Optica Applicata, Vol. XVI, No. 1, 1986

Application of weighted moments to image coding, decoding and processing. Part II. Blurred image recovery by the operations on moment's representation

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This paper deals with one of the possible applications of intensity distribution moments to image processing. The relation between the moments of convolution and those of convolved functions enables the reduction of integral convolution equation to the set of algebraic equations. These equations can be inverted in a very simple manner, allowing the calculation of the moments of convolved function, provided that the moments of convolution and another convolved function are known. The example shows the possibility of partial deblurring of an image recorded out of the focus plane.

1. Introduction

The possibility of 2-D image reconstruction from its intensity moments was discussed in paper [1]. To this end such a representation was orthogonalized and the image reconstructed by the approximation with orthogonal polynomials series. The weighted moments of distribution $f(x, y)$ were defined as

$$M_w^{pq} = \iint\limits_{-\infty}^{\infty} w(x, y) f(x, y) x^p y^q dx dy \tag{1}$$

where: M_w^{pq} — weighted moment of order $p+q$,
$w(x, y)$ — weight function,
and the integral (1) must be convergent.

The set of moments may be subject to various operations affecting the reconstructed image. This fact becomes obvious while taking account of the relation between the moments and Fourier transform (the moment's theorem)

$$M_w^{pq} = \frac{1}{(2\pi i)^{p+q}} \frac{\partial^{p+q} F(u, v)}{\partial u^p \partial v^q}\bigg|_{u=v=0} \tag{2}$$

where $F(u, v)$ is the Fourier transform of $f(x, y)$.

In particular

$$M^{00} = F(u, v)|_{u=v=0}, \tag{3}$$

and setting M^{00} to zero is equivalent to the "high-pass" filtering, or the thresholding of constant component of an image. The values of M^{01} and M^{10}:

$$\begin{aligned} M^{01} &= \iint_{-\infty}^{\infty} f(x, y) y dy, \\ M^{10} &= \iint_{-\infty}^{\infty} f(x, y) x dx \end{aligned} \tag{4}$$

define the position of "image centroid" and their modification enables simple image displacements. By applying higher order moments the image re-orientation is possible.

Due to the above mentioned properties the moments were applied to the construction of image invariants [2, 3].

In this paper we shall present the possibility of a partial recovery of an image degraded by the convolution with quasi-stationary point spread function.

2. Convolution moments

It will be assumed now that the functions $f(x, y)$ and $g(x, y)$ have the moments with weighting function equal to unity, these moments will be referred to as M_f^{pq} and M_g^{pq}, respectively. The convolution of these functions is another function

$$h(x, y) = \iint_{-\infty}^{+\infty} f(x', y') g(x-x', y-y') dx' dy'. \tag{5}$$

If the moments M_f^{pq} and M_g^{pq} exist, then the moments M_h^{pq} of $h(x, y)$ also exist and can be easily calculated. Substituting $h(x, y)$ from Eq. (5) to the definition (1), after some very simple transformations, yields [4]

$$M_h^{pq} = \sum_{i=0}^{p} \sum_{j=0}^{q} \binom{p}{i} \binom{q}{j} M_f^{p-i\, q-j} M_g^{ij}. \tag{6}$$

Thus, the moments of $h(x, y)$ are the combinations of the moments M_f^{pq} and M_g^{pq} of the same and lower orders.

If the distributions are represented by the matrices $\hat{M}_f^{PQ}$, $\hat{M}_g^{PQ}$, $\hat{M}_h^{PQ}$, $p = 0, \ldots, P$, $q = 0, \ldots, Q$, then the matrix $\hat{M}_h^{PQ}$ may be expressed as a specific "product" (6) of matrices $\hat{M}_f^{PQ}$ and $\hat{M}_g^{PQ}$.

CASASENT et al. [4] proposed the application of Eq. (6) to the "correction" of the moments of an image degraded by convolution. In this paper another application of Eq. (6) will be proposed.

3. Inversion of Equation (6)

The set of Equations (6) for various p and q may be inverted, thus allowing the calculation of the moments of convolved function, provided that the moments of convolution and another convolved function are known.

It will be assumed now, that all M_f^{pq} and M_h^{pq} are known and M_g^{pq} are to be found. Three cases of Eq. (6) will be analysed:

i) If $p+q = 0+0$, then

$$M_g^{00} = M_h^{00}/M_f^{00} \tag{7}$$

(if $f(\ldots)$ denotes the intensity distribution, then $M_f^{00} = 0$, for $f(x,y) = 0$, only).

ii) If $p \neq 0, q = 0$ (first column of the matrix M_h^{pq}), then

$$M_h^{p0} = \sum_{i=0}^{p} \binom{p}{i} M_f^{p-i\,0} M_g^{i\,0}. \tag{8}$$

The separation from the above series of the last component which includes M_g^{p0} yields

$$M_g^{p0} = \left[M_h^{p0} - \sum_{i=0}^{p-1} \binom{p}{i} M_f^{p-i\,0} M_g^{i\,0}\right] / M_f^{00}, \tag{9}$$

which enables the calculation of the subsequent moments $M_g^{p0}, p = 1, \ldots, P$.

iii) $p \neq 0, q \neq 0$.

