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Single cemented doublets without primary aberrations

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An optical system in air having 3 refracting surfaces is considered. Conditions are given in order to make all five monochromatic primary aberrations zero. The corrections are obtained by using surfaces of revolution of the second order of which one may be spherical and the two others aspehrical. A method for solving this problem and some examples are given.

1. Introduction

Within the last years some methods have been developed to improve the image quality of optical systems by using aspheric surfaces. Such methods make it possible to reduce the aberrations for a given number of surfaces, or to reduce the number of refracting surfaces by keeping the aberrations constant.

An ideal image can be obtained within the range of third order aberrations, if a small area of a plane around optical axis and perpendicular to it is imaged by using rays, the inclination angles of which with respect to the optical axis are small, i.e., when all five coefficients of Seidel's abeṛrations vanish at once. For that problem examples are given in [1], [2], [3] and [4]; the most recent papers are quoted here. SCHULZ [4] has proved that it suffices to calculate a system of 3 refracting (two aspheric and one spherical) surfaces in air to abolish 5 Seidel's aberrations for a real image of any magnification. A method is introduced by means of which those systems can be calculated. Only surfaces of revolution of the second order are used.

2. Seidel's aberrations of rotational-symmetric surfaces of second order

The vertex of the considered rotational-symmetric surfaces coincides with the origin of the system of Cartesian coordinates. We assume that x-axis is rotative optical axis of the system. Then the rotational-symmetric surfaces of second order are described by

$$\hat{\varrho}x^2+\varrho(y^2+z^2)-2x = 0 \tag{1}$$

where ϱ is the vertex curvature and $\hat{\varrho}$ – a parameter of the asphere. With the choice of $\hat{\varrho}$ the following surfaces are described:

- plane: any $\hat{\varrho}$; $\varrho = 0$,
- sphere: any $\hat{\varrho} = \varrho \neq 0$,
- ellipsoid: $\varrho\hat{\varrho} > 0$,
- hyperboloid: $\varrho\hat{\varrho} < 0$,
- paraboloid: $\hat{\varrho} = 0$, any $\varrho \neq 0$.

A general structure of the 5 aberration coefficients of one surface follows from [5] due to specialization

$$C_j^{(i)} = n' \frac{h'}{s'} \left(\frac{h}{s}\right)^{4-j} \left(\frac{\hat{h}}{p}\right)^{j-1} A_j^{(i)} \tag{2}$$

where

$$i = a,\ b,$$

$$j = \mathrm{I}(1),\ \mathrm{II}(2),\ \mathrm{III}(3),\ \mathrm{IV}(4),$$

with: C_{I}, C_{II}, $C_{\mathrm{III}}^{(a)}$, $C_{\mathrm{III}}^{(b)}$, C_{IV} – aberration coefficients of one surface for spherical aberration, coma, astigmatism, field curvature and distortion, respectively; n, n' – refractive indices, $h, \hat{h}$ – heights from the axis of an aperture ray and a field ray in the vertex, respectively, s, p – respective positions of the object plane and the stopplane with respect to the vertex.

The aberration terms written in detail are:

$$\begin{aligned}
A_{\mathrm{I}} &= \frac{s'}{n'} s^3 (K_s^2 R + T),\\
A_{\mathrm{II}} &= \frac{s'}{n'} s^2 p (K_s K_p R + T),\\
A_{\mathrm{III}}^{(a)} - A_{\mathrm{III}}^{(b)} &= \frac{s'}{n'} s p^2 (K_s - K_p)^2 \varrho \left(\frac{1}{n} - \frac{1}{n'}\right),\\
A_{\mathrm{III}}^{(b)} &= \frac{s'}{n'} s p^2 (K_p^2 R + T),\\
A_{\mathrm{IV}} &= \frac{s'}{n'} p^3 \left\{\frac{K_p}{K_s} K_p^2 R + T + \frac{K_p}{K_s} (K_s - K_p)^2 \varrho \left(\frac{1}{n} - \frac{1}{n'}\right)\right\},
\end{aligned} \tag{3}$$

with

$$K_s = n\left(\frac{1}{s} - \varrho\right); \quad K_p = n\left(\frac{1}{p} - \varrho\right) \quad \text{(Abbe's invariants)},$$

$$R = \frac{1}{n's'} - \frac{1}{ns},$$

$$T = (n' - n)\varrho^2(\hat{\varrho} - \varrho) \quad \text{(aspheric term)}.$$

Sum of the respective aberration coefficients for each surface is equal to Seidel's aberration coefficients of a system composed of different surfaces.

