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Modes in resonators with resonant reflectors*

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The general equation of vibrations for resonators with resonant reflectors is formulated. Solving method and an example of solved resonant system are given. The procedure proposed is rigorous and does not put any restrictions on the dimensions of resonant reflectors. In many cases this procedure allows to formulate the equations in a simple manner, using the symmetry of system examined.

1. Introduction

The frequency of laser oscillation depends on both the fluorescence emission of the laser materials and the resonant frequencies of the resonant modes. Thus, if the fluorescence of the laser materials spreads into a line width broader than the separation between adjacent frequencies of the resonance, the number of resonant modes, resonant frequencies of which are covered by the fluorescence line width, may exhibit at the same time laser oscillations. In fact, this is the case in the usual laser under normal operating conditions. To obtain a single frequency output, the laser action should take place in only one preferred mode of the resonator, all the other undesired modes must be suppressed.

One of the methods used to achieve single frequency output is that of a coupled resonator, having one or more extra reflecting surfaces in addition to the usual two-mirror structure [1, 2]. The extra surface forms another Fabry--Pérot resonator, together with the original one, so that the laser resonator becomes frequency-sensitive. If two or more plane and parallel reflecting surfaces are stacked at one end of the resonator, thus replacing output mirror a *resonator reflector* is formed which can be designed so that the region of high reflectivity is narrow. Resonant reflectors are widely used in pulsed solid-state lasers [3–7] in which the problem of mode selection had not been solved so far. This, in particular, refers to Q-switch lasers.

In some works [7, 8] the resonant reflector was treated as an element of negligible small dimensions, in relation to the total length of resonator. Such

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a treatment is admissible when the length of resonant reflector amounts to a few percent of the total resonator length. Otherwise the calculations are charged with considerable errors.

2. Eigenvalue evaluation

Let us consider a resonator formed by a mirror (the reflectance coefficient r_0) and n plane, parallel layers with various reflective indices (Fig. 1). Separate regions can be of gain or loss properties. The waves in the resonator travel in two directions: $+z$ and $-z$.

We assume that the field in the resonator has the form $e^{\gamma z}$, where γ is propagation constant

$$\gamma = \sigma + i\beta,$$

(σ — attenuation constant, $\beta = 2\pi/\lambda$).

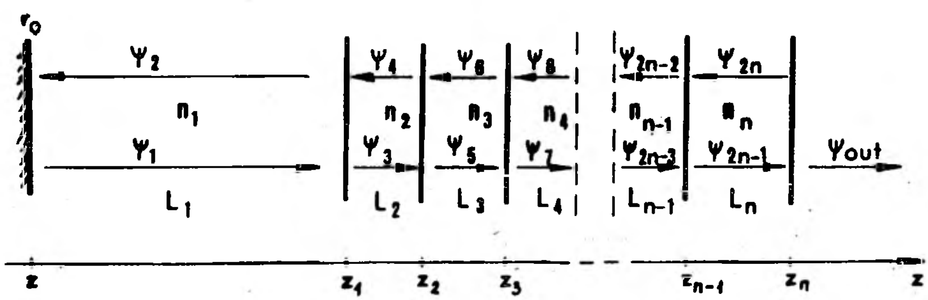

Fig. 1

Boundary conditions can be described by

$$\begin{aligned} &\Psi_{2i+1}(z \to z_i^+, 0) = A_{2i+1}, \quad \Psi_{2i+1}(z \to z_{i+1}^-, 0) = B_{2i+1}, \\ &\Psi_{2i+2}(z \to z_{i+1}^-, 0) = A_{2i+2}, \quad \Psi_{2i+2}(z \to z_i^+, 0) = B_{2i+2} \end{aligned} \tag{1}$$

where A_i and B_i are chosen so that $(A_i)^2$ and $(B_i)^2$ are the equal corresponding powers, or rewritten in the form

