Optica Applicata, Vol. XIV, No. 3, 1984

Heat-sinking process in light-emitting diodes

WŁODZIMIERZ NAKWASKI

nstitute of Physics, Technical University of Łódź, ul. Wólczańska 219, 93–005 Łódź, Poland

In this paper, the heat spreading in the semi-infinite heat-sink of a device with a cylindrical symmetry (e.g., a light-emitting diode) is analysed. The formulae for the spreading thermal resistances, the position-dependent and the mean resistances, are derived for two cases of distributions of the heat flux flowing into the heat-sink: i) the uniform heat flux density within the given circle and the zero heat flux outside it, and ii) the Gaussian shape of the heat flux density.

1. Introduction

The performance of a light-emitting diode is affected by an inside temperature rise which influences emission, modulation, carrier confinement, current-voltage characteristic, reliability and so on. This temperature rise takes place not only in a semiconductor volume but in a heat-sink (Fig. 1) as well.

The thermal sensitivity of a device, i.e., the temperature response to the supplied heat flux Q, is for the steady-state conditions usually described in terms of a thermal resistance Θ defined as

$$\Theta = \Delta T/Q \ [\mathrm{K/W}] \tag{1}$$

where ΔT – temperature rise within a device.

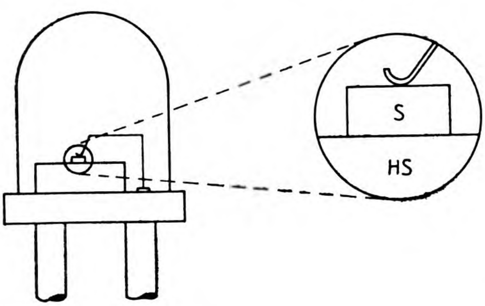

Fig. 1. The light-emitting diode configuration. S – semiconductor crystal, HS – heat-sink

The thermal resistance Θ_{LED} of a light-emitting diode may be divided into two parts:

$$\Theta_{\mathrm{LED}} = \Theta_{\mathrm{SC}} + \Theta_{\mathrm{HS}} \tag{2}$$

where Θ_{SC} and Θ_{HS} are the thermal resistance of a semiconductor crystal and the thermal spreading resistance of the heat-sink, respectively. The forme quantity has been calculated in paper [1] by means of the Green functions. Now we consider the thermal spreading resistance of the heat-sink.

This work is arranged as follows: in Section 2 the simplified formulae (known from the literature) for Θ_{HS} are presented. Section 3 is devoted to the method of determination of the effective radius a_e of the heat flux flowing into the heat-sink from a semiconductor crystal. The position-dependent thermal spreading resistance $\Theta_{\mathrm{HS}}(r)$ for a uniform heat flux is derived in Section 4. The analogous resistance for the case of a position-dependent heat flux is analysed in Section 5. The comparison of the results is given in Section 6.

All the derived formulae may be used for devices with cylindrical symmetry, e.g., for light-emitting diodes with a surface emission.

2. Simplified formulae for the thermal spreading resistance of the heat-sink in the case of devices with cylindrical symmetry

In a typical light-emitting diode, dimensions of the heat-sink are much greater than those of the semiconductor crystal. Therefore the heat-sink is usually treated as semi-infinite.

The generally known formula for the spreading thermal resistance of a semi-infinite heat-sink, in the case of devices with cylindrical symmetry, is position-independent and reads as follows [2]:

$$\Theta_T = (4\lambda a)^{-1} \tag{3}$$

where λ is the thermal conductivity of the heat-sink material. This formula has been derived, assuming a steady heat flow from a circle of a radius a into a half space of constant temperature. For estimative calculations, the radius a_c of the top contact is usually used in the Eq. (3)

$$\Theta_T \approx (4\lambda a_c)^{-1}. \tag{4}$$

The same, but without the assumption of constant temperature of a half space, leads to another formula [3, 4]:

