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Polarizational optical multistability in the nonlinear Fabry-Pérot cavity*

G. G. Adonts, G. P. Djotyan, E. G. Kanetsyan

Research Institute of Condensed Media Physics, Yerevan State University, Kievian Street 1a, 375028 Yerevan, USSR.

The interaction of polarized light with nonlinear Fabry-Pérot cavity filled with resonance medium is studied. Different cases of light polarization, i.e., circular, linear, elliptic ones are investigated. It is shown that in the case of circular input light polarization the output light has the same polarization and the usual bistability takes place. When the light is linearly polarized, asymmetric as well as symmetric solutions occur at the output. The asymmetric solution is due to nonlinear interaction between different circular polarizations of a wave. This polarization results in multistability at the output of interferometer. In the case of elliptic input polarization critical values of intensity, at which ellipticity abruptly increases at output, are obtained.

1. Introduction

In the recent years the phenomenon of optical bistability, widely used in different optical devices ([1–3]) has been intensively investigated. As a rule, in investigations of bi- and multistability polarization of transmitted radiation is neglected. This makes it possible to reveal only the amplitude bistability and hysteresis. On the other hand a similar phenomenon takes place for the polarization degree of the transmitted light [4]. A theoretical analysis, based on the balance equations for the special model of two-photon resonant medium, has shown that a Fabry-Pérot cavity exhibits optical tristability for polarized incident light [5]. Polarizational optical multistability [6] and bistability, asymmetric in direction of the symmetric incident light [7], was investigated in a ring cavity. In as much as the study in [5] has been formulated in terms of population number, a phase relation which can essentially alter the real picture was not taken into account.

In this work the interaction of elliptically polarized light with nonlinear Fabry-Pérot cavity filled with resonant medium is studied and it is shown that there occur both amplitude and polarizational optical multistabilities. Linear

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polarization of a wave at the input into cavity has been analysed in detail and it should be noted that even in this case the polarizational multistability takes place. The amplitude hysteresis undergoes material changes with respect to the results of scalar theory and hysteresis of polarizational degree is obtained.

2. Theoretical analysis

Consider the interaction of elliptically polarized light with Fabry-Pérot cavity filled with a resonant medium. The resonant medium is modelled as a two-level atomic system. In the absence of external field the energy levels of the atoms are degenerated with respect to the projection of the angular momentum and the resonant medium is optically isotropic. A polarized wave removes degeneracy of energy levels and induces anisotropy in the media [8, 9]. Assume the intracavity field consisiting of two counter-propagating plane monochromatic waves with electric vector

$$\boldsymbol{E} = [\boldsymbol{E}_1(z)e^{ikz}+\boldsymbol{E}_2(z)e^{-ikz}]e^{-i\omega t}+c.c., \tag{1}$$

where $\boldsymbol{E}_1(z)$, $\boldsymbol{E}_2(z)$ are slowly varying functions of z compared with the exponential, $k = n(\omega/c)$, n – the linear term of medium refractive index.

Let us represent the elliptically polarized light as a sum of right and left circularly polarized components

$$E_{\pm} = E_x \pm iE_y. \tag{2}$$

In the adiabatic approximation following in the absence of absorption and nonlinear saturation we have Maxwell's equations for circular components of the field (1) in the form

$$\frac{d^2E_{\pm}}{dz^2}+k^2E_{\pm} = kq\beta(\mu_1|E_{\pm}|^2+\mu_2|E_{\mp}|^2)E_{\pm} \tag{3}$$

where $q = \pi N\omega|d|^2/c\hbar\varepsilon$, $\beta = |d|^2/\hbar^2\varepsilon^2$, $\varepsilon = \omega_0-\omega$ is the detuning resonance, d is reduced matrix element of transition, N is atomic density; the coefficients μ_1 and μ_2 are determined by the angular moments j_1 and j_2 of the ground and excited states, respectively [8].

