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Design and raytrace evaluation of polygonal trough solar concentrators

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The design and geometrical-optical performance characteristics of a polygonal trough with a flat absorber are presented. The distribution of local concentration ratio over a flat absorber at the focal plane of the polygonal trough has been determined using both analytical and conventional ray trace techniques. The results have been plotted graphically and discussed.

1. Introduction

Considerable interest has recently been shown in the development of linear solar concentrators providing intermediate concentrations [1, 2]. Such concentrators may be used for solar thermal as well as photovoltaic applications. In the present paper the design and geometrical-optical performance characteristics of two typical polygonal trough designs with flat absorbers have been presented. The first design consists of mirror elements of equal sizes placed at some specified angles (Fig. 1) whereas the second design uses mirror elements of varying sizes in order to produce a comparitively more uniform illumination (Fig. 2). For both the cases, the generalized formulae have been derived for the design parameters associated with each mirror element. The distribution of local concentration ratio (LCR) over a flat absorber placed at the focal plane of the first polygonal trough design has been investigated using an analytical method, whereas for the second design both analytical as well as a conventional ray trace techniques have been used for this purpose.

2. Analysis

For the purpose of the work presented herein, it is assumed that the polygonal trough concentrator is perfectly tracked, the mirror elements are specularly reflecting, solar radiation is incident axially and the intensity of solar disc is uniform.

2.1. First design

Figure 1 shows the cross-sectional geometry of the first design of the polygonal trough concentrator analysed in this paper. The polygonal trough consists of K mirror elements of the width B on each half of the polygonal trough. The flat plate absorber size has been chosen to be equal to the size of the constituent mirror element (i.e., B), and it is placed at a distance F from the central mirror element, so that the normally incident ray at the lower edge of each constituent mirror element is reflected back to one edge of the flat absorber, whereas the ray incident at the upper edge interacts the plane of the flat absorber as per geometrical considerations.

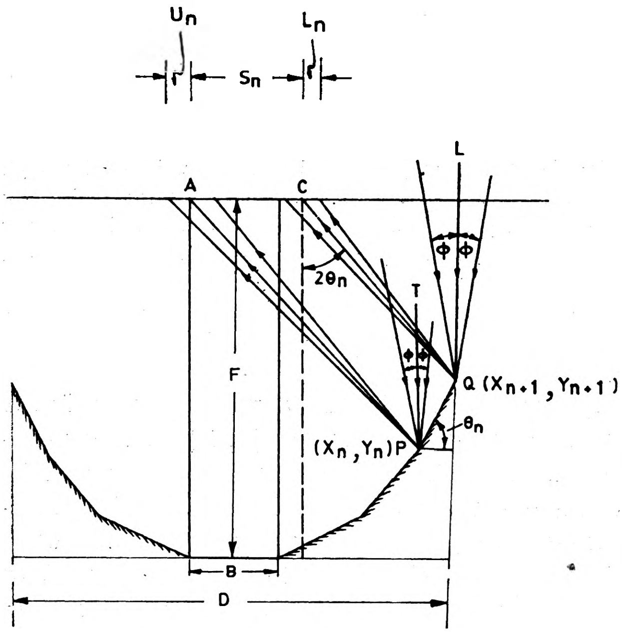

Fig. 1. Cross-sectional geometry of a polygonal trough (design I)

The coordinates (X_1, Y_1) of lower edge of the first mirror element are given by

$$X_1 = X_0 + B\cos\theta_0,$$

$$Y_1 = Y_0 + B\sin\theta_0$$

where $X_0 = -B/2$ and $Y_0 = 0$.

Similarly, the tilt of the first mirror element with the horizontal may be calculated from

$$\theta_1 = \frac{1}{2}\tan^{-1}\left(\frac{B/2+Y_1}{F-Y_1}\right).$$

Obviously, for the central mirror element (if placed) $\theta_0 = 0$.