$$\Theta_G = (\pi\lambda a)^{-1}. \tag{5}$$

For small circle, it is sometimes replaced by a hemisphere of the same radius a. This means that the material within the hemisphere is treated as a perfect conductor. Consequently, the heat flow is radial and the thermal spreading resistance is expressed by [5]:

$$\Theta_R = (2\pi\lambda a)^{-1}. \tag{6}$$

3. Effective heat flux flowing into the heat-sink

In the above formulae (3)–(6), it has been assumed that the heat flow from a circle of a radius a into a half space of the heat-sink is uniform. But the density distribution $q_{\mathrm{HS}}(r)$ of the heat flux flowing into the heat-sink is quite different.

This distribution is influenced by two processes (Fig. 2): i) the current spreading effect between the top contact and the p-n junction, and ii) the heat flux spreading effect between the p-n junction and the heat-sink. As a result, the distribution of $q_{\mathrm{HS}}(r)$ takes approximately the Gaussian shape [6].

LAFF et al. [7] have solved a similar problem for the stripe-geometry lasers by introducing the effective width of the heat flux flowing from a laser diode crystal into its heat-sink. The analogous method for the case of light-emitting diodes will be shown in this section.

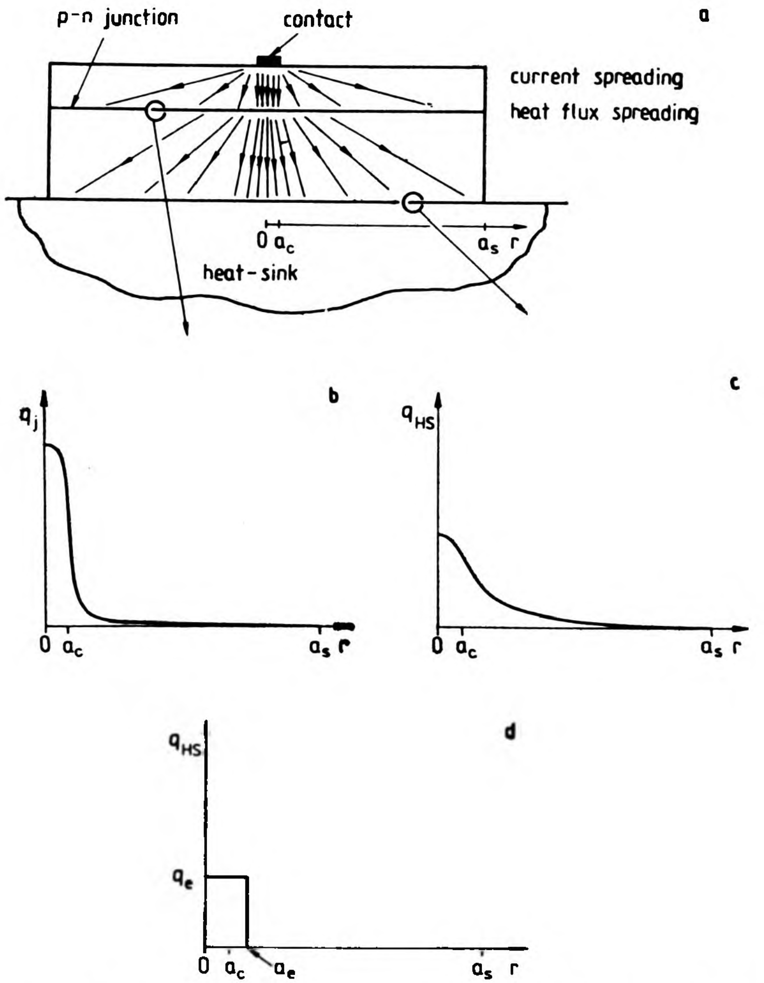

Fig. 2. The current spreading effect and the heat flux spreading effect in a light-emitting diode: light-emitting diode configuration with a location of both the above processes (a), distribution of the heat flux density q_j in the p-n junction plane (b), distribution of the heat flux density q_{HS} in the plane of the semiconductor/metal heteroboundary (c), effective density distribution q_{HS} of the heat flux flowing into the heat-sink (d). a_c, a_s and a_e — contact, structure and effective radii, respectively

We look for the effective radius a_e of the uniform heat flux (Fig. 2d) which eventually gives in the heat-sink the same mean temperature increase (calculated for points within the circle of radius a) as the real heat flux (Fig. 2c).

