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Spatial solitons excited by the second-order Hermite – Gaussian beams

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It is shown that the second-order Hermite – Gaussian solution treated as an initial condition to the nonlinear Schrödinger equation generates an *even* number of solitons in the Kerr medium. The parameters of solitons as amplitudes and angles of propagation are evaluated numerically by the Inverse Scattering Transform. The following soliton generation scenario is presented. For a low incident beam amplitude no solitons are found. As the incident amplitude increases *two* solitons appear and propagate under opposite propagation angles. Further increase of the incident amplitude decreases the propagation angles and above some specific amplitude level solitons form a bound solution. A very good agreement is found between a direct numerical integration of NSE and variational approximation to the problem for incident amplitudes below the solitons excitation level.

It has recently been reported [1] that a linear propagation of Hermite – Gaussians (HG) and a nonlinear propagation of optical solitons have a lot in common, and that the second process can be modelled by the first one, including higher order beams of both types. This contribution follows numerically that conjecture on the second order level and gives a few basic facts on the ($N = 2$) soliton propagation. The propagation of an intense electromagnetic beam in a medium with planar symmetry can be modelled by the normalized nonlinear Schrödinger equation (NSE) [2]

$$i\frac{\partial U}{\partial \zeta}+\frac{1}{2}\frac{\partial^2 U}{\partial x^2}+|U|^2U = 0 \tag{1}$$

where ζ is measured in the Rayleigh (or Fresnel) length z_F units and x in the initial beam width w_0 units. The N-soliton solution to this equation (when the solitons are well separated)

$$U(x,\zeta) = \sum_{i=1}^{N} 2\eta_i e^{2i(\eta_i^2-\xi_i^2)\zeta-2ix\xi_i x}\operatorname{sech}[2\eta_i x+4\eta_i\xi_i\zeta] \tag{2}$$

was obtained by ZAKHAROV and SHABAT [2], who applied the Inverse Scattering Transform (IST). In Equation (2), $2\eta_i$ is the amplitude of the i-th soliton, $\vartheta_i = -\arctan(2\xi_i)$ is the angle of soliton propagation to the ζ-direction and $\lambda_i = \xi_i+i\eta_i$ is an eigenvalue in the Direct Scattering Problem [2].

In the linear medium solution to the propagation equation (*i.e.*, Eq. (1) without the last term of the left-hand side) can be analyzed in terms of symmetric in a complex argument HG functions [3]. The second-order HG solution has the form

$$U(x,\zeta) = \frac{A(\zeta)}{v^3(\zeta)}\left[-1+\left(\frac{x}{v(\zeta)}\right)^2\right]\exp\left(-\frac{x^2}{2v^2(\zeta)}\right), \tag{3}$$

with $A(\zeta) = \text{const}$ and $v(\zeta) = \sqrt{1+i\zeta}$ [4]. In the following we will consider the propagation of the function (3), with $A(0) = q_0$ and $v(0) = 1$, in the nonlinear Kerr medium.

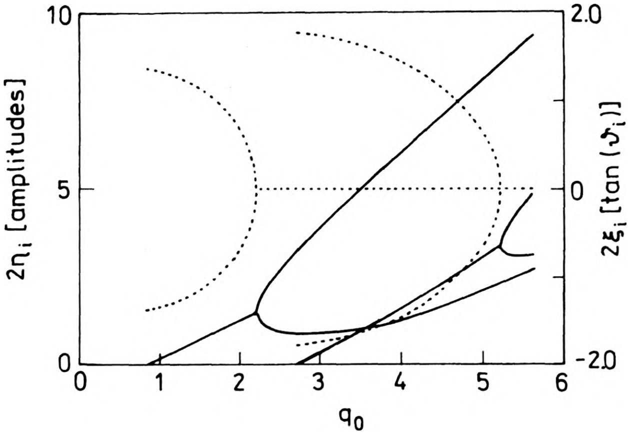

Fig. 1. Soliton parameters excited by the second-order Hermite–Gaussian beam (3). The imaginary parts (solid curves – the left-hand-side axis), and real parts (dotted curves – the right-hand-side axis) of eigenvalues λ_i, $i = 1, ..., 4$ *vs.* the incident beam amplitude q_0

