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Soliton-like solutions to optical Bloch equations

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Description of self-induced transparency, which is lossless propagation of an optical pulse in a resonant medium of two-level atoms, is based on the coupled Maxwell–Bloch equations [1]. These equations are valid under the slowly varying envelope (SVEA) and rotary wave-approximation (RWA). A solution represents an optical soliton, which is represented by sech pulse and is travelling without change of shape.

In this communication, a class of sech solutions to the undamped Bloch equations is presented. The optical Bloch equations in the RWA and SVEA version are represented by

$$\dot{u} = -v\Delta\omega, \tag{1a}$$
$$\dot{v} = u\Delta\omega + \varepsilon w, \tag{1b}$$
$$\dot{w} = -\varepsilon v. \tag{1c}$$

In Equations (1), ε is a pulse of an electric field, $\Delta\omega = \omega - \omega_0$ (ω – carrier frequency, ω_0 – transition frequency), u, v – atomic envelope functions for the dipole variables, w – nversion. A solution to (1) is valid for $\varepsilon(t) = \varepsilon_0 \operatorname{sech}(t/\tau)$, where $\varepsilon_0 = 2N/\tau$ (N – integer). Of course, for $N = 1$ the solution is well known [1]. For $N = 2$, for example, one obtains $\varepsilon_0 = 4/\tau$ and:

$$\varepsilon = (4/\tau)\operatorname{sech}(t/\tau), \tag{2}$$
$$u = (\omega - \omega_0)(a_1\varepsilon + a_2\varepsilon^3), \tag{3a}$$
$$v = -a_1\dot{\varepsilon} - 3a_2\varepsilon^2\dot{\varepsilon}, \tag{3b}$$
$$w = \frac{1}{2}a_1\varepsilon^2 + \frac{3}{4}a_2\varepsilon^4 - 1 \tag{3c}$$

where the coefficients a_1 and a_2 are given by:

$$a_1 = \frac{1}{(\omega - \omega_0)^2 + (1/\tau)^2}, \quad a_2 = -\frac{3}{8}a_1\frac{1}{(\omega - \omega_0)^2 + (3/\tau)^2}.$$

From Equations (3), it follows that twice coherent excitation of two-level atom is possible. For (1) combined with the Maxwell equations one can expect that a solution for $N > 1$ is valid, too.

References

ALLEN L., EBERLY J. H., *Optical Resonance and Two-Level Atoms*, Wiley, New York 1975.