Optica Applicata, Vol. XIII, No. 4, 1983

An intensity calculation program for general rotationally symmetric lens systems*

Jarmo Alander

Department of High Energy Physics, Helsinki University, Siltavourenpenger 20C, SF-00170 Helsinki 17, Finland.

A general and efficient algorithm is introduced that solves the ray-surface intersection problem of general rotationally symmetric lens systems. The noniterative algorithm is based on a *b*-spline expansion and search tree structure of the surface profile. The ray trace data are used with the generalized Coddington equations to evaluate the intensity profile on test apertures of the lens systems.

1. Introduction

A modern optical design program should be able to handle a wide variety of elements like spherical, aspherical, Fresnel etc. lenses. However, even the most recent lens design programs are not able to handle other surfaces than spherical and perhaps some standard aspherical surfaces expressed as even Taylor series. Just a few programs are able to handle more complicated elements like Fresnel lenses and only by making some simplifying approximations.

In this work an efficient, unified and a general method to represent and process rotationally symmetric optical elements is introduced. *B*-spline surface interpolation, as first suggested by Rigler and Vogl [1] is used as a primary surface representation method. The algorithm is applied to some simple lens systems including one Fresnel lens.

2. B-splines

When we do not have any a priori knowledge of the behaviour of the functions to be approximated, the spline interpolation methods are among the best and most simple ones. The basic splines or, shortly, *b*-splines $B_{j,k}(x)$ are piecewise

* This paper has been presented at the European Optical Conference (EOC'83), May 30–June 4, 1983, in Rydzyna, Poland.

polynomials of order k, which can be defined by the following recursive equations [2]:

$$B_{j,1}(x) \doteq \begin{cases} 1, & \text{when } t_j \leqslant x < t_{j+1} \\ 0, & \text{otherwise} \end{cases},$$

$$B_{i,k}(x) = \frac{x-t_i}{t_{i+k-1}-t_i} B_{i,k-1}(x) + \frac{t_{i+k}-x}{t_{i+k}-t_{i+1}} B_{i+1,k-1}(x)$$

where t_i is a set of non-decreasing points called knots, the boundaries of the polynom pieces.

A point set (x_i, y_i) can be interpolated by a k-th order b-spline expansion by solving the linear equation

$$y_i = \sum_{j=1}^{n} a_j B_{j,k}(x_i),$$

assuming that the data points and knots are situated properly so that the equation is non-singular.

For fast access of right polynom piece, the surface is represented as a tree structure, where the leaves are the interpolating polynom pieces (see Fig. 1). The right piece can be accessed in time proportional to $\log(n)$, where n is the number of polynom pieces.

Hall has deduced the error of the spline interpolation $I_k f$ in the case $k = 3$ [3]:

$$|f - I_3 f| \leqslant 5/384\, |x|^4 \, \|f^{(4)}\|$$

where $|x|$ is the maximum length of the approximation intervals and $\|f\|$ denotes $\max_{x \in [x_{min}, x_{max}]} |f|$.

Hall and Meyer have also evaluated the error bounds of the derivatives of I_3 [4]:

$$e' = |f' - I'_3 f| \leqslant 1/24\, |x|^3 \, \|f^{(4)}\|,$$

and

$$e'' = |f'' - I''_3 f| \leqslant 1/8\, |x|^2 \, \|f^{(4)}\|.$$

The curvature k of a plane curve f is given by the formula

$$k = \frac{f''}{(1+f'^2)^{3/2}}.$$

From this equation we can see that a third order interpolation $I_3 f$ is enough to approximate the curvature of f continuously. Assuming that the errors e' in f' and e'' in f'' are small, using the binomial expansion and deleting factors which are of order two or higher with respect to e' and e'', we get approximately for the error of curvature ek:

$$ek \cong -3f'f''e' + (1-3/2f'^2)e''.$$

Assigning the equations of e' and e'' of the third order spline expansion, we get an approximating formula for ek:

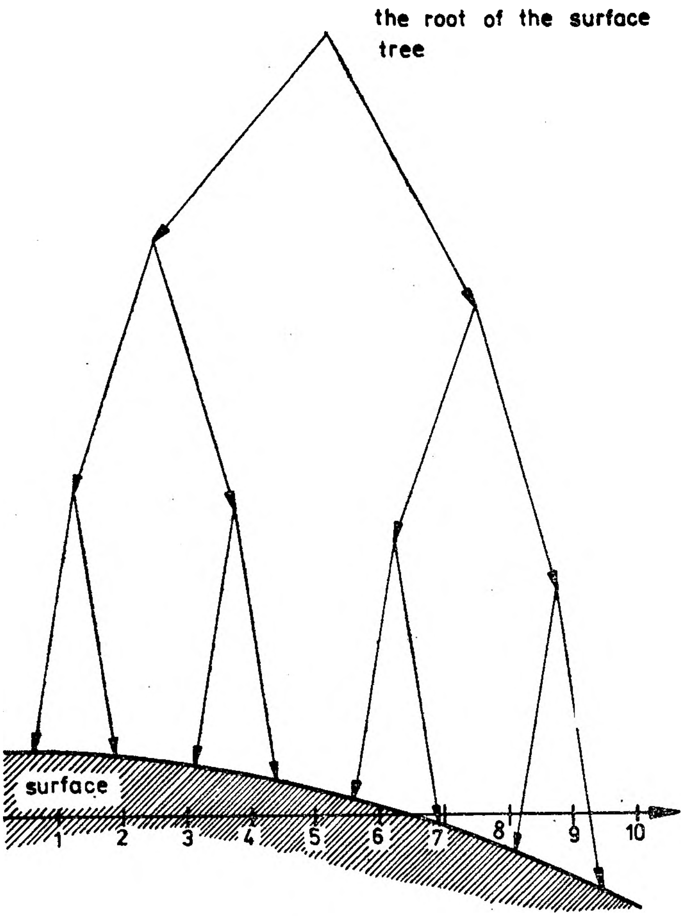

Fig. 1. The arrangement of the polynom pieces into a tree structure

$$ek \cong \left(-f'f''|x| + (1-3/2f'^2)\right)|x|^2 \|f^{(4)}\|/8 =$$
$$= \left(-f'f''|x| + (1-3/2f'^2)\right)e''.$$

For most optical surface profile derivatives $f^{(n)}$, $1 \leqslant n \leqslant 4$, are limited by some finite constant c limiting ek, too. I.e., the third order b-spline interpolation can be used to represent most optical surface profiles. We have used the third order b-splines for primary surface representation I_3f, while the second order least squares fit for I_3f is used for fast analytic ray surface intersection evaluation.

3. The generalized Coddington equations

Burkhard and Shealy have generalized the old Coddington equations, which in the geometrical approximation give the change of principal curvatures k and torsion t of the wave front in reflections and refractions. For refracted wave front the generalized Coddington equations are [5]:

$$\begin{cases} k_+(s) = hk_+(i)+ak_+, \\ k_=(s)\cos^2\varphi(s) = hk_=(i)\cos^2\varphi(i)+ak_=, \\ t(s)\cos\varphi(s) = ht(i)\cos\varphi(i)+at, \end{cases}$$

where: $h = n_i/n_s$ and $a = -h\cos\varphi_i+\cos\varphi_s$, indices i and s refer to incident and refracted rays respectively, and indices $=$ and $+$ refer to the plane of incidence and the plane perpendicular to it, respectively.

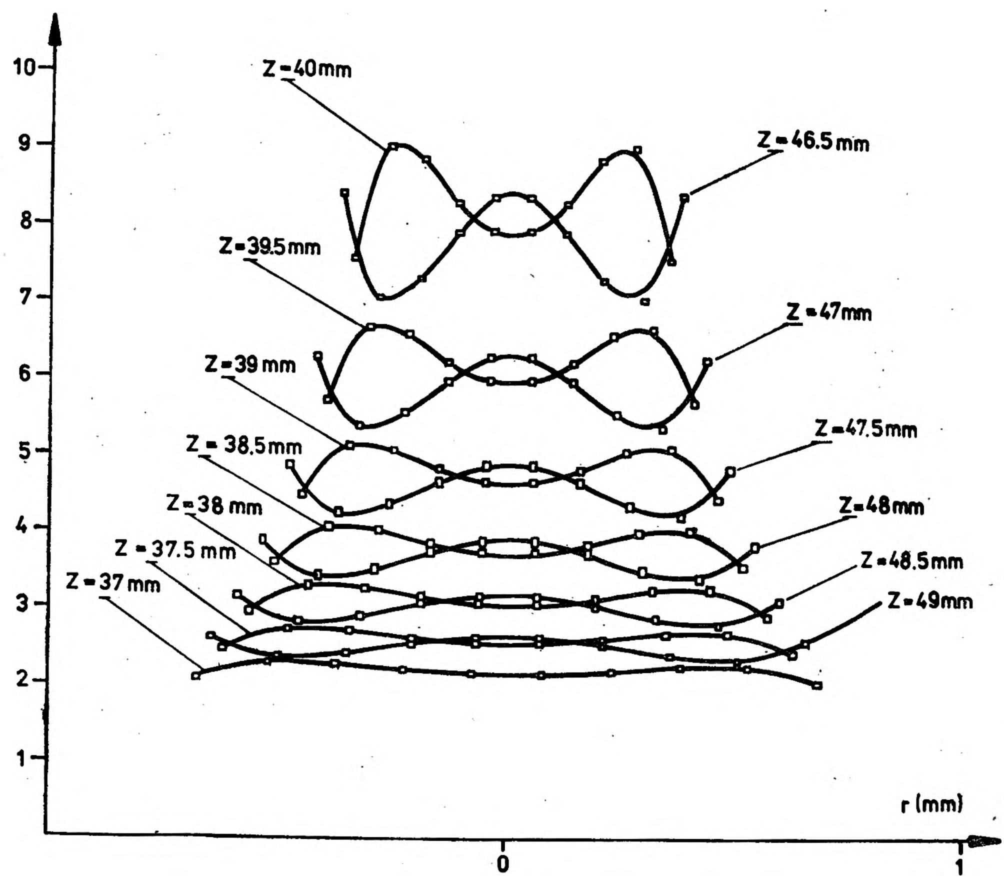

Fig. 2. Illumination profiles in a Cooke triplet system

With the help of these equations the flux can be represented as a product

$$E_n = I_n \prod_{j=1}^{n-1} \cos\varphi_i(j)/\cos\varphi_s(j)(r_{+=}/r_{+=}')(j)$$