An example of our calculations [5] is presented in Figure 1, where the imaginary and real parts of eigenvalues λ_i are shown, respectively, as a function of the incident beam amplitude q_0. For small incident beam amplitudes q_0 no solitons are present in the spectrum of the IST. For amplitudes higher than threshold (numerical) value $q_0^{(\text{th})} \approx 0.86$ (where the superscript "th" indicates threshold) *two* solitons are excited by the incident beam (3). These zero-amplitude solitons ($\eta_1 = \eta_2 = 0$) propagate at angles $\theta_{1,2} = \pm 53.7°$ ($\xi_{1,2} \approx \pm 0.68$) with respect to the ζ-direction. With an increasing incident beam amplitude q_0, the excited soliton amplitudes increase (solid curves in Fig. 1), and the relative angle of propagation between solitons: $\Delta\theta = \arctan(2\xi_1) - \arctan(2\xi_2)$ decreases (dotted curves in Fig. 1). In the far field the solution of NSE is in this case given by the sum of two solitons described by Eq. (2). For the numerical value $q_0^{(b)} \approx 2.22$ both eigenvalues collapse and further increase of q_0 causes the increase of the difference: $\Delta\eta = \eta_1 - \eta_2$ in values of soliton amplitudes, whereas real parts of eigenvalues $\lambda_{1,2}$ are equal to zero: $\xi_1 = \xi_2 = 0$ (see Fig. 1). This means that both solitons form a bound solution which propagates along the ζ-axis (and so the superscript b denotes bound). The (approximate) period of spatial oscillations of the breather soliton is given by: $Z_p = \pi/(\eta_1^2 - \eta_2^2)$, provided $\eta_1 > \eta_2$ [6]. Thus for the incident beam amplitude equal to $q_0^{(b)}$ ($\eta_1 = \eta_2 \approx 0.75$) the period Z_p is infinite and solution of the NSE has the form of two solitons propagating in parallel with each other, separated in space (Figure not shown), each with the same space. The increase of q_0 above the $q_0^{(b)}$ level decreases value of Z_p and the breather soliton oscillates, *i.e.*, periodically changes its amplitude and width. With further increase of the incident beam amplitude q_0 the process of soliton excitation repeats.

Variational approach to nonlinear beam propagation was originally developed by ANDERSON [7]. In the framework of this approach the beam is assumed to take a Hermite—Gaussian form (3). The shape parameters as complex amplitude $A(\zeta)$ and complex width $v(\zeta)$ are allowed to evolve as beam propagates down the medium. By taking variations of the appropriate Lagrangian with respect to each parameter of the beam, the evolution (ordinary differential) equations for the beam parameters are determined (ODE model) along the lines analogical to the procedure [7], [8] presented for the fundamental HG beam excitation. In Figure 2, we compare the presented variational approximation to beam propagation, as obtained from numerical solution of ODE model, with an exact, numerical solution of NSE.

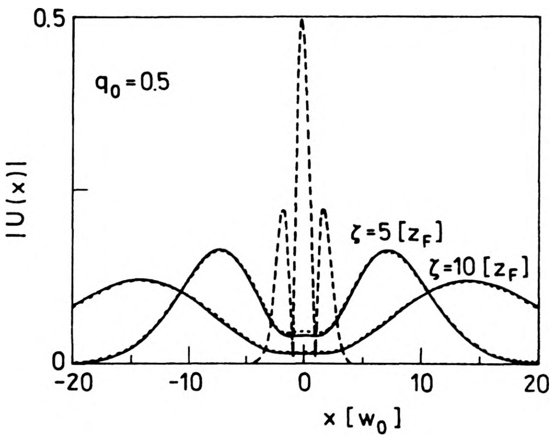

Fig. 2. Numerical (solid lines) and variational (dotted lines) profiles of solutions of NSE at $\zeta = 5z_F$ and $\zeta = 10z_F$ for the incident beam amplitude $q_0 = 0.5$. (Profiles: . . . . . variational, —— numerical, — — — incident)

The transverse cross-sections of the field are shown at $\zeta = 5z_F$ and $\zeta = 10z_F$ (solid lines), in comparison to the incident beam profile at $\zeta = 0$ (dashed line). For the incident beam amplitude $q_0 = 0.5$, *i.e.*, below the solitons excitation threshold (comp. Fig. 1), an excellent agreement between both methods is shown. The incident beam undergoes nonlinear diffraction and preserves *exactly* its (Hermite—Gaussian of the second-order) form.

Above the soliton excitation threshold $q_0^{(\mathrm{th})}$ the approximation is getting worse and above the bound solution formation level $q_0^{(v)}$ no quantitative agreement has been found between predictions of both methods. However, some qualitative similarities still can be noticed. Numerical calculations show that there exist the transverse cross-sections of the beam profiles where, alternatively, the field magnitude of the soliton or the propagation direction of the gravity points of the side lobes preserve the actual values. Results of these calculations will be presented elsewhere.

Finally, the nonlinear propagation of Hermite–Gaussian beams of the second order in Kerr medium has been investigated. The second-order Hermite–Gaussian function excites an even number of solitons in Kerr medium. It has been shown that in some cases the nonlinear propagation of the incident second-order HG profile can be reasonably well approximated by the propagation of second-order Hermite–Gaussian beam with variationally modified parameters.

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