3. Conditions of correction for a cemented lens

Using Equations (3), we may separate h_μ, h'_μ and $\hat{h}_\mu$ from the expressions of the sum of the optical system. In order to correct the system the 5 aberration terms resulting from the separation given above should be reduced to zero:

$$\hat{C}_j = \sum_{\mu=1}^{3} (\gamma_\mu^{j-1} c_\mu + a_\mu^{5-j} \delta_\mu^{j-1} T_\mu) + r_j = 0,$$

$$j = \mathrm{I}(1),\ \mathrm{II}(2),\ \mathrm{III}(3),\ \mathrm{IV}(4), \qquad (4a,\ 4b,\ 4c,\ 4d)$$

$$\hat{C}^{(b)}_{\mathrm{III}} = \sum_{\mu=1}^{3} \delta_\mu^2 \left(1 - \frac{a_\mu \gamma_\mu}{\delta_\mu}\right)^2 \bar{K}^2_{s\mu} \varrho_\mu \left(\frac{1}{n_\mu} - \frac{1}{n_{\mu+1}}\right) = 0, \qquad (4e)$$

with:

$$r_{\mathrm{I}} = r_{\mathrm{II}} = r_{\mathrm{III}} = 0,$$

$$r_{\mathrm{IV}} = \sum_{\mu=1}^{3} \delta_\mu^2 \gamma_\mu \left(1 - \frac{a_\mu \gamma_\mu}{\delta_\mu}\right)^2 \bar{K}^2_{s\mu} \varrho_\mu \left(\frac{1}{n_\mu} - \frac{1}{n_{\mu+1}}\right),$$

$$c_\mu = a_\mu \bar{K}^2_{s\mu} \bar{R}_\mu,$$

$$\gamma_\mu = \bar{K}_{p\mu} / \bar{K}_{s\mu},$$

$$a_1 = s'_1/s_2,\ a_2 = 1,\ a_3 = s_3/s'_2,$$

$$\delta_1 = p'_1/p_2,\ \delta_2 = 1,\ \delta_3 = p_3/p'_2,$$

$$\bar{K}_{s\mu} = a_\mu K_{s\mu},\ \bar{K}_{p\mu} = \delta_\mu K_{p\mu},\ \bar{R}_\mu = a_\mu R_\mu.$$

The 5 Equations (4) guarantee an image free of Seidel's aberrations.

An optical system in air, containing 3 aspheric surfaces is determined uniquely by 12 quantities: 3 vertex curvatures ϱ_μ, 3 parameters of the aspheres $\hat{\varrho}_\mu$, 2 lens thicknesses e_1, e_2, 2 refractive indices n_2, n_3 and the positions of the object s_1 and the stop p_1.

As all Seidel's aberrations should be corrected at once, the position of the stop cannot be used for correction [6]. For technological reasons as the second surface an aspherical one is chosen. Now there remain 10 parameters. The following ones are given (see [4]): 2 refractive indices n_2 and n_3, thickness of the first lens e_1 and – for reasons of symmetry – s_1 and s'_3 (instead of s_1 and β'), the thickness of the second lens e_2 (with restriction $e_2 > 0$); the parameters T_1 and T_3 of the aspheres and 2 terms X and Y are left for correction. X and Y are introduced to simplify the description. Finally, we obtain a set of conditions of correction, depending of 5 mentioned above terms:

$$\sum_{\mu=1}^{3}(\varepsilon_\mu^{j-1}c_\mu + a_\mu^{5-j}\eta_\mu^{j-1}T_\mu) + r_j^* = 0, \quad (j = 1, 2, 3, 4), \qquad \text{(5a, 5b, 5c, 5d)}$$