where: $I_n = I_0 \cos\varphi_i(n)$,

$$(r_{+=}/r_{+=}')(j) = r_{+i}(j)r_{=i}(j)/r_{+s}(j)r_{=s}(j).$$

In Figure 2 there are shown the illumination profiles on some test apertures of a Cooke triplet system ([6], pp. 492–93) on the both sides of the focal plane, when the point source is located on the optical axis.

The flux is singular at the caustic surfaces where at least one of the curvatures of the wave front is zero. This causes some numerical problems because the flux varies rapidly. In Figure 3 there is an example of the flexibility of the ray tracing algorithm. On the left-hand side there is a ray trace of a parabolic lens and on the right-hand side there is the ray trace of the corresponding Fresnel lens. Every groove of the Fresnel lens is separately approximated by a b-spline expansion.

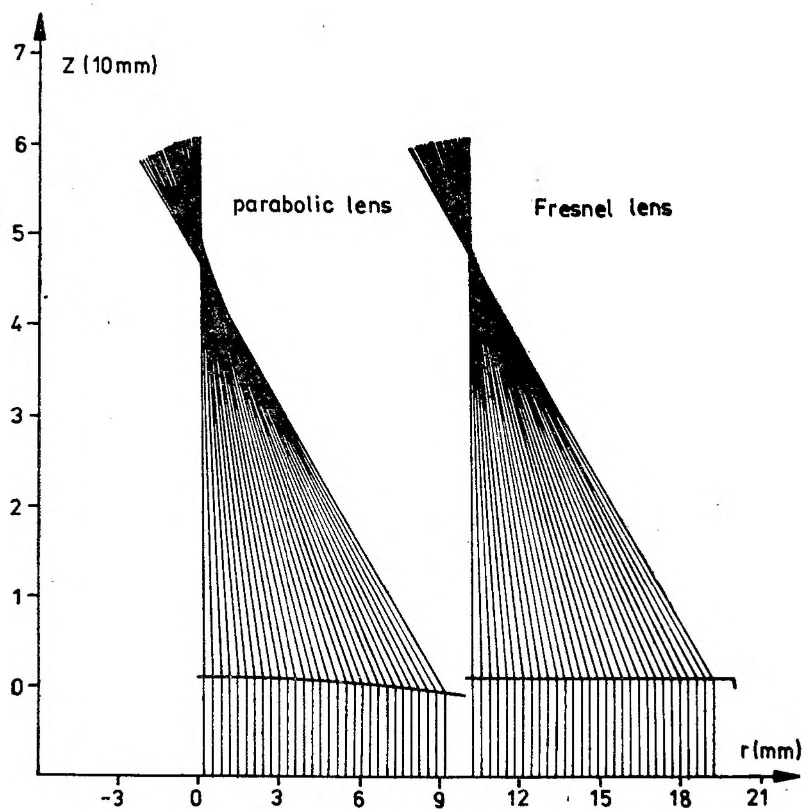

Fig. 3. A ray trace of a parabolic and a corresponding Fresnel lens

4. Summary

The test run was done by the Burroughs B7800 computer of the University of Helsinki using single precision (48 bit/word). The (unoptimized) efficiency was about 1000 ray traces/s with 50 rays/system including initialization, io etc. processing too. The algorithm was written in Extended Algol language [7].

Further work is done to generalize the algorithm to be able to handle general non-rotationally symmetric spline surfaces, too [8].

References

[1] Rigler A., Vogl T., Appl. Opt. **10** (1971), 1648–1651.
[2] De Boor C., *A practical guide to splines*, Springer-Verlag, Berlin 1978.
[3] Hall C., J. Approximation Theory **1** (1968), 209–218.
[4] Hall C., Meyer W., J. Approximation Theory **16** (1976), 105–122.
[5] Burkhard D., Shealy D., Appl. Opt. **20** (1981), 3299–3306.
[6] Levi L., *A guide to optical system design*, [in] *Applied Optics*, Vol. 1, John Wiley and Sons Ltd., New York 1968.
[7] BURROUGHS, B5000/B6000/B7000 Series ALGOL, Burroughs Corporation, Detroit 1981.
[8] Alander J., Mäntylä M., Rantanen T., *Solid modelling parametric sufraces*, [in] *Proceedings of the Eurographics*, 1983, Zagreb 1983.

Received August 25, 1983