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Self-imaging phenomenon of tilted linear periodic objects

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Self-imaging phenomenon of a cosinusoidal, amplitude type linear diffraction grating illuminated by a plane beam tilted with respect to the grating normal is investigated. Equations describing the case of simultaneous tilt in the plane perpendicular and parallel to grating lines are derived, they include the special cases recently described in the literature.

1. Introduction

In the recent paper [1] an attention has been drawn to the properties of the Fresnel diffraction field of linear diffraction grating being tilted with respect to the optical axis or, equivalently, being illuminated by the oblique plane wavefront. The special cases of the grating tilt about the axis either parallel or perpendicular to grating lines have been treated separately. It has been shown that in both the cases the well defined diffraction images of the grating can be found in the observation planes parallel to the grating plane. When changing the observation distance the diffraction images are periodically detected. The above characteristics have been obtained by calculating or heuristically interpreting the intensity distribution patterns in the planes perpendicular to the direction of the illuminating beam. The experimental verification has been given. The established properties of the self-imaging phenomenon under oblique illumination are of practical importance in the shadow Moiré technique described in [2].

In this report we would like to present the investigation of a general case of the plane wavefront oblique illumination. Incidence plane of the illuminating beam does not coincide with the plane either parallel or perpendicular to the grating lines. The analytical formulae will be derived using the concept of an angular spectrum of plane waves. Additionally, the simplified calculation model pertinent to one of the special cases treated in [1] (the incidence plane being perpendicular to grating lines) will be presented. This model enables direct calculation of the intensity distributions in the planes parallel to grating plane.

Therefore, direct analytical estimation of the lateral period of diffraction images is possible in this case.

2. Analysis

The calculation of the Fresnel field pattern of obliquely illuminated linear periodic object could be done using Kirchhoff integral [3, 4]. However, we will adopt (as in the referenced paper [1]) the approach based on the concept of an angular spectrum of plane diffracted waves [5, 6]. These plane waves, when summed in both amplitude and phase, give the desired field distribution, the diffraction pattern can be considered as an interference pattern of all plane waves originating at the grating.

2.1. Calculation of the propagation directions of diffraction orders

In order to perform the summation of diffracted beams it is necessary to know their propagation directions. For this purpose the Fresnel-Kirchhoff calculation model will be used. Let us introduce, after [7], the general diffraction arrangement, its notation is schematically shown in Fig. 1. The point

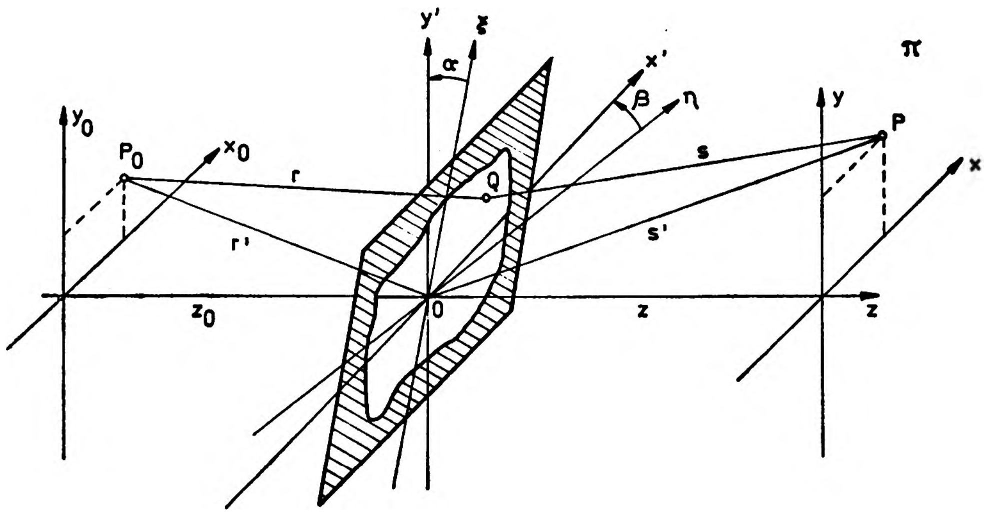

Fig. 1. Fresnel-Kirchhoff diffraction geometry for calculating the propagation angles of diffracted beams