Let us add a thin layer of a thickness d_t between the light-emitting diode chip and its heat-sink. Then the increase in the thermal resistance Θ_{LED} may be expressed as follows:

$$d\Theta_{\mathrm{LED}} = \frac{\partial \Theta_{\mathrm{LED}}}{\partial d_t}\,\delta d_t. \tag{7}$$

On the other hand, assuming the uniform density distribution q_e of the heat flux (Fig. 2d) flowing into the heat-sink from a circle of a radius a_e, the above presented increase in the thermal resistance may be written as

$$d\Theta_{\mathrm{LED}} = \frac{\delta d_t}{\pi a_e^2 \lambda_t} \tag{8}$$

where λ_t — thermal conductivity of the additional layer considered. Taking both the Eqs. (7) and (8) together, we obtain

$$a_e = \left(\pi\lambda_t \frac{\partial \Theta_{\mathrm{LED}}}{\partial d_t}\right)^{-1/2} \tag{9a}$$

or

$$a_e = \lim_{\Delta d_t \to 0} \left(\pi\lambda_t \frac{\Delta \Theta_{\mathrm{LED}}}{\Delta d_t}\right)^{-1/2}. \tag{9b}$$

For more precise calculations the radius a_c of the top contact in Eqs. (4)–(6) should be replaced by the effective radius a_e.

4. The position-dependent thermal spreading resistance of the heat-sink in the case of the uniform heat flux $q_{HS}(r)$

Let us consider a semi-infinite region $z \geqslant 0$, $r \geqslant 0$, into which the uniform heat flux of the density q_e is flowing from a circle of the radius a_e. The circle is situated in the plane $z = 0$ (Fig. 3a). The remaining area of the plane $z = 0$, i.e., for $r > a_e$, is assumed to be thermally isolated. In this case, the heat spreading is governed by the thermal conduction equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = 0 \tag{10}$$

with the following boundary conditions:

$$T(r = \infty) = 0, \tag{11}$$

$$T(z = \infty) = 0, \tag{12}$$

$$-\lambda \frac{\partial T}{\partial z}\bigg|_{z=0} = \begin{cases} q_e & \text{for} \quad r \in \langle 0, a_e \rangle \\ 0 & \text{for} \quad r > a_e \end{cases} \tag{13}$$

where T — temperature, and λ — thermal conductivity of the heat-sink material.

Let us apply the Hankel transform of the temperature

$$H_0[T(r)] \equiv V(\sigma) = \int_0^\infty rJ_0(\sigma r)T(r)\,dr \tag{14}$$

where J_0 is the zero order Bessel function of the first kind.

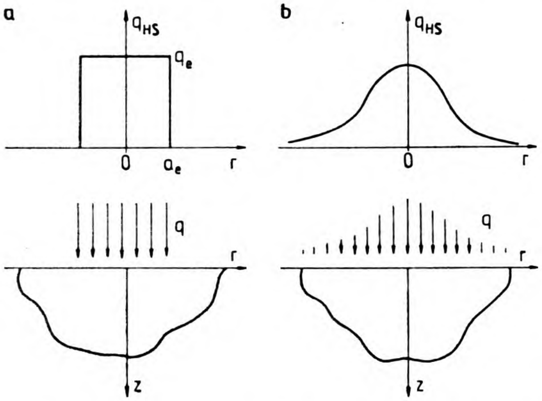

Fig. 3. The flow of the heat flux generated in the active region of a light-emitting diode into the semi-infinite heat-sink: the uniform heat flux density $q(r) = q_e$ within the circle of the radius a_e and the zero heat flux outside the circle (a), the Gaussian shape of the heat flux density (b)