$$\begin{aligned}
A_1 &= A_2 r_0 \frac{1}{h_1} e^{-\gamma L_1 - i\theta_1}, \\
A_2 &= A_1 r_{12} \frac{1}{h_1} e^{-\gamma L_1 - i\theta_1} + A_4 \tau_{21} \frac{1}{h_2} e^{-\gamma L_2 - i\theta_2}, \\
A_3 &= A_1 \tau_{12} \frac{1}{h_1} e^{-\gamma L_1 - i\theta_1} + A_4 r_{21} \frac{1}{h_2} e^{-\gamma L_2 - i\theta_2}, \\
&\text{-- -- -- -- -- -- -- -- -- -- --} \\
A_{2n-1} &= A_{2n-3} \tau_{n-1n} \frac{1}{h_{n-1}} e^{-\gamma L_{n-1} - i\theta_n} + A_{2n} r_{n\,n-1} \frac{1}{h_n} e^{-\gamma L_n - i\theta_n}, \\
A_{2n} &= A_{2n-1} r_{n\,n+1} \frac{1}{h_n} e^{-\gamma L_n - i\theta_n}
\end{aligned} \tag{2}$$

where L_i — optical length of i-region,
τ_{ij} — transmission coefficient,
θ_i — constant phase shift,
$r_{ij} = (n_i - n_j)/(n_i + n_j)$ — reflection coefficient,
$(h_i)^2 = (A_i)^2/(B_i)^2$ — fractional power gain (loss) per one-way pass through i-region.

Relation (2) can be written in determinant form (3) presented on page 366 where $S_i = e^{-\gamma L_i}$, $K_i = e^{-i\theta_i}$.

Based on the energy conservation law for the wave propagating from i- to j-region the following expression can be obtained [9]

$$(r_{ij})^2 + \frac{n_j}{n_i}(\tau_{ij})^2 = 1, \text{ for } j = i \pm 1. \tag{4}$$

It can be rewritten in the form

$$(r_{ij})^2 + \tau_{ij}\tau_{ji} = 1, \text{ for } j = i \pm 1. \tag{5}$$

This relation allows us to eliminate τ_{ij}.

Putting determinant D equal to zero we get the eigenvalue equation for the resonator being considered

$$D = 0. \tag{6}$$

3. Solution

The general solution of the Equation (6) is complicated but in many cases the determinant reduces to a polynomial with integer exponents. This happens when all the lengths of resonator regions are exact multiples of some relative unit L_0

$$L_i = m_i L_0.$$

Then the Eq. (6) becomes polynomial equation of order m

$$m = \sum_{i=1}^{n} m_i. \tag{7}$$

The solution of the equation is a complex number of the form

$$z = e^{2(\gamma L_0 + i\theta)}. \tag{8}$$

and refers to the L_0-region as being a unit region. The figures which determine the parameters of the waves propagating in the resonator are complex. The plane waves can be expressed in the form

$$\Psi = A e^{i2\varphi}. \tag{9}$$

Comparing (8) and (9) we can obtain expressions for wave parameters

$$A = e^{2\sigma L_0}, \varphi = \theta + \beta L_0 = \theta + \frac{2\pi}{\lambda} L_0.$$

$$D = \left|\begin{array}{cccc|ccc}
1 & -\frac{1}{h_1} r_0 S_1 K_1 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{h_1} r_{12} S_1 K_1 & 1 & 0 & -\frac{1}{h_2} \tau_{12} S_2 K_2 & 0 & 0 & 0 \\
-\frac{1}{h_1} \tau_{12} S_1 K_1 & 0 & 1 & -\frac{1}{h_2} r_{21} S_2 K_2 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & -\frac{1}{h_{n-1}} \tau_{n-1n} S_{n-1} K_{n-1} & 1 & -\frac{1}{h_n} r_{nn-1} S_n K_n \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{h_n} r_{nn+1} S_n K_n & 1
\end{array}\right| \tag{3}$$

4. Numerical example

Some properties of resonators with one-plate resonant reflector that cannot be determined by other methods will be shown in the example below.

Assuming $n_3 = n_4 = \dots n_{n+1} = 1$ and $n_2 > n_1$, $n_2 > 1$ as shown in Fig. 1, the equation for eigenvalue becomes

$$r_0 r_{23} \frac{1}{h_1^2 h_2^2} S_1^2 S_2^2 K_1^2 K_2^2 + r_0 r_{12} \frac{1}{h_1^2} S_1^2 K_1^2 + r_{21} r_{23} \frac{1}{h_2^2} S_2^2 K_2^2 - 1 = 0.$$

Figure 2 shows the distribution of longitudinal modes as a function φ, provided that the parameters h_1 and h_2 are constant within the total spectral range. The calculations were carried out for the following data $r_0 = 1$, $L_1/L_2 = 7$, $h_1 = 5$, $h_2 = 0.2$, $\theta_1 = \theta_2 = 0$. The results are given in Table 1.

