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Mode propagation in a perturbed slab dielectric waveguide*

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A simple perturbation method for prediction of the mode propagation constant shift and the power loss due to the change of any waveguide parameter is introduced. Formula to evaluate the mode losses is derived and the applicability limitations of the method are discussed.

1. Introduction

The problems of mode propagation, power loss and mode to mode coupling in non-perfect slab dielectric waveguides have been seriously considered since the very origin of integrated optics. Among many works dealing with this subject [1–18] there are books and papers by Marcuse [16-18] devoted to the theory of mode coupling in perturbed waveguides in terms of local normal modes and in terms of normal modes of an ideal guide. These approaches are nowadays considered to be the best tools of integrated optics. They are, however, unconvenient for fast handy estimation of the waveguide quality.

A very simple approach to the problem of power loss in a waveguide of lossy materials was reported by Kane and Osterberg [19]. These authors treated the loss tangents of waveguide materials as a component perturbing their refractive indices, deriving in this way the perturbation method to evaluate the power loss of waveguide modes.

An analogical perturbation method was applied by Ulrich [20] to derive the theory of a prism coupler. The presence of the prism was treated there as the perturbation of the refractive index of cover material.

These examples lead to the conclusion that a simple perturbation method to evaluate power loss and guiding properties of a waveguide with an arbitrary parameter perturbed can be derived. Such a method will be presented in this paper.

2. Fundamentals of the theory of an ideal waveguides

Let us consider an ideal waveguide (Fig. 1) that consists of a guiding dielectric film of width $2l$ and bulk refractive index n_f sandwiched between substrate and cover with lower refractive indices n_s and n_c. The width $2l$ is of the same order of magnitude as the guided light wavelength.

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Light distribution in such a structure, once a monochromatic beam has been coupled into it, can be described as a result of the superposition of two plane waves incident at the film-substrate and the film-cover boundaries at an angle θ (Fig. 1). The phase factors of these waves are

$$\exp[-ik_0 n_f(\pm x\cos\theta + z\sin\theta)]. \tag{1}$$

Then the wave resulting in their superposition is travelling down the guide with a propagation constant

$$\beta = k_0 n_f \sin\theta \tag{2}$$

where k_0 denotes the wavenumber in vacuum. In analogy to the classical optics relation (2) can be rewritten as

$$\beta = k_0 N \tag{3}$$

where $N = n_f \sin\theta$ is called the waveguide effective refractive index for the wave with propagation constant β.

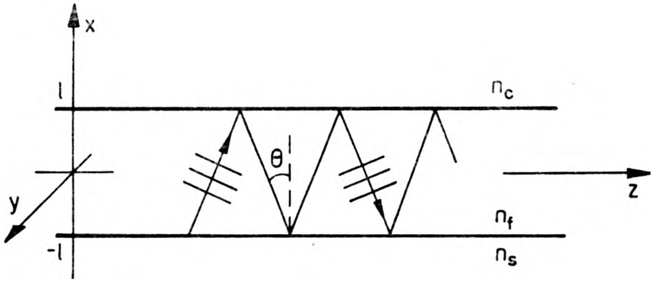

Fig. 1. An ideal dielectric slab waveguide – choice of coordinates

The condition of light confinement in the waveguiding structure is that the angle θ should be greater than both the critical angles for total internal reflection at the film-substrate and the film-cover boundaries.

The condition of phase consistency is expressed by well-known transversal resonance conditions or by the waveguide characteristic equations, which can be derived directly from the Maxwell's equations [18]. They have different forms for TE and TM modes:

$$\text{TE}: 4k_0 l\sqrt{n_f^2 - N^2} - 2\tan^{-1}\sqrt{\frac{N^2 - n_c^2}{n_f^2 - N^2}} - 2\tan^{-1}\sqrt{\frac{N^2 - n_s^2}{n_f^2 - N^2}} = 2\,m\pi, \tag{4}$$

$$\text{TM}: 4k_0 l\sqrt{n_f^2 - N^2} - 2\tan^{-1}\left(\sqrt{\frac{N^2 - n_c^2}{n_f^2 - N^2}}\,\frac{n_f^2}{n_c^2}\right) - 2\tan^{-1}\left(\sqrt{\frac{N^2 - n_s^2}{n_f^2 - N^2}}\,\frac{n_f^2}{n_s^2}\right) = 2\,m\pi,$$

$m = 0, 1, 2 \ldots$.