Using simple geometry, the generalized expressions for the coordinates (X_n, Y_n), and tilt (θ_n) of the n-th mirror element are given by:

$$X_n = X_{n-1} + B\cos\theta_{n-1}, \tag{1}$$

$$Y_n = Y_{n-1} + B\sin\theta_{n-1}, \tag{2}$$

and

$$\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{B/2+X_n}{F-Y_n}\right), \tag{3}$$

respectively.

Figure 1 shows the geometry necessary for calculation of the distribution of LCR over the plane of the flat absorber. From the geometry of the concentrator it is clear that the width of the intercept produced by each constituent mirror element over the plane of the flat absorber will be somewhat larger than the size of the flat absorber (which in this case is equal to the size of the constituent mirror elements). The intercept width (S_n) produced by the rays LQ and TP incident normally on the n-th mirror element may be calculated as

$$S_n = X_n + B/2 + B\cos\theta_n - [F-(Y_n + B\sin\theta_n)]\tan 2\theta_n, \tag{4}$$

with $S_0 = B$.

A further spread in the width of the region of non-zero intensity over the plane of the flat absorber is due to the finite angular subtense of the sun. Thus, if 2Φ is the angle subtended by sun disc at any point on the concentrator, the additional increase in the illumination on its both sides may be given by

$$L_n = \frac{[F-(Y_n + B\sin\theta_n]\sec 2\theta_n \sin\Phi}{\cos(2\theta_n - \Phi)}, \tag{5}$$

and

$$U_n = \frac{(F-Y_n)\sec 2\theta_n \sin\Phi}{\cos(2\theta_n + \Phi)}, \tag{6}$$

respectively.

In order to study the distribution of local concentration ratio over the flat absorber, we use the procedure described by Singh et al. [3]. The contribution of the n-th mirror element to the distribution of LCR over the flat absorber may be calculated as

$$CI_n = \frac{B\cos\theta_n}{L_n + S_n + U_n}. \tag{7}$$

If it is assumed that no mirror element is placed in the central region owing to the shading caused by the absorber, then the total contribution of all the mirror elements to the LCR over the flat absorber may be calculated as

$$C = 2\sum_{n=1}^{K} CI_n. \tag{8}$$

2.2. Second design

As mentioned earlier, the second design analysed in this work consists of mirror elements of varying width (Fig. 2). The size of each constituent mirror element is chosen so that for normally incident rays the intercept produced on the plane of the absorber coincides with the size (B') of the flat absorber.

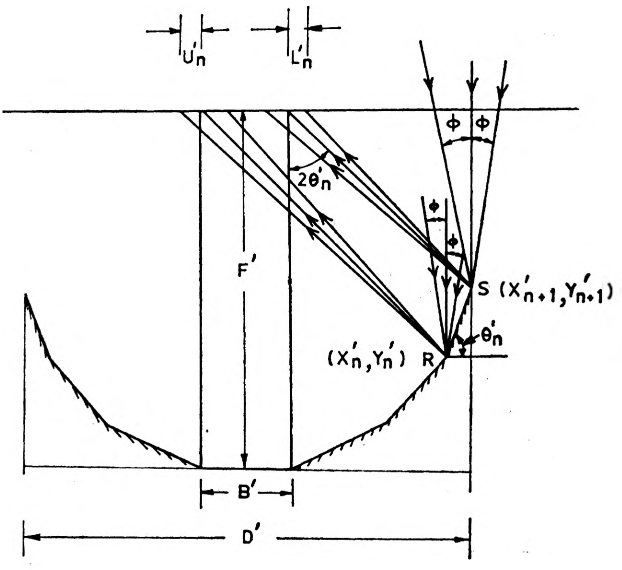

Fig. 2. Cross-sectional geometry of a polygonal trough (design II)