The formulae (10)–(13) are then transformed into

$$\frac{d^2V}{dz^2} - \sigma^2 V = 0, \tag{15}$$

$$\lim_{z\to\infty} V = 0, \tag{16}$$

$$-\lambda \frac{dV}{dz}\bigg|_{z=0} = \frac{q_e a_e}{\sigma} J_1(\sigma a_e) \tag{17}$$

where the integral (A1) is taken from the Appendix, J_1 is the first order Bessel function of the first kind.

The solution of the above mathematical problem may be presented in a form

$$V(\sigma, z) = \frac{a_e q_e}{\lambda\sigma^2} J_1(\sigma a_e) e^{-\sigma z}. \tag{18}$$

On the other hand, the solution of the Eq. (10), with the boundary conditions

[(11)–(13)], found from (18) by the inverse transformation

$$T(r,z) = \int_0^{\infty} \sigma J_0(\sigma r) V(\sigma, z) d\sigma, \tag{19}$$

after an integration takes the following form [8]:

$$T_1(r, z=0) = \frac{a_e q_e}{\lambda} \Omega(r) \tag{20}$$

with

$$\Omega(r) = \begin{cases} 1 & \text{for } r = 0 \\ F(1/2, -1/2, 1, r^2/a_e^2) & \text{for } r \in \langle 0, a_e \rangle \\ 2/\pi & \text{for } r = a_e \\ \dfrac{a_e}{2r} F(1/2, 1/2, 2, a_e^2/r^2) & \text{for } r \geqslant a_e \end{cases} \tag{21}$$

where F is the hypergeometric function [9]. In the above calculations we have used the integrals (A3), (A5) and (A6) from the Appendix. The hypergeometric function F may be given in the form of a following series [9]:

$$F(a, b; c; z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)} \frac{z^n}{n!}, \tag{22a}$$

where Γ is the gamma function, or simply by the expression

$$F(a, b; c; z) = 1 + \frac{ab}{c \cdot 1} + \frac{a(a+1)b(b+1)}{c(c+1)1\cdot 2} z^2 + \frac{a(a+1)(a+2)b(b+1)(b+2)}{c(c+1)(c+2)1\cdot 2\cdot 3} z^3 + \ldots \, . \tag{22b}$$

The mean temperature within the circle of the radius a_e is in turn equal to

$$T_{m_1} = \frac{4\int\limits_0^{\pi/2} d\varphi \int\limits_0^{a_e} T_1(r, z=0) r dr}{4\int\limits_0^{\pi/2} d\varphi \int\limits_0^{a_e} r dr} = \frac{2q_e}{\lambda} \int_0^{\infty} J_1^2(\sigma a_e) \frac{d\sigma}{\sigma^2}. \tag{23}$$

Using the integral (A4) from the Appendix, we obtain finally the following relation [2]:

$$T_{m_1} = \frac{8 a_e q_e}{3\pi\lambda}. \tag{24}$$

For the heat flux $Q = 0.175$ W, what corresponds to a current of 250 mA, and the copper heat-sink ($\lambda = 400$ W/mK), the temperature $T_1(r, z = 0)$ as well as the mean temperature T_{m_1} are plotted in Fig. 4 for three values of the radius a_e (50 μm, 75 μm and 100 μm).