source plane x_0y_0, the untilted object plane $x'y'$ and the observation plane xy are separated by z_0 and z, respectively. The plane object shown as an aperture in Fig. 1 is rotated about the x' and y' axes by the angles α and β, respectively. Using the usual approximations assumed when calculating the Fresnel-Kirchhoff integral [7] we can calculate the complex amplitude at the point P in the observation plane in the form

$$U(P) \propto -\frac{Ai\cos\delta}{r's'} \iint\limits_S \exp\{ik(r+s)\} dS \tag{1}$$

where A denotes the amplitude of the wave emitted by the point source P_0, δ is the angle between the line P_0P and the z axis, S corresponds to the object surface, dS is the object element and k denotes the wave number. Other symbols are shown in Fig. 1. After expanding r and s into the power series including the linear terms only [7] and performing simple calculations, Eq. (1) becomes

$$U(P) = -\frac{i\cos\delta}{\lambda} \frac{A\exp\{ik(r'+s')\}}{r's'} \iint\limits_S \exp\{ikf(\xi,\eta)\} d\xi d\eta \tag{2}$$

where

$$\begin{aligned} f(\xi,\eta) = {} & (l_0-l)\,\xi\cos a+(m_0-m)\eta\cos\beta+(\xi\sin a+\eta\sin\beta) \\ & \times \left[\sqrt{1-(l^2+m^2)}-\sqrt{1-(l_0^2+m_0^2)}\right], \end{aligned} \tag{3}$$

$$\left.\begin{aligned} l_0 &= -y_0/r', & l &= y/s' \\ m_0 &= -x_0/r', & m &= x/s' \end{aligned}\right\}. \tag{4}$$

Let us consider the case of plane beam illumination; in this case $l_0 = m_0 = 0$. Introducing the notations

$$\left.\begin{aligned} L &= l\cos a+\sin a\left[1-\sqrt{1-(l^2+m^2)}\right] \\ M &= m\cos\beta+\sin\beta\left[1-\sqrt{1-(l^2+m^2)}\right] \end{aligned}\right\} \tag{5}$$

we obtain

$$U(P) = C\int\limits_S \exp\{-ik(L\xi+M\eta)\} d\xi d\eta \tag{6}$$

where C is the proportionality constant. Now, let us introduce a periodic structure of spatial period d as the object in the $\xi\eta$ plane. The object lines will be assumed as being parallel to η axis. The amplitude transmittance of the singular linear element of the structure will be denoted by $F(\xi)$. Using Eq. (6) and the calculation model shown in [7] the amplitude at P is

$$U(L) = U_0(L)\sum_{n=0}^{N-1} \exp\{-ikndL\} \tag{7}$$

where N is the number of linear elements in the object and

$$U_0(L) = C \int_S F(\xi)\exp\{-ikL\}d\xi. \tag{8}$$

The intensity becomes

$$I(L) = I_0 H\left(N, \frac{kdL}{2}\right) = I_0\left(\frac{\sin(NkdL/2)}{\sin(kdL/2)}\right)^2. \tag{9}$$

From the last equation we obtain the condition for the angular localization of diffraction orders, i.e.

$$L = n\frac{\lambda}{d}. \tag{10}$$

Since the amplitude transmittance of periodic object is constant along the η direction we have

$$U(\eta) = \int_S \exp\{-ikM\}, \tag{11}$$

what imposes

$$M = 0. \tag{12}$$

Inserting Equations (10) and (11) into Equation (5) we obtain the following system of two equations with two unknown l and m:

$$\left.\begin{aligned} l\cos\alpha + \sin\alpha\left[1-\sqrt{1-(l^2+m^2)}\right] &= n\frac{\lambda}{d} \\ m\cos\beta + \sin\beta\left[1-\sqrt{1-(l^2+m^2)}\right] &= 0 \end{aligned}\right\}. \tag{13}$$