Table 1

(a)	0.828	0.982	1.130	1.184	1.130	0.982	0.828
$A = (a)^2$	0.685	0.967	1.276	1.406	1.276	0.967	0.685
$\varphi\left[\frac{\text{rad}}{2\pi}\right]$	0.074	0.219	0.360	0.500	0.640	0.781	0.926

In the calculated resonator only those modes will be generated which amplitude satisfying the condition given below

$$A_i \geqslant 1.$$

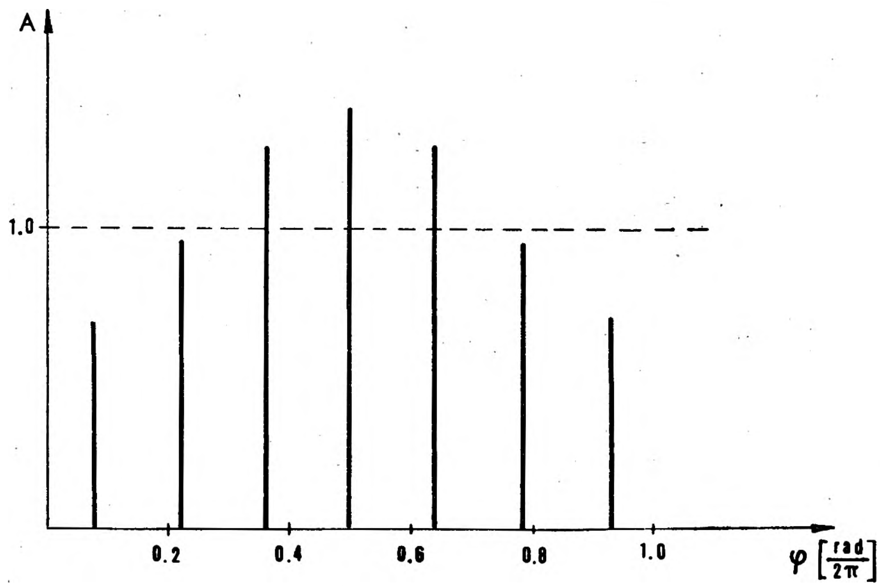

Fig. 2

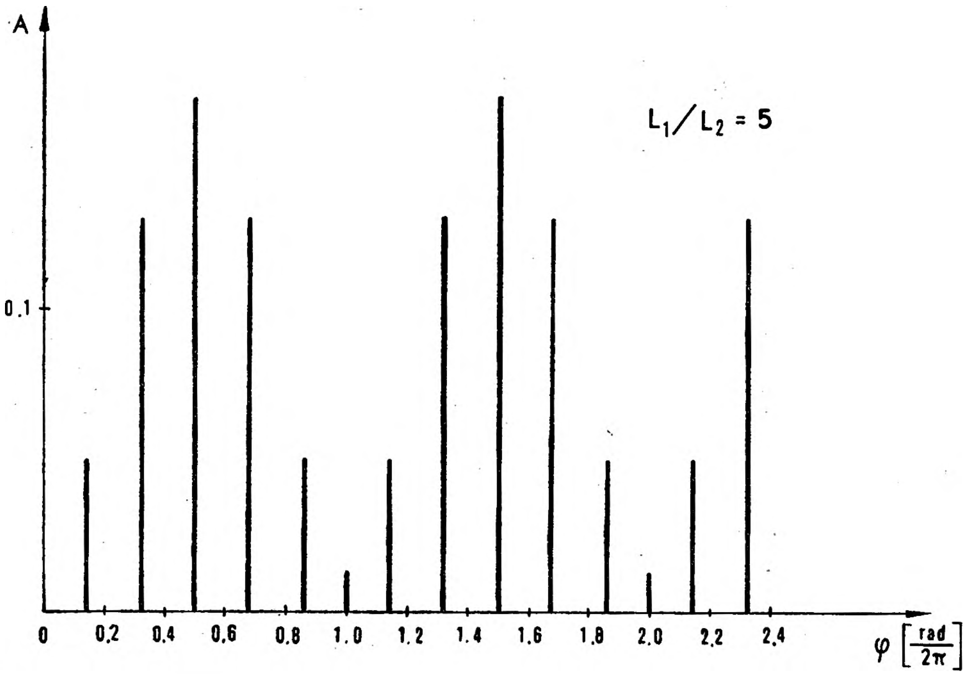
A
0.1
$L_1/L_2 = 5$
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
φ [rad/2π]