As in the case ii), the last component of series (6) with the term M_g^{pq} can be separated to obtain

$$M_g^{pq} = \left[M_h^{pq} = \sum_{i=0}^{p} \sum_{j=0}^{G} \binom{p}{i}\binom{q}{j} M_f^{p-i\,q-j} M_g^{ij}\right] / M_f^{00}, \tag{10}$$

which is also a set of recurrence equations. The limit of summation equals

$$G = \begin{cases} q & \text{for } i \neq p \\ q-1 & \text{for } i = p. \end{cases} \tag{11}$$

Thus, a set of Equations (6) can be inverted and the moments of convolved function obtained. This function can be then reconstructed with Legendre polynomials series by means of the procedure described in paper [1].

In recurrence Equations (9) and (10) in order to calculate the subsequent moments of $g(x, y)$ its moments of lower orders are applied, which were calculated previously, using the same equation. This creates the risk of avalanche-like increase of calculation errors and their cummulation. However, it should be pointed out that for the reasons related to the reconstruction procedure, the area over which the distributions are defined is always limited to $|x| < 1$, $|y| < 1$ (this can always be realized by a simple rescaling of coordinates). The values of distributions are also limited to unity. This accounts for the fact that the values of moments decrease to zero, when their orders increase. This assures the stability of the above procedure.

4. Experimental verification of the method

In order to testify the described method of deconvolution, some experimental convolutions were realized, which included the reconstruction of the expected "ideal" geometrical image if the real image was obtained in an optical setup (Fig. 1) outside the focus plane. In the case of incoherent illumination the mechanism of imaging can by described by an integral

$$I'(x_i', y_i') = \iint_{-\infty}^{\infty} s(x_0, y_0, x_i', y_i') I_0(x_0, y_0) dx_0 dy_0 \tag{12}$$

where: $I'(x_i', y_i')$ — blurred image (outside the focus),
$I_0(x_0, y_0)$ — "ideal" geometrical image (to be reconstructed),
$s(x_0, y_0, x_0', y_0')$ — incoherent point spread function.

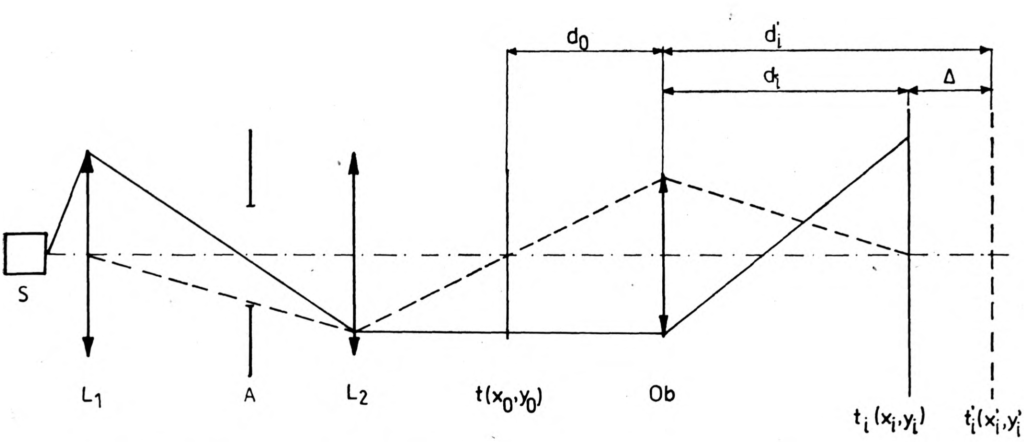

Fig. 1. Optical setup used for the blurred image detection. S — incoherent source, A — aperture, L_1, L_2 — collective and condenser lenses, t — transparency to be imaged, Ob — objective, d_0 — object-to-objective distance, d_i — image plane distance, d_i' — detection plane distance, Δ — focus plane-to-image plane distance