Neglecting the parametric interaction between different circularly polarized components of counter propagating waves we obtain the following solutions for wave amplitudes:

$$\begin{aligned} E_{1\pm}(z) &= E_{1\pm}(0)e^{-\frac{i\beta q}{2}\varkappa_{\pm}z}, \\ E_{2\pm}(z) &= E_{2\pm}(0)e^{\frac{iq\beta}{2}\varrho_{\pm}z} \end{aligned} \tag{4}$$

where

$$\varkappa_{\pm} = \mu_1[|E_{1\pm}(0)|^2+2|E_{2\pm}(0)|^2]+\mu_2[|E_{1\mp}(0)|^2+|E_{2\mp}(0)|^2],$$

$$\varrho_{\pm} = \mu_1[|E_{2\pm}(0)|^2+2|E_{1\pm}(0)|^2]+\mu_2[|E_{1\mp}(0)^2|+|E_{2\mp}(0)|^2].$$

Taking into account the boundary conditions on cavity mirrors we have the following set of equations:

$$
\begin{aligned}
I_{T+} &= I_{i+}T^2\left[1+R^2-2R\cos(a_1 I_{T+}+a_2 I_{T-}+\varphi_n)\right]^{-1},\\
I_{T-} &= I_{i-}T^2\left[1+R^2-2R\cos(a_1 I_{T-}+a_2 I_{T+}+\varphi_n)\right]^{-1}
\end{aligned}
\tag{5}
$$

where: $a_1 = 3/2q\mu_1 \dfrac{1+R}{1-R}\, l$, $a_2 = \mu_2 q \dfrac{1+R}{1-R}\, l$; $\quad I_{i\pm} = \beta|E_{i\pm}|^2$, $\quad I_{T\pm} = \beta|E_{T\pm}|^2$ are the respective dimensionless intensity parameters of input and output waves with normal angles of the cavity entry, R is the mirror reflectivity, $T = 1-R$ is the coefficient of transmission, l – the length of the cell with medium in cavity, φ_n – the phase linear running-on.

The set of transcendental Eqs. (5) defines the output intensity $I_{T\pm}$ of circularly polarized components as a function of the input one. A particular interest arises in the case when in resonant media the coefficient $\mu_2 \neq 0$, i.e., there is nonlinear coupling between different circular wave components in media.

Thus, for atomic transitions with $j_1 = j_2 = 1/2$ the coefficient $\mu_2 = 0$ and the set of Eqs. (5) is reduced to the equation completely similar to the one in scalar case considered in literature in detail. It is connected with the fact

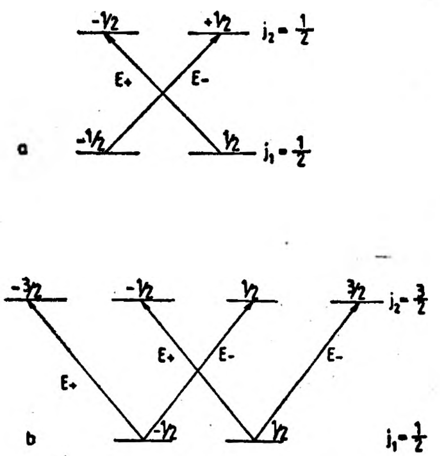

Fig. 1. Interaction of elliptically polarized light with two-level atom: $j_1 = j_2 = 1/2$ (a), $j_1 = 1/2$, $j_2 = 3/2$ (b)

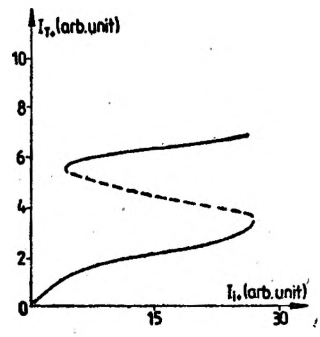

Fig. 2. Output intensity vs. circularly polarized input one

that in such a system the right- and left-polarized wave components propagate independently (Fig. 1a). For all atomic systems with the momentum of one of the levels $j_i \geqslant 1$ the coefficient $\mu_2 \pm 0$. For obvious reason the bistability features arising in such systems are analysed on the example of the system with $j_1 = 1/2$

and $j_2 = 3/2$ (Fig. 1b), for which we have [8]:

$$\begin{aligned} a_1 &= 3.75\,\frac{1+R}{1-R}\,(n-1)\,\frac{\omega}{c}\,l, \\ a_2 &= 1.5\,\frac{1+R}{1-R}\,(n-1)\,\frac{\omega}{c}\,l. \end{aligned} \tag{6}$$

The cases of circular, linear and elliptical polarizations of the input wave will be considered.