Fig. 3

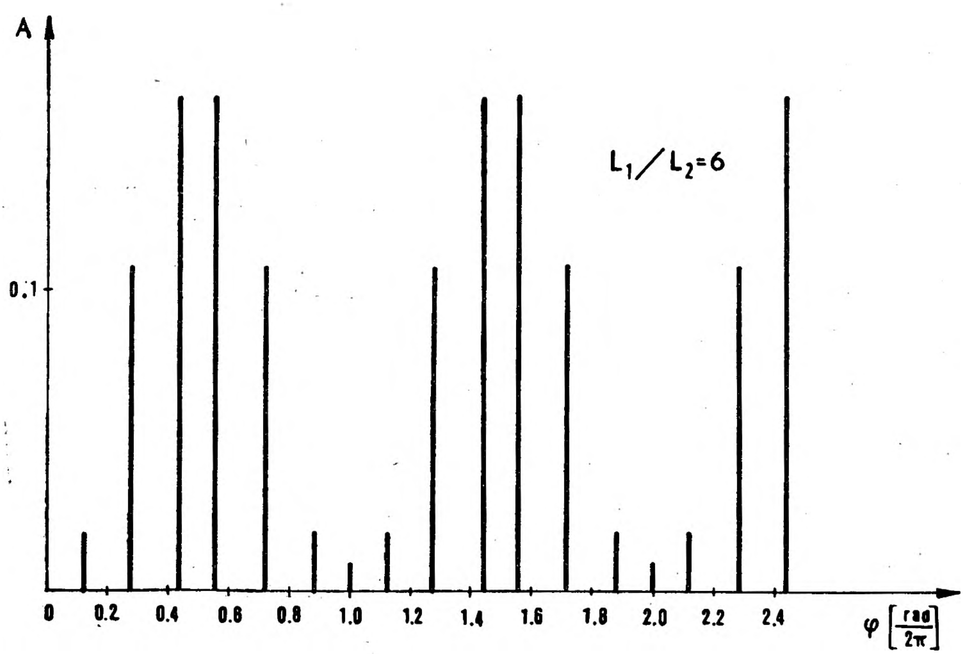
A
0.1
$L_1/L_2=6$
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
φ [rad/2π]

Fig. 4

The other modes will be damped. It should be noted that the intervals between the modes are different. Amplitude envelope of longitudinal modes resembles in the shape the curve of reflectivity coefficient of the flat plate given by [10]

$$r = \frac{r_{12}+r_{23}e^{2i\varphi}}{1+r_{12}r_{23}e^{2i\varphi}}.$$

Let us consider the influence of relation L_1/L_2 upon the distributons of the longitudinal modes. Figures 3 and 4 present the distributions for $L_1/L_2 = 5$ and $L_1/L_2 = 6$ at $r_0 = 1$.

As it results from Figure 4 the ratio L_1/L_2 being odd number is useless from the viewpoint of mode selection, because there exist two modes having the same value of their maximum amplitude. Thus in technical design such cases should be avoided.

It appears that when L_1/L_2 is an even number, the value of dominant mode amplitude depends on the value of ratio L_1/L_2. This dependence plotted on the base of the results given in Table 2 is illustrated in Fig. 5. A_{max} decreases asymptotically to a constant value being equal to the maximum value of reflection coefficient of the one-plate resonant reflector [11]

$$r_{max} = \frac{n^2-1}{n^2+1}, \text{ for } n = 1.5, r_{max} = 0.385.$$

For $L_1/L_2 \to \infty$ the resonator under consideration becomes an origin resonator in which resonant reflector plays the role of a flat mirror with appropriate reflecting characteristics.

In the method presented the dependence of A_{max} on L_1/L_2 can be explained by taking into account spatial and time relations between the waves interfering in resonator.

