

The reflectivity of the incident radiation on the multilayer system can be then obtained by multiplying the expressions of R and R^* given by Eqs. (15), (21).

References

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Расчет коэффициента отражения в системе непоглощающих слоев

Выведено выражение для расчета коэффициента отражения электромагнитного излучения для многослойной системы при помощи матрицы перехода.