In the case of a relatively small misfocus ($\varepsilon = 1/d_0 + 1/d_i - 1/f \approx 0$, but $\varepsilon \neq 0$), the incoherent point spread function is stationary: $s(x_0, y_0, x'_i, y'_i) = s(x_0 - x'_i, y_0 - y'_i)$. This results from the fact that

$$s(x_0, y_0, x'_i, y'_i) = h(x_0, y_0, x'_i, y'_i) h^*(x_0, y_0, x'_i, y'_i), \tag{13}$$

and that the coherent point spread $h(...)$ function is stationary (provided that vignetting and aberrations are neglected)

$$h(x_0, x'_i, y_0, y'_0) = h(x_0 - x'_i, y_0 - y'_i) = K \int\int_{-\infty}^{\infty} P(\lambda d'_i \tilde{x}_1, \lambda_i d' \tilde{y}_i)$$

$$\times \exp\left\{\frac{ik\varepsilon}{2} \lambda^2 d_i'^2 (\tilde{x}_i^2 + \tilde{y}_l^2)\right\} \exp\{-2\pi i [\tilde{x}_l (x'_i - \tilde{x}_0) + (y'_i - \tilde{y}_0)\tilde{y}_l]\} d\tilde{x}_l d\tilde{y}_l \tag{14}$$

where: $\tilde{x}_l = x_l/\lambda d'_i$, $\tilde{y}_l = y_l/\lambda d'_i$, $\tilde{x}_0 = -(d'_i/d_0)x_0$, $\tilde{y}_0 = -(d'_i/d_0)y_0$, and $P(...)$ is the pupil function of imaging lens.

The stationarity of the spread function assures the convolution-type process of blurring, and the presented method of deconvolving may be applied. This was realized in the optical setup illustrated in Fig. 1. The deconvolution and reconstruction were calculated by means of a minicomputer.

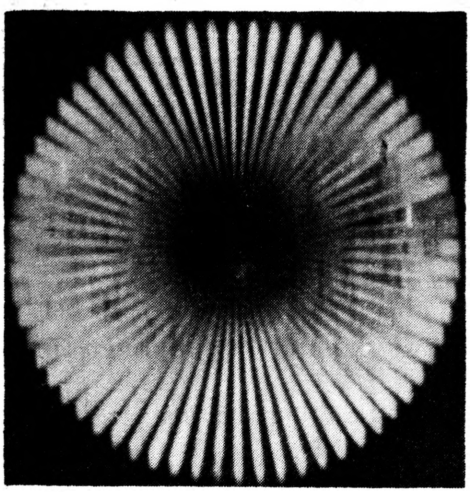

Fig. 2. Blurred spoke target image

The exemplary results for $d_0 = 300$ mm, $d'_i = 600$ mm, $f = 180$ mm, $\varepsilon = -1/1800$ mm^{-1} are presented below. Figure 2 shows the spoke target image. The non-cylindrical shape of optical transfer function results from the "prolate" shape of the light source and the fact that Köhler's conditions in the disfocused setup were neglected. While calculating the moments of point spread function the influence of its shape was automatically taken into account.

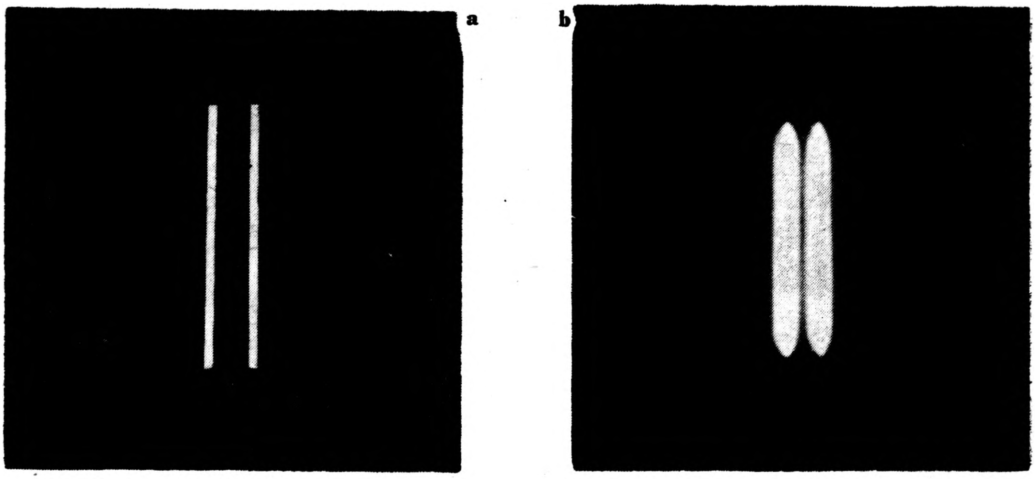

Fig. 3. Two-slit aperture in position I (**a**), and its blurred image (**b**)