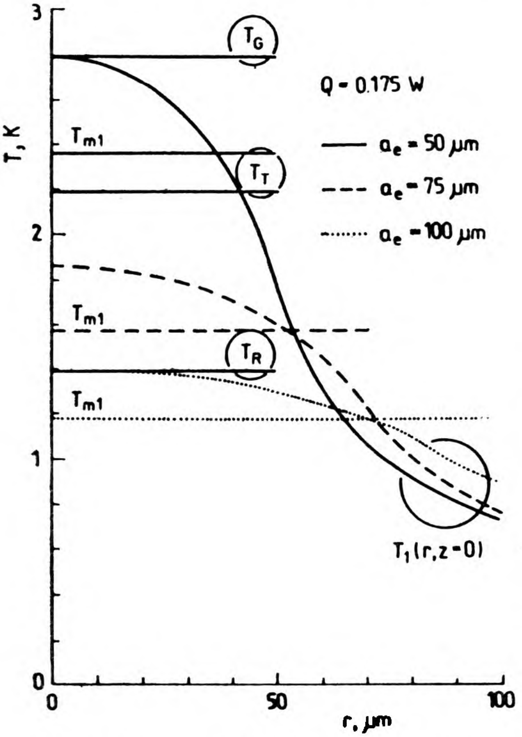

Fig. 4. The position-dependent temperature increases in the semi-infinite copper heat-sink for the uniform heat flux density q_e within the circle of the radius a_e. The curves have been plotted for the heat flux $Q = 0.175$ W (which corresponds to a current 250 mA) and for three values of a_e: 50μm, 75 μm and 100 μm. T_{m_1} – the mean temperature inside the circle. The lines denoted by symbols in circles show the mean temperature calculated (for $a_e = 50$ μm) with the aid of the hitherto known formulae (3), (5) and (6) – see subscripts

The resultant thermal spreading resistance, i.e., the position-dependent resistance $\Theta_1(r)$ and the mean resistance Θ_{m1}, obtained immediately from the relations (20) and (24), respectively, take the following forms:

$$\Theta_1(r) = \frac{1}{\pi a_e \lambda}\,\Omega(r), \tag{25}$$

$$\Theta_{m_1} = \frac{8}{3\pi^2 a_e \lambda}. \tag{26}$$

5. The position-dependent thermal spreading resistance of the heat-sink in the case of the Gaussian shape of the heat flux q_{HS} (r)

For more precise calculations the heat flux flowing into the heat-sink should be assumed to be of position-dependent Gaussian shape (Fig. 3b) [6]:

$$q_{\mathrm{HS}}(r) = q_A \exp\left(-\frac{r^2}{b^2}\right) \tag{27}$$

in contrast to the previous section where a uniform heat flux through the circle radius was considered. In Eq. (27), q_A may be determined by means of the power balance (using the integral (A2) from the Appendix)

$$Q = \int_0^{2\pi} d\varphi \int_0^{r_s} r q_A \exp\left(-\frac{r^2}{b^2}\right) dr, \tag{28}$$

giving

$$q_A = \frac{Q}{\pi b^2 [1 - \exp(-r_s^2/b^2)]} \approx \frac{Q}{\pi b^2} \tag{29}$$

where Q is the power of the heat source, r_s is the radius of the diode structure and the parameter b depends on the current spreading between the top contact and the active region as well as on the heat flux spreading between the active region and the heat-sink.

The problem reduces to solving the thermal conduction Eq. (10) with the boundary conditions:

$$T(r = \infty) = 0, \tag{30}$$

$$T(z = \infty) = 0, \tag{31}$$

$$-\lambda \frac{\partial T}{\partial z}\bigg|_{z=0} = q_A \exp\left(-\frac{r^2}{b^2}\right). \tag{32}$$

After the Hankel transformation (14), the condition (32) takes the following form:

$$-\lambda \frac{dV}{dz}\bigg|_{z=0} = q_A \exp\left(-\frac{1}{4}\sigma^2 b^2\right)\frac{b^2}{2}, \tag{33}$$

which has been derived using the integral (A9) from the Appendix. Then the solution of Eq. (15) with the boundary conditions (16) and (33) is given by

$$V(\sigma, z) = \frac{b^2 q_A}{2\lambda\sigma} \exp\left(-\frac{1}{4}\sigma^2 b^2 - \sigma z\right). \tag{34}$$