These transcendental equations express the condition that the structure described by parameters n_f, n_s, n_c, l works as a guide for a discrete set of N_m values only. Waves with propagation constants β_m

$$\beta_m = k_0 N_m \tag{5}$$

are the guided modes of this structure.

Generally, each of the characteristic equations (4) is a function of the waveguide effective refractive index and other parameters, that changes its value by 2π, while the mode order changes by 1

$$\Psi(N_m, n_f, n_s, n_c, l) = 2m\pi. \tag{6}$$

This notation is equivalent to

$$\exp[i\Psi(N_m, n_f, n_s, n_c, l)] = 1. \tag{7}$$

Solutions of Eq. (6) or Eq. (7) form a discrete set of guided modes propagation constants

$$\{\beta_m : \beta_m = k_0 N_m, m = 0, 1, 2, \ldots\}. \tag{8}$$

3. Light propagation in a perturbed waveguide

Now, let the parameters of an ideal slab waveguide be denoted by a^0, b^0, c^0, .., and its effective refractive index by N^0, and let us cosider a given mode propagating in this structure with a propagation constant

$$\beta^0 = k_0 N^0 \tag{9}$$

being the solution of the waveguide characteristic equation

$$\exp[i\Psi(a^0, b^0, c^0, \ldots)] = 1. \tag{10}$$

If this mode is incident on a part of the waveguide, where the parameter is a slightly disturbed

$$a = a^0 + \Delta a, \quad \left|\frac{\Delta a}{a^0}\right| \leqslant 1, \tag{11}$$

then its propagation constant will be forced to change, so as to satisfy the characteristic equation of the disturbed waveguide

$$\beta = \beta^0 + \Delta\beta = \beta^0 + k_0 \Delta N = \beta^0 + k_0(A + iB), \tag{12a}$$

$$\exp[i\Psi(N^0 + \Delta N, a^0 + \Delta a, b^0, c^0, \ldots)] = \exp[i\Psi(N, a, b^0, c^0, \ldots)] = 1. \tag{12b}$$

As we have assumed that the considered perturbation is small, the characteristic function of the perturbed waveguide can be approximated by the characteristic function of the ideal guide expanded in a double Taylor series in point (N^0, a^0, b^0, ...)

$$\exp[i\Psi(N, a, b^0, c^0, \ldots)] = \exp[i\Psi(N^0, a^0, b^0, c^0, \ldots)] \times \exp\left\{i\left[\left(\frac{\partial\Psi}{\partial N}\right)_0 \Delta N + \left(\frac{\partial\Psi}{\partial a}\right)_0 \Delta a\right]\right\} \ldots \tag{13}$$

where

$$\left(\frac{\partial\Psi}{\partial N}\right)_0 = \left(\frac{\partial\Psi}{\partial N}\right)_{(N=N^0,\, a=a^0,\, b=b^0,\, \ldots)}, \quad \left(\frac{\partial\Psi}{\partial a}\right)_0 = \left(\frac{\partial\Psi}{\partial a}\right)_{(N=N^0,\, a=a^0,\, b=b^0,\, \ldots)} \tag{14}$$

are the derivatives of the characteristic function of the ideal guide taken in point (N^0, a^0, b^0, c^0, ...).

Only the zeroth and the first terms of the expansion (13) will be taken into account. As the term of the zeroth order is a characteristic function of an ideal guide and is equal to unity, then in order to satisfy Eq. (12) the first term must be also equal to unity

$$\exp\left[i\left(\frac{\partial\Psi}{\partial N}\right)_0 \Delta N + i\left(\frac{\partial\Psi}{\partial a}\right)_0 \Delta a\right] = 1. \tag{15}$$

This expression can be further approximated as follows:

$$\exp\left\{i\left[\left(\frac{\partial\Psi}{\partial N}\right)_0 \Delta N + \left(\frac{\partial\Psi}{\partial a}\right)_0 \Delta a\right]\right\} \approx \left[1 + i\left(\frac{\partial\Psi}{\partial N}\right)_0 \Delta N\right]\left[1 + i\left(\frac{\partial\Psi}{\partial a}\right)_0 \Delta a\right] = 1. \tag{16}$$