$$\sum_{\mu=1}^{3} r_{\mathrm{IV}\mu}^* = 0, \qquad \text{(5e)}$$

with

$$r_1^* = r_2^* = r_3^* = 0, \; r_4^* = \sum_{\mu=1}^{3}\eta_\mu r_{\mathrm{IV}\mu}^*,$$

$$r_{\mathrm{IV}\mu}^* = \varrho_\mu\left(\frac{1}{n_\mu} - \frac{1}{n_{\mu+1}}\right),$$

$$\eta_1 = \frac{e_1}{n_2}, \; \eta_2 = 0, \; \eta_3 = -\frac{e_2}{n_3},$$

$$a_1 = 1 + \eta_1(Y - X), \; a_2 = 1, \; a_3 = 1 + \eta_3 Y,$$

$$\varepsilon_\mu = \frac{1 + \eta_\mu \bar{K}_{s\mu}}{a_\mu \bar{K}_{s\mu}},$$

$$\bar{K}_{s1} = \frac{1}{n_2 - 1}\left[\frac{n_2}{s_1} - (Y - X)\left(1 - \frac{e_1}{s_1}\right)\right], \quad \bar{K}_{s2} = Y - \frac{n_2}{n_3 - n_2}X,$$

$$\bar{K}_{s3} = \frac{1}{n_3 - 1}\left[\frac{n_3}{s_3'} + Y\left(1 + \frac{e_2}{s_3'}\right)\right],$$

$$\bar{R}_1 = \frac{1}{n_2}(Y - X)\left(\frac{1}{n_2} - \frac{e_1}{s_1}\right) - \frac{1}{s_1}, \quad \bar{R}_2 = \frac{1}{n_3^2}Y - \frac{1}{n_2^2}(Y - X),$$

$$\bar{R}_3 = \frac{1}{n_3}Y\left(\frac{1}{n_3} + \frac{e_2}{s_3'}\right) - \frac{1}{s_3'},$$

$$\varrho_1 = \frac{1}{n_2 - 1}\left(\frac{Y - X}{a_1} - \frac{1}{s_1}\right), \quad \varrho_2 = \frac{1}{n_3 - n_2}X, \quad \varrho_3 = \frac{1}{n_3 - 1}\left(\frac{Y}{a_3} - \frac{1}{s_3'}\right).$$

4. Solution of the system of equations

First, one determines e_2 by solving the system of Eqs. (5) so that Petzval's sum (5e) becomes zero

$$e_2 = n_3\left(\frac{1}{Y} - \frac{1}{Q + 1/s_3'}\right) \qquad (6)$$

where

$$Q = n_3(r^*_{\mathrm{IV}1} + r^*_{\mathrm{IV}2}).$$

The parameters T_1 and T_3 of the asphere are determined so that astigmatism and coma vanish:

$$T_1 = -\frac{a_3 R - \eta_3 K}{a_1^2 \beta \eta_1}, \tag{7a}$$

$$T_3 = \frac{a_1 R - \eta_1 K}{a_3^2 \beta \eta_3}, \tag{7b}$$

with

$$K = \sum_{\mu=1}^{3} \bar{K}_{s\mu} R_\mu, \quad R = \sum_{\mu=1}^{3} \varepsilon_\mu^2 c_\mu, \quad \beta = \eta_1 - \eta_3 + \eta_1 \eta_3 X.$$

In consequence, e_2, T_1 and T_3 are expressed by two variables X and Y ((6), (7a), (7b)). Two equations are left for determining X and Y:

$$\sum_{\mu=1}^{3} F_\mu = 0 \text{ (spherical aberration)}, \tag{8a}$$

$$\sum_{\mu=1}^{3} \varepsilon_\mu F_\mu = 0 \text{ (distortion)}, \tag{8b}$$

with

$$F_\mu = \begin{cases} c_\mu (a_{4-\mu} \varepsilon_\mu - \eta_{4-\mu}) / \bar{K}_{s\mu} + r^*_{\mathrm{IV}\mu}, & \mu = 1, 3, \\ c_2 (a_3 \varepsilon_3 - \eta_3)(a_1 \varepsilon_2 - \eta_1) + r^*_{\mathrm{IV}2}, & \mu = 2. \end{cases}$$