Substituting the above relation (34) into the inverse transformation formula (19) and using the integral (A7) from the Appendix, we get the following expression for the temperature at the plane $z = 0$ [10]:

$$T_2(r, z = 0) = \frac{\sqrt{\pi} b q_A}{2\lambda} \zeta(r), \tag{35}$$

with

$$\zeta(r) = \exp\left(-\frac{1}{2}\frac{r^2}{b^2}\right) I_0\left(\frac{1}{2}\frac{r^2}{b^2}\right) \tag{36}$$

where I_0 is the modified Bessel function of the zero order.

Using the integral (A8) from the Appendix, the mean temperature inside the circle of the radius a_e may be calculated by integrating (see Eq. (23)), to give

$$T_{m2} = \frac{\sqrt{\pi}\, b q_A}{2\lambda} M(1/2, 2, -a_e^2/b^2) \tag{37}$$

where M is the Kummer's confluent hypergeometric function [11] which may be expressed as follows:

$$M(a, b, z) = 1 + \frac{a}{b}\frac{z}{1!} + \frac{a(a+1)}{b(b+1)}\frac{z^2}{2!} + \frac{a(a+1)(a+2)}{b(b+1)(b+2)}\frac{z^3}{3!} + \ldots \,. \tag{38}$$

For the same heat flux $Q = 0.175$ W, as in the previous example, and the copper heat-sink, the temperature $T_2(r, z = 0)$ and the mean temperatures T_{m2} are plotted in Fig. 5 for various values of the parameter b.

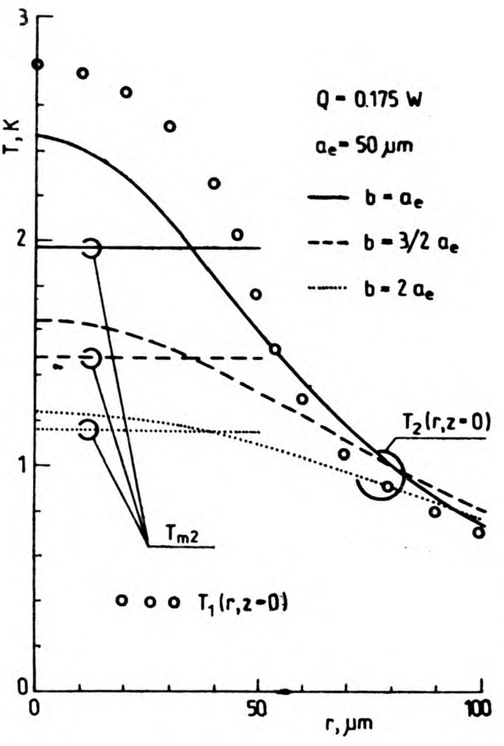

Fig. 5. The position-dependent temperature increases in the semi-infinite copper heat-sink for the Gaussian shape of the heat flux denisty $q_{HS}(r)$. Solid, dashed and dotted lines correspond to $b = a_e$, $b = 3/2\, a_e$ and $b = 2a_e$, respectively. The curves have been calculated for the same heat flux $Q = 0.175$ W as in the previous case (see Fig. 4). In this figure the curve $T_1(r, z = 0)$ (small circles) for the uniform heat flux density q_e within the circle of the radius a_e is shown for comparison

The position-dependent thermal spreading resistance $\Theta_2(r)$ and the mean thermal resistance Θ_{m2} of the semi-infinite heat-sink, obtained from the Eqs.