The parameters l and m correspond to the propagation directions of diffraction orders of the periodic object in the coordinate system xyz. They are calculated from (13) as:

$$\begin{aligned} l = \frac{n\lambda}{d\cos\alpha} - \frac{\sin\alpha\cos\beta}{1-\sin^2\alpha\sin^2\beta}\Bigg[\cos\alpha\cos\beta + \frac{n\lambda}{d}\frac{\sin\alpha\cos\beta}{\cos\alpha} \\ -\left(\cos^2\alpha\cos^2\beta + 2\frac{n\lambda}{d}\sin\alpha\cos^2\beta - \frac{n^2\lambda^2}{d^2}\right)^{1/2}\Bigg], \end{aligned} \tag{14}$$

$$\begin{aligned} m = -\frac{\sin\alpha\cos\beta}{1-\sin^2\alpha\sin^2\beta}\Bigg[\cos\alpha\cos\beta + \frac{n\lambda}{d}\frac{\sin\alpha\cos\beta}{\cos\alpha} \\ -\left(\cos^2\alpha\cos^2\beta + 2\frac{n\lambda}{d}\sin\alpha\cos^2\beta - \frac{n^2\lambda^2}{d^2}\right)^{1/2}\Bigg]. \end{aligned} \tag{15}$$

From these equations describing a general case of oblique illumination $\alpha \neq 0$, $\beta \neq 0$ the expressions relevant to special case [8–10] are readily obtained:

i) $\alpha = 0$

$$\left.\begin{aligned} l &= n\frac{\lambda}{d} \\ m &= -\sin\beta\left[\cos\beta - \sqrt{\cos^2\beta - n\lambda/d)^2}\right] \end{aligned}\right\}. \tag{16}$$

ii) $\beta = 0$

$$\left.\begin{aligned} l &= \cos\alpha\left(\frac{n\lambda}{d} - \sin\alpha\right) + \sin\alpha\sqrt{1-(\sin\alpha - n\lambda/d)^2} \\ m &= 0 \end{aligned}\right\}. \tag{17}$$

iii) $\alpha = \beta = 0$

$$\left.\begin{aligned} l &= n\frac{\lambda}{d} \\ m &= 0 \end{aligned}\right\}. \tag{18}$$

2.2. Calculation of the Fresnel diffraction field

Knowing the propagation directions of diffraction orders of obliquely illuminated periodic structure we are ready to calculate the Fresnel diffraction field using

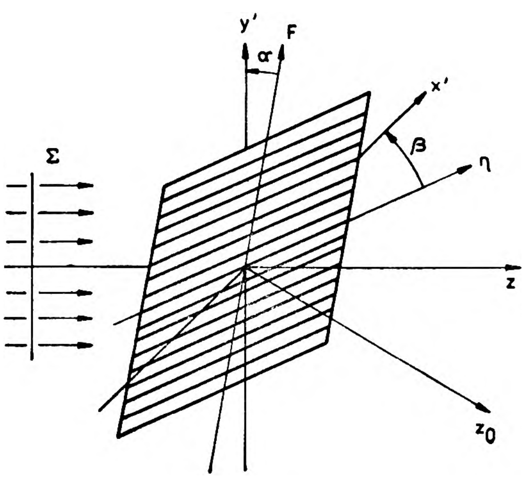

Fig. 2. Diffraction geometry for calculating the Fresnel field of linear periodic structure having the normal z_0 tilted with respect to the illuminating beam propagation direction. Plane beam impignes along the z direction. The object plane $\xi\eta$ is tilted by the angles α and β with respect to the plane $x'y'$ normal to z

the concept of an angular spectrum [5, 6]. It has been found from many trials that for the convenience and simplicity of calculation of a general case $\alpha \neq 0$, $\beta \neq 0$ it is good to assume the z axis of the coordinate system as coinciding with the illuminating beam propagation direction. It passes through the centre of the plane $\xi\eta$. This choice is much better than the assumption that the z axis is coinciding with the grating normal. The diffraction geometry, corresponding to the calculation in the following, is schematically shown in Fig. 2. It is readily seen that the incidence angles α and β of the illuminating beam correspond now to the tilt angles α and β of the grating. However, according to the convention of the signing angles, assumed before, we have to change signs of the grating tilt angles.