Substituting $\Delta N = A + iB$ into (16) (see Eq. (12b)) and comparing its real and imaginary parts we obtain a set of equations

$$\left.\begin{aligned} \left(\frac{\partial\Psi}{\partial N}\right)_0 A + \left(\frac{\partial\Psi}{\partial a}\right)_0 \left(\frac{\partial\Psi}{\partial N}\right)_0 \Delta a B &= 0, \\ \left(\frac{\partial\Psi}{\partial a}\right)_0 \Delta a + \left(\frac{\partial\Psi}{\partial N}\right)_0 B - \left(\frac{\partial\Psi}{\partial a}\right)_0 \left(\frac{\partial\Psi}{\partial N}\right)_0 \Delta a A &= 0, \end{aligned}\right\} \tag{17}$$

from which we can derive

$$\left.\begin{aligned} A &= -\frac{\left[\left(\frac{\partial\Psi}{\partial a}\right)_0 \Delta a\right]^2}{\left(\frac{\partial\Psi}{\partial N}\right)_0}, \\ B &= -\frac{\left(\frac{\partial\Psi}{\partial a}\right)_0 \Delta a}{\left(\frac{\partial\Psi}{\partial N}\right)_0 \left[1 + \left(\frac{\partial\Psi}{\partial a}\right)_0^2 (\Delta a)^2\right]}. \end{aligned}\right| \tag{18}$$

Since we have

$$\left(\frac{\partial \Psi}{\partial a}\right)_0 \Delta a \leqslant 1,$$

then the set of Eqs. (18) can be finally rewritten as

$$A = -\frac{\left[\left(\frac{\partial \Psi}{\partial a}\right)_0 \Delta a\right]^2}{\left(\frac{\partial \Psi}{\partial N}\right)_0}, \tag{19a}$$

$$B = -\frac{\left(\frac{\partial \Psi}{\partial a}\right)_0 \Delta a}{\left(\frac{\partial \Psi}{\partial N}\right)_0}. \tag{19b}$$

These expressions determine the propagation constant of the considered mode in a perturbed waveguide and allow us to describe its field distribution as

$$E = E_0 \exp(-k_0 A z) \exp[i(\beta^0 + k_0 B) z] \tag{20}$$

where A is the attenuation constant and B – the propagation constant shift.

As it follows from Eqs. (19) the mode propagation in a perturbed waveguide can be determined if only the ideal waveguide characteristic function and the value of deflection of the perturbed parameter are known. Let us note that the Ψ derivative over N being always negative, the mode attenuation due to the waveguide imperfections is always greater than 0, while the sign of the propagation constant shift depends on the sign of the characteristic function derivative over the disturbed parameter. Both the expressions (19a), (19b) depend on the mode order through the characteristic function derivatives, the analytical forms of which are functions of the waveguide effective refractive index.

4. Discussion

We considered a single mode propagation in an ideal waveguide and incident on its imperfect section, where one of the waveguide parameters was slightly disturbed. Further propagation of light in a perturbed waveguide is described by Eq. (20). We can say that the disturbance of the waveguide caused the power loss given by factor A (Eq. (19a)) and the guided mode adjustment to the disturbed waveguide given by factor B (Eq. (19b)).

Equations (19a), (19b) and (20) were derived under three assumptions:

i) the characteristic function of an imperfect waveguide can be approximated by the expansion of the characteristic function of an ideal guide,

ii) in the disturbed waveguide a mode of the same order and polarization but with a changed propagation constant will be supported,

iii) the imaginary part, responsible for the mode attenuation, will appear in the new propagation constant.

These assumptions yield the following conditions limiting the applicability of Eq. (19a), (19b) and (20):

i) the deflection of the disturbed parameter should be very small and change slowly,

ii) the band of spatial frequencies of the perturbation should be limited in order not to cause the mode order shift,

iii) modes near the *cut-off* cannot be analysed, since they may be cut off by even a very small imperfection of guiding structure.