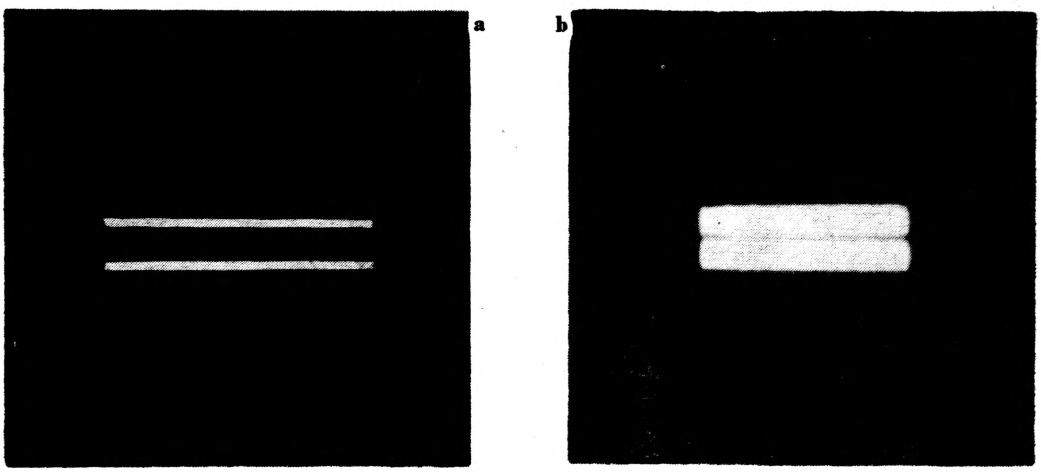

Fig. 4. Two-slit aperture in position II (**a**), and its blurred image (**b**)

The example of two-slit aperture with two orientations is presented in Figs. 3 and 4.

After the evaluation of the moments of convolution (blurred image $I'(x_i', y_i')$) and point spread function $s(\ldots)$ (the image of a pinhole, $\varnothing \approx 10$ microns), the moments of the "ideal" $I_0(x_0, y_0)$ were calculated. Next, from its moments $I_0(x_0, y_0)$ was reconstructed. The cross-sections of this reconstructions (perpendicular to the direction of the slits, in the middle of their length) are shown in Fig. 5. In these reconstructions various orders $P+Q$ of the representations were applied. In the first case (I) the improvement is relatively small, but the blurred image quality was quite good. In the case II the improvement is quite substantial.

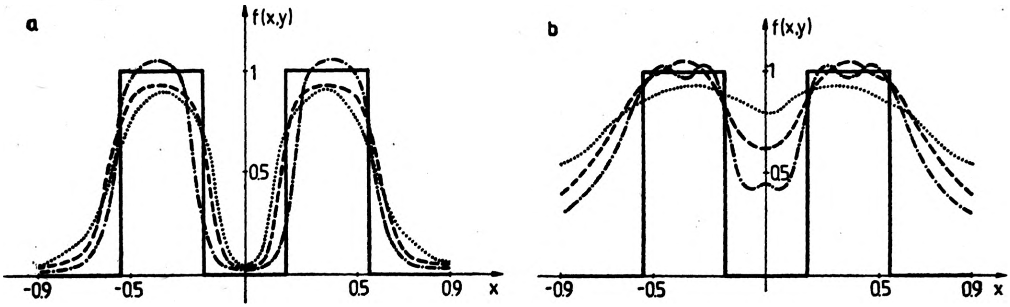

Fig. 5. Cross-section of reconstructed images: I (**a**) and II (**b**). The reconstructions with various orders of representations $P+Q$ (——— image in the focus plane, ... blurred image, – – – $P+Q = 8+8$, –.–.–. $P+Q = 12+12$)

In order to evaluate quantitatively the possible improvement, the contrast K of reconstructed slits was calculated

$$K = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}. \tag{15}$$

For the case of ideal imaging $K = 1$ and in the case of blurred imaging we have:
case I $- K = 0.89$,
case II $- K = 0.10$.

Values of contrast K for various reconstructions of the two-slit aperture

	Ideal image	Blurred image	Reconstruction order (12 + 12)	Reconstruction order (8 + 8)
Case I	1.0	0.89	0.93	0.92
Case II	1.0	0.10	0.38	0.20

The Table shows the improvement of contrast K for various orders of reconstruction. This improvement is particularly obvious in the case II — nearly $4\times$ for the reconstruction of order of $12+12$.

These experiments carried out for a number of various distributions gave similar results, thus proving the importance of nonorthogonal representations in some operations in the image.

References

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Received June 25, 1985

Применение взвешиваемых моментов для кодирования, декодирования и преобразования изображения. Ч. II. Реконструкция размытого изображения при помощи преобразования представления моментов

Работа касается возможности применения моментов распределения напряжения для преобразования изображения. Соотношение между моментами сплетения и моментами сплетаемых функций позволяет привести интегральное уравнение сплетения к форме системы алгебраических уравнений. Следовательно, эти уравнения возможно обратить очень просто, что позволяет рассчитать моменты одной из ,,сплетенных'' функций, при условии, что моменты сплетения и второй функции известны. Пример показывает возможность частичного удаления размыва изображения, зарегистрированного вне фокальной плоскости.