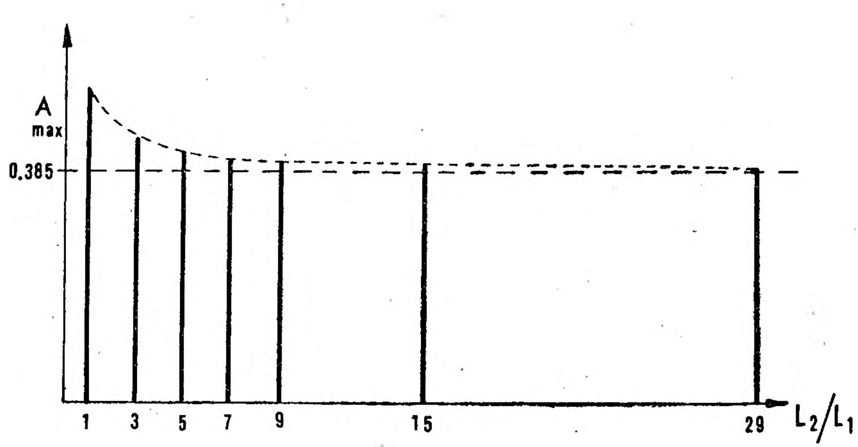

Fig. 5

Table 2

L_1/L_2	1	3	5	7	9	15	29	99
A_{max}	0.543	0.440	0.418	0.409	0.403	0.396	0.391	0.386

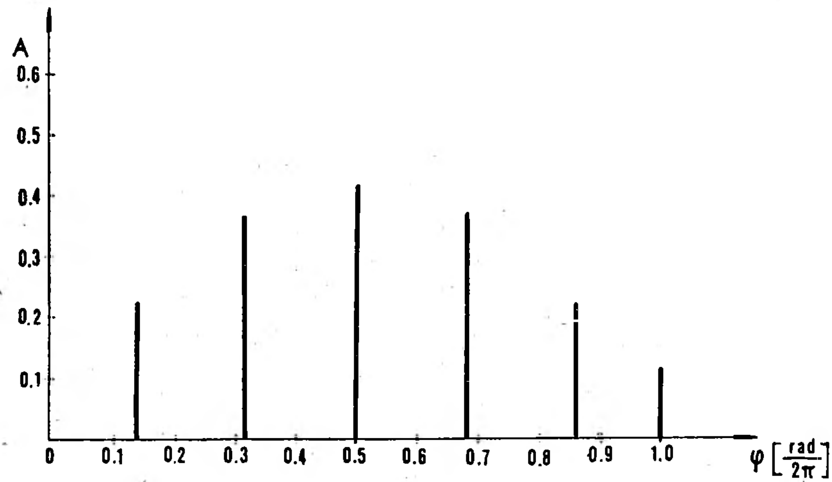

Fig. 6

Finally, let us examine the influence of changes in the length of resonator region on the distribution of longitudinal modes. The change of length L_1 by $\lambda/4$ is equivalent to the change of the sign of reflection coefficient r_0. It will cause the change of the longitudinal modes distribution. Figures 6 and 7 represent the mode distribution for $r_0 = 1$ and $r_0 = -1$, respectively.

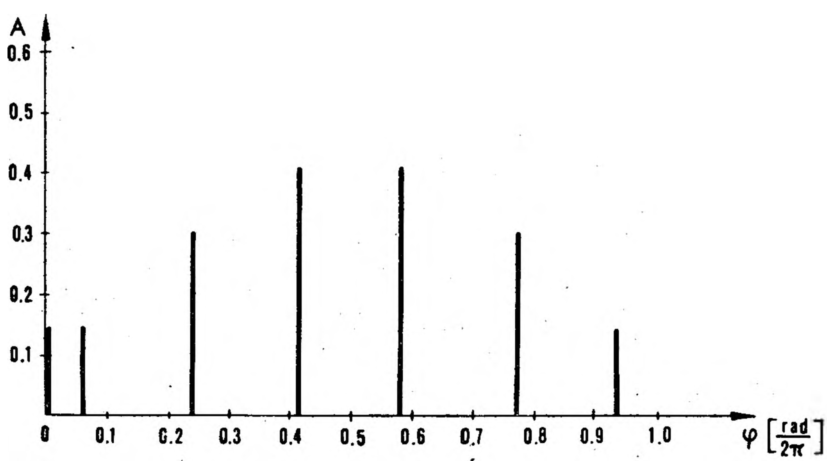

Fig. 7

From these figures it can be seen that the resonator is sensitive to variations of the lengths of its regions. A stable single-mode operation is possible if the changes of resonator regions lengths are smaller than $\lambda/4$.

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