Simple geometrical considerations led to the following expression for the parameters characterizing the position and size of the first mirror element, respectively:

$$X' = X_0' + W_0 \cos \theta_0',$$
$$Y_1' = Y_0' + W_0 \sin \theta_0',$$
$$W_1 = B' \cos 2\theta_1' \sec \theta_1'$$

where: $X_0' = -B'/2$, $Y_0 = 0$, $W_0 = B'$, and to the following generalized expressions for the position $(X_n' Y_n')$, and width (W_n) of the mirror element

$$X_n = X_{n-1}' + W_{n-1} \cos \theta_{n-1}',$$
$$Y_n' = Y_{n-1}' + W_{n-1} \sin \theta_{n-1}'$$

and

$$W_n = B' \cos 2\theta_n' \sec \theta_n.$$

To calculate the tilt of the constituent n-th mirror element (θ_n') with horizontal we may use the expression similar to Eq. (3)

$$\theta_n' = \frac{1}{2} \tan^{-1} \left(\frac{B'/2 + X_n'}{F' - Y_n'} \right) \tag{12}$$

where $\theta_0 = 0$ and F' is the distance of the flat absorber from the central mirror element.

The expressions for spreads L'_n and U'_L (Fig. 2) caused by the finite angular subtense of the sun on both the sides of the specified absorber size may be expressed by the formulae:

$$L'_n = \frac{[F' - (Y'_n + W_n \sin\theta'_n)]\sec 2\theta'_n \sin\Phi}{\cos(2\theta'_n - \Phi)}, \tag{13}$$

$$U'_n = \frac{(F' - Y'_n)\sec 2\theta'_n \sin\Phi}{\cos(2\theta'_n + \Phi)}. \tag{14}$$

In this case, the contribution of the n-th mirror element to the distribution of LCR over the flat absorber may be calculated as