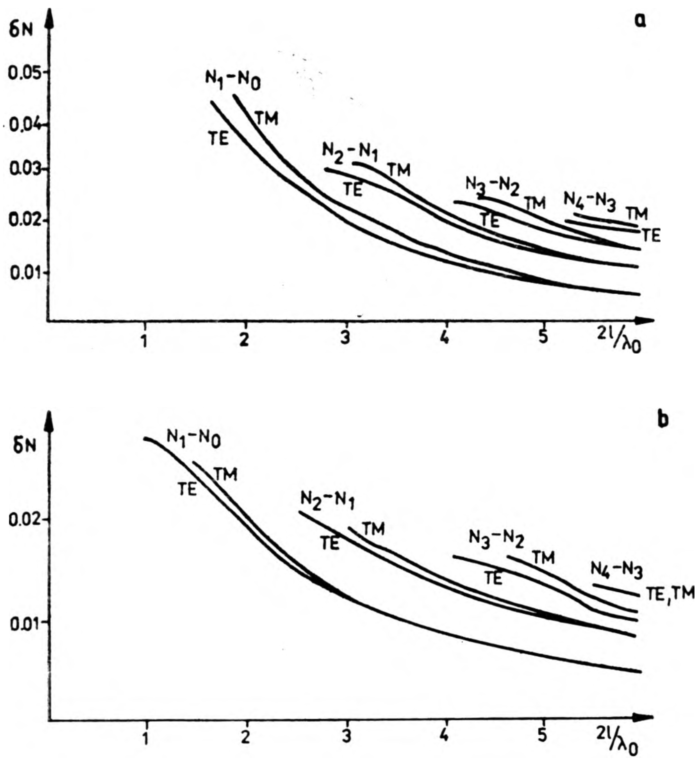

Fig. 2. Exemplary plots of $(N_m - N_{m-1})$ differences vs. $2l/\lambda_0$ for: $n_f = 1.5$, $n_s = 1.44$, $n_c = 1.0$ (a), and $n_f = 1.5$, $n_s = 1.46$, $n_c = 1.0$ (b)

The magnitudes of limitations resulting from the second condition can be estimated from the plots of $(N_m - N_{m-1})$ vs. $2l/\lambda_0$ (Fig. 2). Marcuse [18] (p. 134) derived the following approximated dependence of the difference between propagation constants of two neighbour modes on the waveguide thickness $2l$

$$\delta\beta_{m,m+1} = -\frac{2\pi^2(m+1)}{(2l)^2}. \tag{21}$$

It is very important to note that the perturbation of any waveguide parameter is projected on N by means of the guide characteristics $N(a)$ derived from the characteristic Eq. (4), see Fig. 3. Thus, the choice of the working point on the linear part of this characteristics allows us to find the distribution $N(z)$ directly from the distribution $a(z)$.

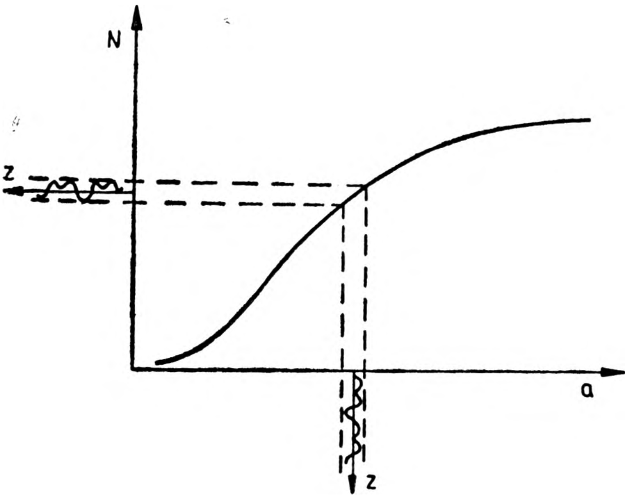

Fig. 3. The perturbed waveguide works as a modulator projecting the $a(z)$ distribution on $N(z)$

5. Power loss in a perturbed slab

The factor $\exp(-k_0 A_m z)$ in Eq. (20) describes the m-th order mode attenuation due to the waveguide imperfections. Thus, following Eq. (19a), the m-th mode constant of attenuation due to the parameter a perturbation is

$$a_m^a = -k_0 \frac{\left(\frac{\partial \Psi}{\partial a}\right)_0^2}{\left(\frac{\partial \Psi}{N}\right)_0} (\Delta a)^2, \tag{22}$$

which can be rewritten as

$$a_m^a = k_0 D_m^a (\Delta a)^2 \tag{23}$$

where factor D_m^a is a constant dependent only on the guide parameters, the order and polarization of the mode. This constant can be called a relative waveguide loss for the m-th mode.