In general, Equations (7a) and (7b) cannot be solved by algebraic methods, but numerical methods must be applied to get pairs of solutions (X, Y). Then, all the data can be determined for a cemented lens with three surfaces free of Seidel's aberrations. While performing the calculations the condition $e_2 > 0$ should be taken into account.

5. Numerical analysis

In order to compare the above system of equations with that calculated by SCHULZ [4] the following data are assumed:

$$n_2 = 1.7, \; n_3 = 1.5, \; e_1 = 1, \; s_1 = -2, \; s_3' = 2.511. \tag{9}$$

The pairs (X, Y) can be separated by elementary methods under the prerequisite $e_2 \leqslant 0$.

Figure 1 shows the result. To prevent a too large vertex curvature, especially of the surface in the middle of the cemented lens X should vary from -0.2 to $+0.2$, Y being arbitrary. This situation is shown in Fig. 2, which is a part of Fig. 1, for this range of the values of X. Equations (8a) and (8b) have no

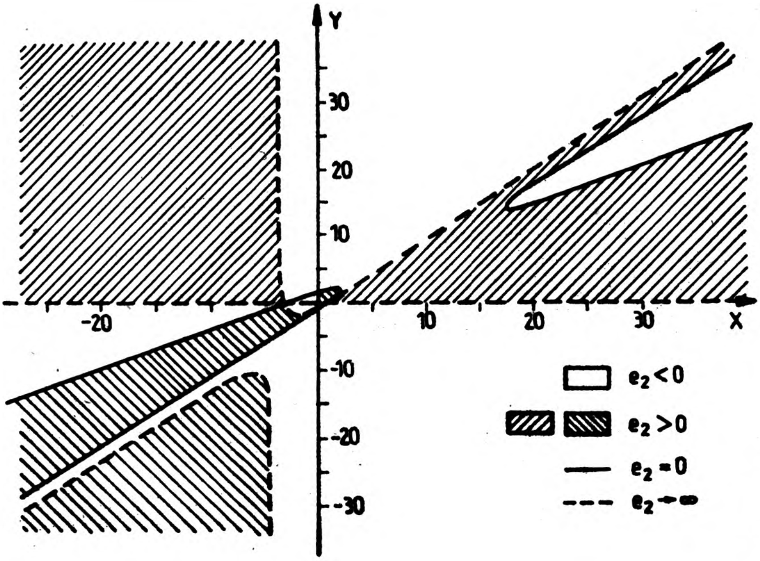

Fig: 1: Areas in the X-Y plane, where $e_2 > 0$, respectively $e_2 < 0$

solution. Without this restriction a vertex curvature can be determined by setting the spherical aberration and distortion zero within Seidel's range. A computer was used for both a systematic searching, root tracing for one equation and simplex method for the root finding of the system of equations. For the given parameters (9) 3 systems of solutions are obtained. One system is very similar to that given by SCHULZ [4] (see Table).