$$CI_n = \frac{B'\cos 2\theta_n}{L'_n + B' + U'_n}. \tag{15}$$

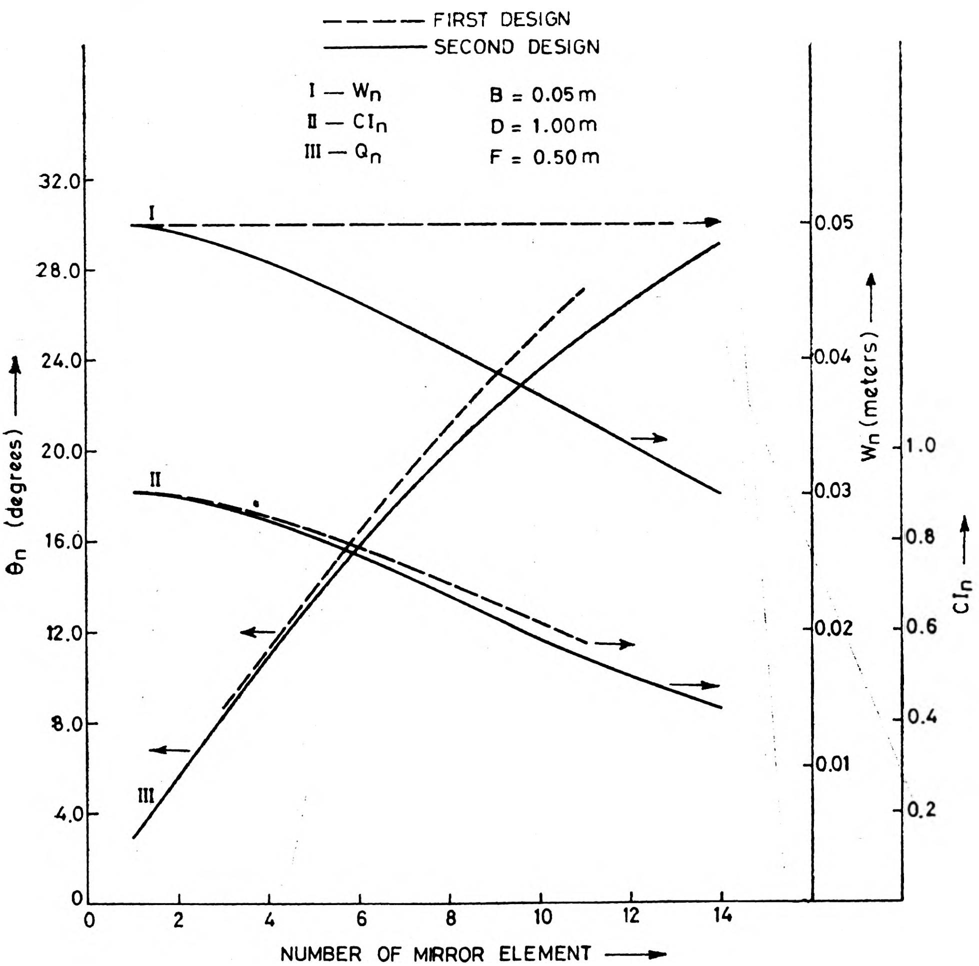

Fig. 3. Variations of tilt, width and contribution (CI_n) with the number of mirror element

Finally, the procedure outlined in paper [3] is used to obtain the distribution of LCR over the absorber plane.

To study the distribution of LCR over the flat absorber of the second design of the polygonal trough, a conventional ray trace technique was also used. Four hundred equidistant points were taken along the aperture of the polygonal trough and at each points seventeen rays were considered at an angular distance of two minutes of arc. For the central ray originating from the sun disc the angular distance is obviously zero. The intersection points of these rays with the flat absorber were calculated by using a simple geometrical optics. Then, by counting the number of rays falling within a small segment of the flat absorber and the number of rays within the segment of the entrance aperture of the equal size, we have obtained the distribution of LCR over the flat absorber.

3. Results and discussion

The results of some typical numerical calculations based on the above analysis are shown in Figs. 3–6. Figure 3 shows the variations of the tilt of the constituent mirror elements of a typical polygonal trough for both the designs. The variation