The complex amplitude of the Fresnel light field in the plane perpendicular to the illumination direction will be calculated. For the simplicity of analysis we will assume the cosinusoidal amplitude transmittance of the periodic object in the form of linear, amplitude-type diffraction grating

$$T(\xi) = V_0 + V\cos(2\pi\xi/d) \tag{19}$$

where V_0 and V denote the amplitude modulation parameters. Using the concept of an angular spectrum of plane waves [1, 6] the light field in the observation plane xy perpendicular to z axis is expressed by

$$\begin{aligned} U(x, y, z) = {} & V_0\exp\{ikz\} + \frac{V}{2}\exp\left\{ik\left[xm_{+1} + yl_{+1} + z\sqrt{1-(l_{+1}^2 + m_{+1}^2)}\right]\right\} \\ & + \frac{V}{2}\exp\left\{ik\left[xm_{-1} + yl_{-1} + z\sqrt{1-(l_{-1}^2 + m_{-1}^2)}\right]\right\} \end{aligned} \tag{20}$$

where l_{+1}, m_{+1}, l_{-1} and m_{-1} denote the propagation directions of the $+1$ and -1 diffraction orders, respectively; $l_0 = m_0 = 0$. In the following we will be concerned with small values of l and m, this is usually the case when low frequency gratings are used for the self-imaging applications. For example, when the grating of maximum frequency of 50 l/mm is considered we have $l = 0.05$ and $m = 0.0005$ for $\lambda = 0.633$ μm. In such a case Eq. (20) becomes

$$\begin{aligned} U(x, y, z) = {} & \exp\{ikz\}\left\{V_0 + \frac{V}{2}\exp\left[ik\left\{xm_{+1} + yl_{+1} - \frac{z}{2}(l_{+1}^2 + m_{+1}^2)\right\}\right]\right. \\ & \left. + \frac{V}{2}\exp\left[ik\left\{xm_{-1} + yl_{-1} - \frac{z}{2}(l_{-1}^2 + m_{-1}^2)\right\}\right]\right\}. \end{aligned} \tag{21}$$

The intensity distribution is

$$I(x,y,z) = V_0^2+\frac{V^2}{2}+\frac{V^2}{2}\cos k\Big\{x(m_{+1}-m_{-1})+y(l_{+1}-l_{-1})$$
$$-\frac{z}{2}(l_{+1}^2-l_{-1}^2+m_{+1}^2-m_{-1}^2)\Big\}+2V_0V\cos k\Big\{\frac{x}{2}(m_{+1}+m_{-1})$$
$$+\frac{y}{2}(l_{+1}+l_{-1})-\frac{z}{4}(l_{+1}^2+l_{-1}^2+m_{+1}^2+m_{-1}^2)\Big\}\cos k\Big\{\frac{x}{2}(m_{+1}-m_{-1})$$
$$+\frac{y}{2}(l_{+1}-l_{-1})-\frac{z}{4}(l_{+1}^2-l_{-1}^2+m_{+1}^2-m_{-1}^2)\Big\}. \tag{22}$$

It is seen that the intensity distribution consists of three basic terms: the background term, the fundamental and the second harmonic. The fundamental is of primary importance, it is composed of the following terms:

i) The term describing the intensity distribution in the xy plane, that is

$$\cos k\Big\{\frac{x}{2}(m_{+1}-m_{-1})+\frac{y}{2}(l_{+1}-l_{-1})-\frac{z}{4}(l_{+1}^2-l_{-1}^2+m_{+1}^2-m_{-1}^2)\Big\}. \tag{23}$$

It expresses a periodic linear intensity pattern, the normal to the fringes is inclined with respect to the y axis (Fig. 3) by an angle ω given by