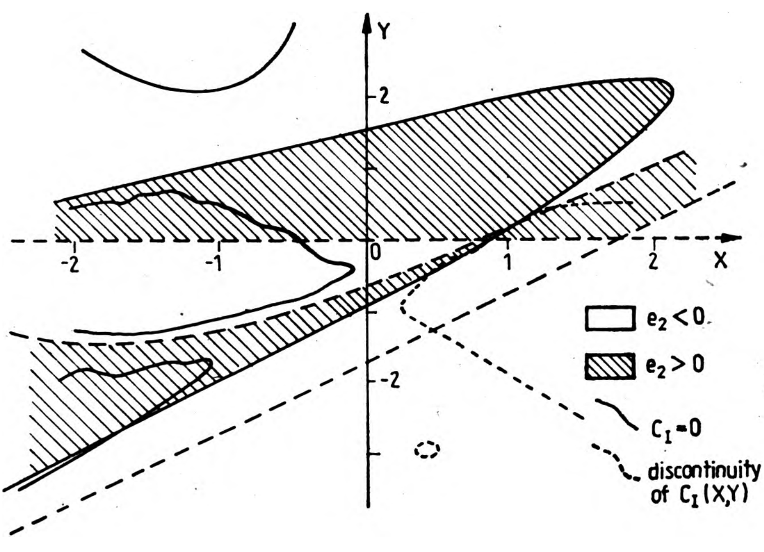

Fig. 2. Part of Fig. 1 in the range $-0.2 \leqslant X \leqslant +0.2$

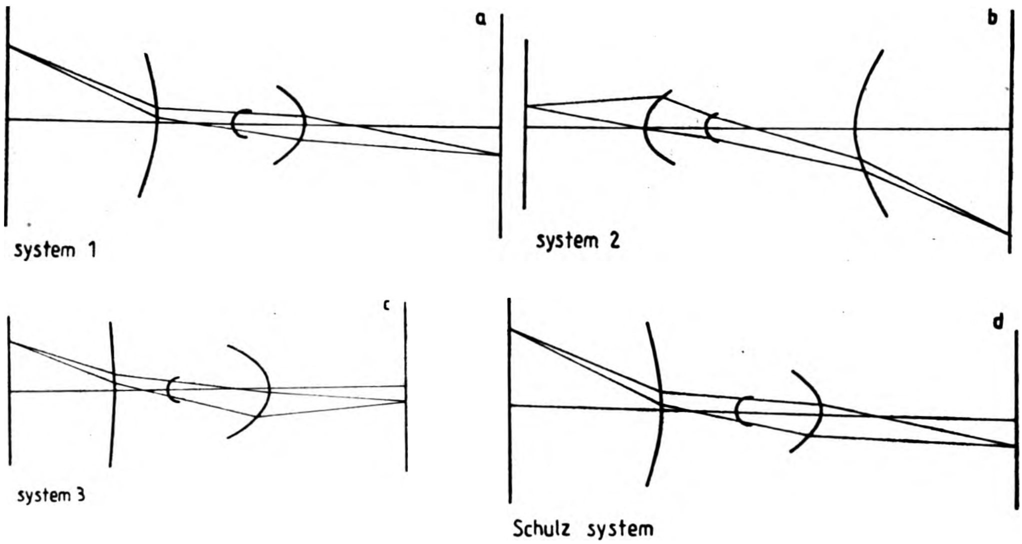

Fig. 3. Illustration of system 1 (a), system 2 (b), system 3 (c), and Schulz system [4] (d)

Figures 3a–d show those systems. It should be not iced that in case of hyper bolic or parabolic surfaces aspheric ones with small vertex curvatures may be of large free diameter. In all 3 systems radii of the refracting surface in the

	$1/\varrho_1$	$1/\varrho_2$	$1/\varrho_3$	$\hat{\varrho}_1$	$\hat{\varrho}_3$	e_2	β'
System 1	−1.0312	0.1814	−0.4045	9.8009	−1.2609	0.9545	−0.3478
System 2	0.5685	0.2196	0.9083	0.5459	−6.0964	2.5410	−5.6180
System 3	−2.2779	0.1766	−0.5333	98.3363	−0.3312	1.8880	−0.3312
Schulz system [4]	−1.182	0.184	−0.430	14.321	−1.009	1.106	−0.333

middle are very small. Here, the limited applications of a system with only 3 surfaces are obvious, because all the possible parameters are already used for correction and none are left for shaping the system.

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Одиночный клеевой дублет без аберрации первого порядка

Обсуждена оптическая система в воздухе с тремя преломляющими поверхностями. Представлены условия для зануления всех пяти монохроматических аберраций. Коррекция была получена путем применения вращательных поверхностей второй степени. Одна из них может быть сферической, но две остальные должны быть асферическими. Представлен метод решения этого вопроса и он иллюстрирован примером.