Generally, the mode attenuation constant is a function of variable z

$$a = a(z). \tag{24}$$

The mean value of a over the distance L can be evaluated as

$$\bar{a} = \frac{1}{L} \int_{z'}^{z'+L} a(z)\, dz. \tag{25}$$

Then the field amplitude at the distance L will be

$$E = E_0 \exp(\bar{a}L), \tag{26}$$

and the power

$$P = E_0^2 \exp(2\bar{a}L), \tag{27}$$

whence

$$\ln \frac{P_0}{P} = -2\bar{a}L \tag{28}$$

As $\ln \frac{P_0}{P} = 2.303 \lg \frac{P_0}{P}$, then

$$B = \frac{10}{2.303} \ln \frac{P_0}{P} \tag{29}$$

describes the power loss in decibels on the distance L.

In view of Equation (23) and taking $f(z) = [\Delta a(z)]^2$ the mean value of attenuation constant over the distance L can be written as

$$\bar{a}_m^a = k_0 D_m^a \bar{f} \tag{30}$$

where f denotes the mean value of a function describing the square of the perturbation of the parameter a.

According to Eqs. (29) and (30) the following formula for the mode power loss over the unit distance can be derived

$$\frac{B}{L} = \frac{20}{2.303} k_0 D_m^a \bar{f}. \tag{31}$$

Thus, the loss in a perturbed waveguide can be quickly estimated due to the knowledge of the mean value of a square of the perturbation distribution.

6. Conclusions

A very simple perturbation method allowing the prediction of the mode propagation constant and its power loss in a non-perfect waveguide has been derived for the case when one waveguide parameter is perturbed. For more than one disturbed parameter an analogous method can be introduced.

The method developed is not concerned with the mode conversion and does not give the spectrum of radiation modes but only the global power loss. It is convenient for both fast handy calculations and numerical computations. The knowledge of the waveguide imperfections distribution is not required, only the average value of its square. In the first approximation, when only the real part of the propagation constant is taken into account, there occurs the mode tuning, which is in agreement with the conception of the local eigenvalue approximation.

As this method considers only small and slowly changing distributions of perturbations it may be applied to an analysis of planar optical elements, such as planar gradient lenses. Nevertheless, random distributions of waveguide inhomogeneities can be also analysed.

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References

[1] Weber H., Dunn F., Liebolt W., Appl. Opt. **12** (1973), 755–757.
[2] Neuman E. G., Nouv. Rev. d'Opt. **6** (1975), 263–271.
[3] Imai M., et al., IEEE J. Quant. Electron. **QE-13** (1977), 255–262.
[4] Walter D. J., Thin Solid Films **52** (1978), 461–476.
[5] Lutter A., Ferencz K., Thin Solid Films **57** (1979), 185–189.
[6] Ohlidal J., Narratil K., Lukes F., Thin Solid Films **57** (1979), 179–184.
[7] Kishii T., Opt. and Laser Techn. **11** (1979), 197–202.
[8] Lee W.-H., Streifer W., J. Opt. Soc. Am. **69** (1979), 1671–1676.
[9] Kishii T., Opt. and Laser Techn. **12** (1980), 99–102.
[10] Lin W.-G., IEEE Trans. Micr. Theory and Techn. **MTT-28** (1980), 339–348.
[11] Boyd T. J. M., Moshkun I., Stephenson I. M., Opt. Quant. Electron. **12** (1980), 143–158.
[12] Glass A. M., Kaminow I. P., et al., Apl. Opt. **19** (1980), 276–281.
[13] Miyanaga S., Asakura T., Atti Fondazione Giorgio Ronchi, No. 4 (1980), 449–455.
[14] Danielsen P., Yevick D., Appl. Opt. **21** (1982), 4188–4189.
[15] Harris M., Macleod H. A., Ogura S., Thin Solid Films **57** (1979), 173–178.
[16] Marcuse D., *Light transmission optics*, Van Nostrand Reinhold Co., 1972.
[17] Marcuse D. (Ed.), *Integrated optics*, IEEE Press, New York 1973.
[18] Maicuse D., *Theory of the dielectric optical waveguides*, Academic Press, New York 1974.
[19] Kane J., Osterberg H., J. Opt. Soc. Am. **54** (1964), 347–352.
[20] Ulrich R., J. Opt. Soc. Am. **60** (1970).

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