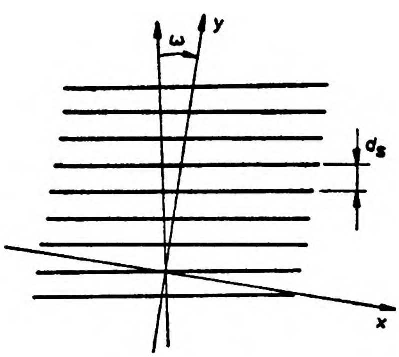

Fig. 3. Schematic representation of the periodic intensity pattern in the xy plane (without showing the contrast modulation effect)

$$\tan\omega = \frac{m_{+1}-m_{-1}}{l_{+1}-l_{-1}}. \tag{24}$$

The spatial period d_s of the intensity pattern is

$$d_s = 2\lambda[(l_{+1}-l_{-1})^2+(m_{+1}-m_{-1})^2]^{-1/2}. \tag{25}$$

The lateral shift p_0 of the periodic intensity distribution with respect to the origin of the x, y plane (measured along direction specified by the angle ω) is

$$p_0 = z(l_{+1}^2 - l_{-1}^2 + m_{+1}^2 - m_{-1}^2)d_s/4\lambda. \tag{26}$$

ii) The term describing contrast modulation of the periodic intensity pattern expressed by Eq. (23). It is proportional to

$$\cos k\left\{\frac{x}{2}(m_{+1}+m_{-1})+\frac{y}{2}(l_{+1}+l_{-1})-\frac{z}{4}(l_{+1}^2+l_{-1}^2+m_{+1}^2+m_{-1}^2)\right\}. \tag{27}$$

The maximum contrast is obtained when

$$x(m_{+1}+m_{-1})+y(l_{+1}+l_{-1})-\frac{z}{2}(l_{+1}^2+l_{-1}^2+m_{+1}^2+m_{-1}^2) = 0. \tag{28}$$

Last equation provides the conditions under which the constant contrast is observed in the detection plane

$$\tan\alpha' = \frac{2(l_{+1}+l_{-1})}{l_{+1}^2+l_{-1}^2+m_{+1}^2+m_{-1}^2} = \frac{z}{y}, \quad \text{for} \quad x = 0, \tag{29}$$

$$\tan\beta' = \frac{2(m_{+1}+m_{-1})}{l_{+1}^2+l_{-1}^2+m_{+1}^2+m_{-1}^2} = \frac{z}{x}, \quad \text{for} \quad y = 0$$

where α' and β' denote the angles between the detection plane and the plane perpendicular to z axis. From Eqs. (27) and (29) we have

$$z = \frac{2N\lambda}{l_{+1}^2+l_{-1}^2+m_{+1}^2+m_{-1}^2} \tag{30}$$

where $N = 1, 2, 3, \ldots$ is a positive integer. For the propagation distances determined by Eq. (30) the value of the contrast modulation term (Eq. (27)) is equal to $(-1)^N$.

We have performed numerical calculations of the above derived equations for various values of the grating spatial frequencies (25 and 40 l/mm) and tilt angles. From the results obtained the following conclusions have been formulated:

— Planes of constant spatial contrast modulation are parallel to the grating plane, i.e., $\alpha = \alpha'$, $\beta = \beta'$.

— Lines in the diffraction images are parallel to grating lines.

— Spatial period of the diffraction images detected in the planes of constant contrast modulation is equal to the object grating spatial period.

— Longitudinal separation distance z between the planes of constant modulation depends on the object tilt angles α and β.

It follows from the above conclusions that the diffraction images in the planes of constant spatial contrast modulation (planes parallel to the object-grating plane) can be named as the self-images of the object grating under oblique illumination.

2.3. Fresnel diffraction field — special cases

After having presented a general solution (Eq. (22)) corresponding to the simultaneous grating tilt by the angles α and β, let us give the special cases, when the tilt occurs in one direction only.