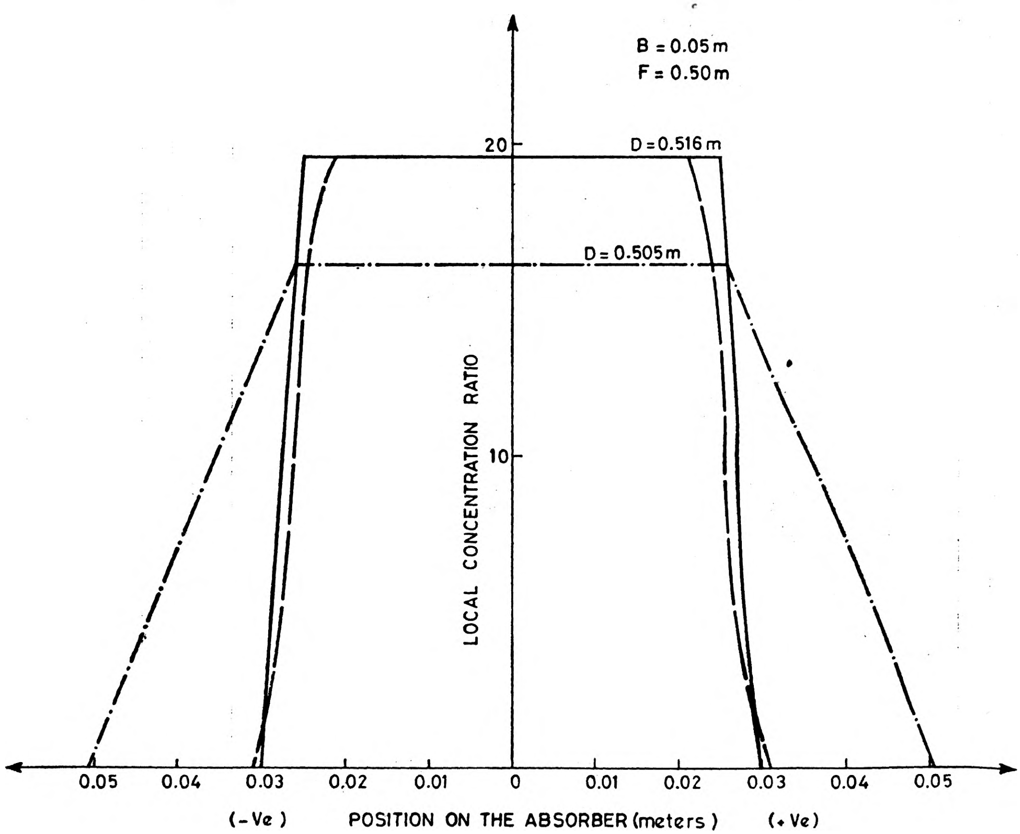

Fig. 4. Distribution of LCR over the plane of a flat absorber (F = 0.50 m)

of the width of each constituent mirror element for the second design with the number of element (as we move towards the edge of the concentrator) is also shown in the same graph. This figure shows, moreover, the variation in the contribution of each mirror element for both the designs. From this graph it may be seen that for both designs, the tilt increases, whereas the contribution over the absorber plane gradually decreases as one moves from the centre towards the edge of the polygonal trough. It should be noted that owing to the finite size of the constituent mirror elements the final aperture diameters of the two designs are different.

From Figure 4, which shows the distribution of LCR, it is clear that the second design, which allows the variation in the constituent mirror sizes, provides a more uniform illumination than that given by the first design consisting of mirror elements of equal sizes. The comparison of Fig. 4, 5 and 6, which represent the LCR distribution for three different values of F, indicates that the relative benefit of the second design is not that prominent at higher values