(35) and (37), respectively, may be expressed in the following forms:

$$\Theta_2(r) = \frac{1}{2\sqrt{\pi b \lambda}} \zeta(r), \tag{39}$$

$$\Theta_{m2} = \frac{1}{2\sqrt{\pi b \lambda}} M(1/2, 2, -a_e^2/b^2). \tag{40}$$

6. Comparison of the models

Main results of the calculations for the standard construction of a light-emitting diode and the heat flux $Q = 0.175$ W are shown in Fig. 4 and Fig. 5. These figures represent the respective temperature increase distributions in the semi-infinite copper heat-sink, for the uniform heat flux density q_e within the circle of the radius a_e, and for the Gaussian shape of the heat flux density $q_{HS}(r)$.

It is shown that within the circle the rise of temperature varies considerably, e.g., for $a_e = 50 \mu m$ (Fig. 4) it changes by about 1 K, whereas the total increase is less than 3 K. The distribution of the temperature increase in the heat-sink is a strong function of the radius a_e, i.e., it depends to a large extent on two phenomena in a light-emitting diode: the current-spreading effect between the top contact and the active region as well as the heat flux spreading effect between the active region (a heat source) and the heat-sink.

The assumption of the Gaussian shape of the heat flux density $q_{HS}(r)$ flowing into the heat-sink is more reasonable than that of the uniform heat flux density within the circle of the radius a_c or even than the assumption of the effective uniform heat flux density q_e within the circle of the radius a_e. In the last case, only the mean temperature within the circle is calculated exactly, whereas the temperature distribution may be inaccurate. The influence of the parameter b, describing the Gaussian function (see Eq. (27)), on the distribution of the temperature increase in the heat-sink is shown in Fig. 5.

A more general case of the laser beam induced temperature rise in a semi-infinite solid has been analysed by Lax [12]. The solution for a general laser intensity distribution is specified for the case of a Gaussian beam.

7. Conclusions

In this paper the exact formulae for the thermal spreading resistance of the heat-sink in a light-emitting diode have been derived for two cases:

i) the position-dependent spreading resistance,

ii) the mean spreading resistance.

The problem has been solved by means of the Hankel transform for two cases of distributions $q_{HS}(r)$ of the heat flux flowing into the heat-sink:

i) the uniform heat flux density within the given circle and the zero flux outside it,

ii) the Gaussian shape of the heat flux density.

The calculation method for the effective, uniform heat flux distribution has been shown.

Appendix

The integrals useful in the analysis of the heat-sinking in light-emitting diodes [13]:

5.52.1

$$\int x^{p+1} J_p(x)\,dx = x^{p+1} J_{p+1}(x), \tag{A1}$$

3.461.3

$$\int_0^\infty x^{2n+1} e^{-px^2}\,dx = \frac{n!}{2p^{2+1}}, \qquad p > 0, \tag{A2}$$

6.561.17

$$\int_0^\infty \frac{J_u(ax)}{x^{u-q}}\,dx = \frac{\Gamma\left(\frac{1}{2}q+\frac{1}{2}\right)}{2^{u-q}a^{q-u+1}\Gamma\left(u-\frac{1}{2}q+\frac{1}{2}\right)}, \qquad -1 < q < u-1/2 \tag{A3}$$

6.575.2

$$\int_0^\infty \frac{J_n(x)J_m(x)}{x^{n+1}}\,dx = \frac{\sqrt{\pi}\,\Gamma(n+m)}{2^{n+m}\Gamma\left(n+m+\frac{1}{2}\right)\Gamma\left(n+\frac{1}{2}\right)\Gamma\left(m+\frac{1}{2}\right)}, \quad n+m > 0 \tag{A4}$$

6.574.2

$$\int_0^\infty J_n(at)J_m(at)t^{-l}\,dt = \frac{a^{l-1}\Gamma(l)\Gamma\left(\frac{n+m-l+1}{2}\right)}{2^l\Gamma\left(\frac{-n+m+l+1}{2}\right)\Gamma\left(\frac{n+m+l+1}{2}\right)\Gamma\left(\frac{n-m+l+1}{2}\right)} \tag{A5}$$