1. $\alpha \neq 0, \quad \beta = 0.$

Now, the propagation directions of diffraction orders are given by Eq (17); the formerly introduced parameters describing the intensity distribution of the Fresnel field become:

$$\tan\omega = 0, \tag{31a}$$

$$\frac{1}{d_s} = \frac{l_{+1}-l_{-1}}{2\lambda}, \tag{31b}$$

$$p_0 = \frac{z}{4\lambda}(l_{+1}^2-l_{-1}^2), \tag{31c}$$

$$\tan\alpha' = \frac{2(l_{+1}+l_{-1})}{l_{+1}^2+l_{-1}^2}, \tag{31d}$$

$$\tan\beta' = 0, \tag{31e}$$

$$z = \frac{2N\lambda}{l_{+1}^2+l_{-1}^2}. \tag{31f}$$

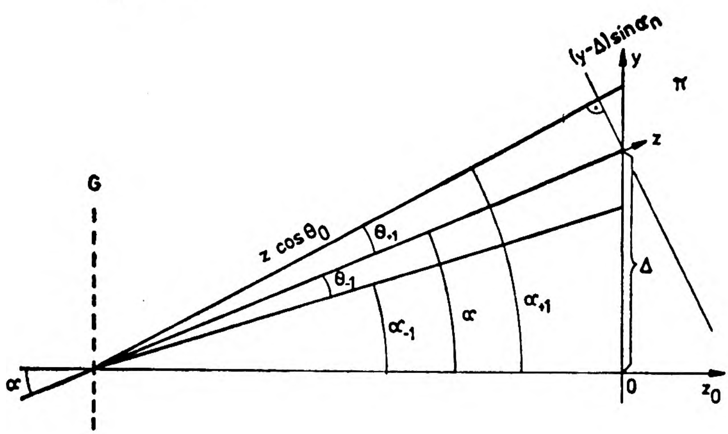

Fig. 4. Symbols used for the derivation of intensity distribution in the plane parallel to the grating, the incidence plane of illuminating beams is perpendicular to grating lines

In this special case another calculation procedure can be employed giving directly the intensity distribution in the plane parallel to the grating plane. The notation is shown in Fig. 4.

Similarly to Eq. (20) we can write

$$U'(y, z) = V_0 \exp\left\{ik[z+y\sin\alpha] + \frac{V}{2}\exp\{ik[z\cos\Theta_{+1}+(y-\Delta)\sin\alpha_{+1}]\right\}$$
$$+\frac{V}{2}\exp\{ik[z\cos\Theta_{-1}+(y-\Delta)\sin\alpha_{-1}]\} \tag{32}$$

where:

$$\left.\begin{aligned} \sin\alpha_{+1} &= \sin\alpha+\lambda/d \\ \sin\alpha_{-1} &= \sin\alpha-\lambda/d \\ \Theta_{+1} &= \alpha_{+1}-\alpha \\ \Theta_{-1} &= \alpha_{-1}-\alpha \end{aligned}\right\}. \tag{33}$$

$$\left.\begin{aligned} z &= z_0/\cos\alpha \\ \Delta &= z_0/\tan\alpha \end{aligned}\right\}. \tag{34}$$

Assuming as before, small values of diffraction angles (that is when the approximation $\cos\Theta = 1-0.5\sin^2\Theta$ can be used) and noting that $\sin\Theta_{+1} = l_{+1}$ and $\sin\Theta_{-1} = l_{-1}$, we can rewrite Eq. (32) in the form

$$U'(y, z) = \exp\{ik[z+(y-\Delta)\sin\alpha]\}\left[V_0+\frac{V}{2}\exp\left\{ik\left[(y-\Delta)\frac{\lambda}{d}\right.\right.\right.$$
$$\left.\left.\left.-\frac{z}{2}l_{+1}^2\right]\right\}+\frac{V}{2}\exp\left\{ik\left[-(y-\Delta)\frac{\lambda}{d}-\frac{z}{2}l_{-1}^2\right]\right\}\right]. \tag{35}$$

The intensity is calculated as

$$I'(y, z) = V_0^2+\frac{V^2}{2}+2V_0\cos\frac{2\pi}{d}\left[y-\Delta-\frac{zd}{4\lambda}(l_{+1}^2-l_{-1}^2)\right]$$
$$\times\cos 2\pi\left[\frac{z}{4\lambda}(l_{+1}^2+l_{-1}^2)\right]+\frac{V^2}{2}\cos\frac{2\pi}{d}\left[2y-2\Delta\frac{zd}{2\lambda}(l_{+1}^2-l_{-1}^2)\right]. \tag{36}$$