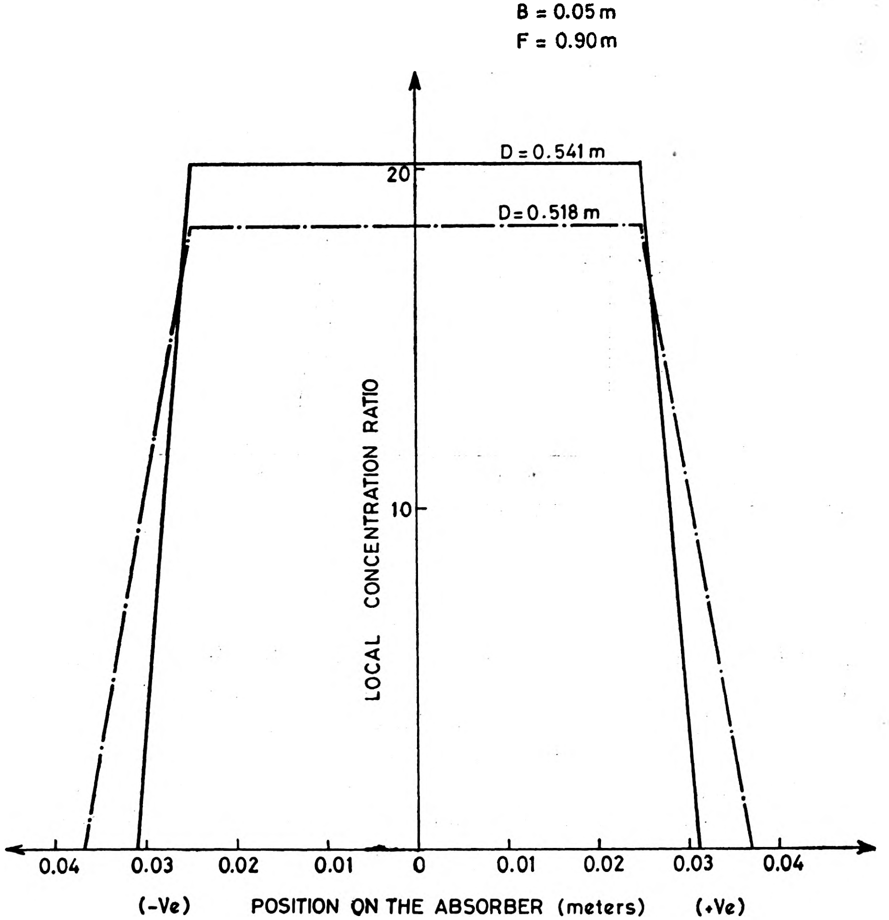

Fig. 5. Same as Fig. 4, but $F = 0.9$ m

of F. However, in all these cases the second design always provides higher concentration than the first design. For example, the central maximum concentration ratio for the first design in these three cases are: 16.3, 17.5 and 16.7 while in the second design the corresponding values are: 19.2, 20.1 and 18.1, respectively. The results of the ray trace analysis made for a typical case of

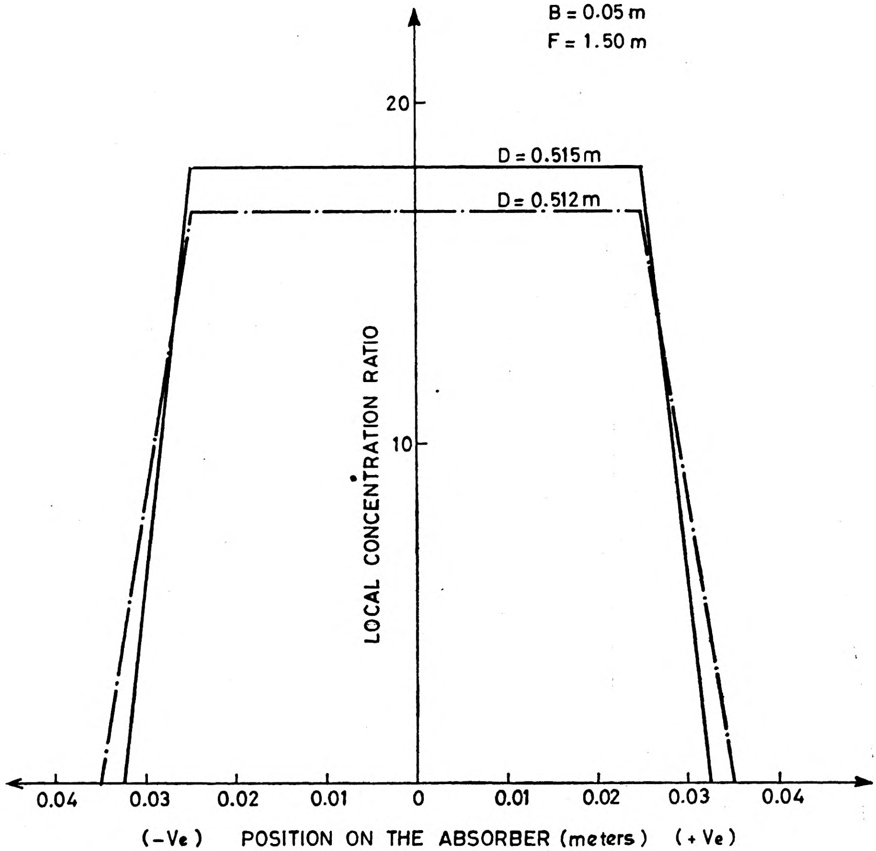

Fig. 6. Same as Fig. 4, but $F = 1.5$ m

the second design are also shown in Fig. 4. The dashed curve represents the distribution of LCR over the flat absorber, determined by this procedure. It may be seen that these results are in a good agreement with those obtained by analytical method within practical limitations.

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Конструкция и перерасчет хода световых лучей многоуглового корытообразного солнечного собирателя (концентратора)

Представлены конструкция и геометрически-оптические характеристики многоугловного корытообразного оптического собирателя (концентратора) с плоским абсорбером. Определено распределение местного коэффициента концентрации на плоском абсорбере, при применении как аналитических, так и конвенциональных техник перерасчета хода лучей. Результаты представлены графически и они обсуждены.