6.574.1

$$\int_0^\infty J_n(at)J_m(bt)t^{-l}dt$$

$$= \frac{a^n \Gamma\left(\frac{n+m-l+1}{2}\right)}{2^l b^{n-l+1}\Gamma\left(\frac{-n+m+l+1}{2}\right)\Gamma(n+1)}$$

$$\times F\left(\frac{n+m-l+1}{2}, \frac{n-m-l+1}{2}; n+1; \frac{a^2}{b^2}\right),$$

$$n+m-l+1 > 0, l > -1, 0 < a < b \qquad \text{(A6)}$$

6.618.1

$$\int_0^\infty \exp(-ax^2)J_n(bx)dx$$

$$= \frac{\sqrt{\pi}}{2\sqrt{a}}\exp\left(-\frac{b^2}{8a}\right)I_{n/2}\left(\frac{b^2}{8a}\right), \quad a > 0, b > 0, n > -1 \qquad \text{(A7)}$$

6.631.1

$$\int_0^\infty x^m \exp(-ax^2)J_n(bx)dx$$

$$= \frac{b^n \Gamma\left(\frac{n}{2}+\frac{m}{2}+\frac{1}{2}\right)}{2^{n+1}a^{1/2(m+n+1)}\Gamma(n+1)} M\left(\frac{n+m+1}{2}, n+1, -\frac{b^2}{4a}\right),$$

$$a > 0, \ m+n > -1, \ b > 0 \qquad \text{(A8)}$$

6.631.4

$$\int_0^\infty x^{n+1}\exp(-ax)^2 J_n(bx)dx = \frac{b^n}{(2a)^{n+1}}\exp\left(-\frac{b^2}{4a}\right), \ a > 0, \ b > 0, \ n > -1.$$

References

[1] NAKWASKI W., KONTKIEWICZ A. M., [in] *Radiative recombination and related phenomena in III–V compound semiconductors*, Prague 1983, B15.

[2] CARSLAW H. S., JAEGER J. C., *Conduction of heat in solids*, Clarendon Press, Oxford 1959, p. 216.

[3] GIBBONS G., Solid-St. Electron. **13** (1970), 799.

[4] DIEBOLD E. J., Trans. AIEE, Part I, Commun. Electron. **76** (1957), 593.
[5] Ref. [2], p. 217.
[6] BROZI A., NAKWASKI W., in preparation.
[7] LAFF R. A., COMERFORD L. D., CROW J. D., BRADY M. J., Appl. Opt. **17** (1978), 778.
[8] NAKWASKI W., [in] *Radiative recombination and related phenomena in III–V compound semiconductors,* Prague 1983, B14.
[9] *Handbook of mathematical functions,* Ed. by M. Abramowitz and I. A. Stegun, Dover Publ., Inc., New York, p. 556.
[10] NAKWASKI W., Solid-St. Electron., in print.
[11] Ref. [9], p. 504.
[12] LAX M., J. Appl. Phys. **48** (1977), 3919.
[13] GRADSHTEJN I. S., RYZHIK I. M., *Tablitsy integralov, summ, ryadov i proizvedenii* (in Russian), Gosudarstvennoe Izdatelstvo Fiziko-Matematicheskoi Literatury, Moskva 1962.

Received January 19, 1984

Процесс теплоотвода в электролюминесцентных диодах

В статье произведен анализ распределения теплоты в корпусе приборов с цилиндрической симметрией, особенно электролюминесцентиых диодов с поверхностной эмиссией. На основе этого анализа выведены зависимости, определяющие термическое сопротивление корпуса: термическое сопротивление, которое зависит от положения, а также среднее термическое сопротивление, для двух случаев: а) однородного теплового потока, проникающего в корпус через круг с радиусом a_e, а также б) гауссового распределения плотности этого потока. Решение уравнения теплопроводности получено для обоих случаев с помощью преобразования Ганкеля. Представлен, кроме того, метод определения эффективного однородного распределения плотности мощности обсуждаемого теплового потока.

Перевела Малгожата Хейдрих