It follows that the intensity distribution has a spatial period d in the y direction. The contrast modulation in the whole observation plane is determined by the cosine term $\cos[2\pi z(l_{+1}^2 - l_{-1}^2)/4\lambda]$. For the observation distances

$$z = \frac{2N\lambda}{l_{+1}^2 + l_{-1}^2} \tag{37}$$

the contrast factor equals $(-1)^N$. This agrees with Eq. (30), where we have to put $m_{+1} = m_{-1} = 0$ for the case under discussion. Therefore, it follows that in the case $\beta = 0$ the results obtained by an indirect method, see Chap. 2.2 and [1] (calculation of intensity distribution in the planes perpendicular to the illuminating beam and the subsequent derivation of the properties of diffraction images in the planes parallel to grating plane) and the method just presented are identical. Certainly, the latter method is simpler and faster.

2. $\alpha = 0, \quad \beta \neq 0.$

In this case propagation directions of diffraction orders are given by Eq. (16) and the parameters characterizing the Fresnel field intensity distribution become:

$$\tan\omega = 0, \tag{38a}$$

$$d_s = d, \tag{38b}$$

$$p_0 = \lambda z/d, \tag{38c}$$

$$\tan\alpha' = 0, \tag{38d}$$

$$\tan\beta' = \frac{m}{l^2 + m^2}, \tag{38e}$$

$$z = \frac{N\lambda}{l^2 + m^2} \tag{38f}$$

where $l = |l_{+1}| = |l_{-1}|$ and $m = m_{+1} = m_{-1}$.

Unfortunately, in this special case we cannot perform the direct calculation of intensity distribution in the plane parallel to the grating plane, as in the case $\alpha \neq 0$, $\beta = 0$. This is due to the fact that the interfering diffraction orders do not lie in a single plane and the calculation model presented in Chap. 2.2 must be used. A heuristic explanation of this case, based on the incoherent Moiré-addition of the two beam interference patterns has been presented in [1]. The results obtained coincide with Eq. (36) of the present paper.

3. Conclusions

Analytical expressions, describing the intensity distribution in the Fresnel diffraction field of a linear amplitude type diffraction grating illuminated by a plane spatially coherent beam, have been derived and discussed. General case of the tilt of grating normal in an arbitrary direction with respect to the illumination direction has been investigated. It has been found that (under usual approximations of relatively small diffraction angles of the object-gratings used for the self-imaging effect applications) the object-grating selfimages are found in the planes parallel to the grating plane. On the other hand, when the observation is conducted in the planes perpendicular to the illuminating beam direction the periodic bands of intensity contrast modulation are observed.

From the expressions corresponding to an arbitrary object tilt angle the formulae describing the special cases of unidirectional tilts have been derived. They are in full agreement with the expressions obtained before in a different way [1] and experimentally verified.

The analysis and its results can be easily extended to phase diffraction gratings as well as to two-dimensional amplitude and phase periodic structures. As it has been mentioned before [1] relatively simple analytical expressions describing the Fresnel field intensity patterns can be obtained for periodic structures generating three diffraction orders (three beam interference). This is due to the fact that there is no equality of angular separation between the adjacent diffraction orders and, consequently, there is no phase coincidence of all orders in the self-image planes. In the case of gratings with higher harmonics the numerical solution is required. However, we have performed many experiments with square wave amplitude type diffraction grating and very close coincidence of the observed Fresnel field properties with the characteristics derived in this paper has been noted. This is because of missing the even orders in the square wave grating amplitude transmittance and the predominance of the first order diffraction beam.

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Явление самоизображения наклоненных линейных периодических предметов

Анализируется явление самоизображения косинусоидальной, амплитудной, линейной, дифракционной решетки, освещенной плоской волной. Направление падения светового пучка является отличается от направления нормали к плоскости решетки. Выведены формулы, описывающие общий случай одновременного наклона в плоскости вертикальной и горизонтальной по отношению к линии решетки. Эти формулы содержат описание особых случаев, анализированных в научной литературе.