2.1. Circular polarization ($I_{i+} \neq 0,\ I_{i-} = 0$)

In this case we have at output

$$I_{T+} = I_{i+}T^2[1+R^2-2R\cos(a_1 I_{T+}+\varphi_n)]^{-1},\ I_{T-} = 0, \tag{7}$$

i.e., the output radiation is polarized by the same circles as the input one. In such a situation we have the usual multistable curve (Fig. 2 — the dotted line corresponds to the unstable branches). In the above and in all other pictures the parameters $\varphi_n = 0$, $R = 0.5$.

2.2. Linear polarization ($I_{i+} = I_{i-} = I_i$)

Here both symmetric ($I_{T+} = I_{T-}$) and asymmetric ($I_{T+} \neq I_{T-}$) solutions occur.

For symmetric solutions the set of Eqs. (5) is reduced to one equation

$$I_{T\pm} = I_i T^2\{1+R^2-2R\cos[(a_1+a_2)I_{T\pm}+\varphi_n]\}^{-1}. \tag{8}$$

This solution corresponds to linear polarization of output radiation.

The asymmetric solutions of the set of Eqs. (5) were analysed numerically (Figs. 3–5). The dependence of circularly polarized component output intensity $I_{T+}(I_{T-})$ on the input one is shown in Fig. 3. As it is seen from this figure, in each period there occur three pairs of asymmetrical solutions, namely: the curves 1, 1a; 2, 2a; 3, 3a in the first period, and — similar curves denoted by dashes — in the second one. If the branches 1–3 correspond to the right (left) circular components the 1a–3a ones correspond to the left (right) components, respectively. Thus as far as intensity is concerned there occur two degenerate solutions at the output, corresponding to two elliptically polarized waves with the same ellipses of polarization rotating in opposite directions.

In Figure 4 the dependence of total intensity $I_T = I_{T+}+I_{T-}$ on the input intensity is given. The curves 1–3 and 1′–3′ are degenerated twice with respect to the intensity and correspond to the ellipses mentioned previously. The variation in the ellipticity degree $\eta = (I_{T+}-I_{T-})/(I_{T+}+I_{T-})$ as a function of input intensity is represented in Fig. 5. Figures 4 and 5 illustrate initiation of polarizational multistability in a Fabry-Pérot cavity. In fact, taking account

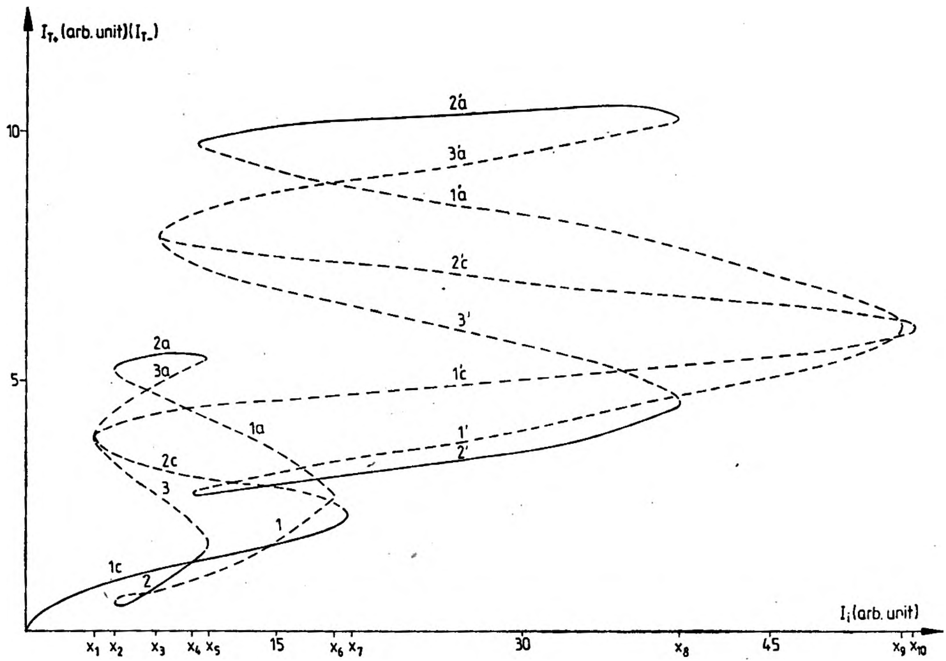

Fig. 3. Circularly polarized component output intensity $I_{T+}(I_{T-})$ vs. linearly polarized input one

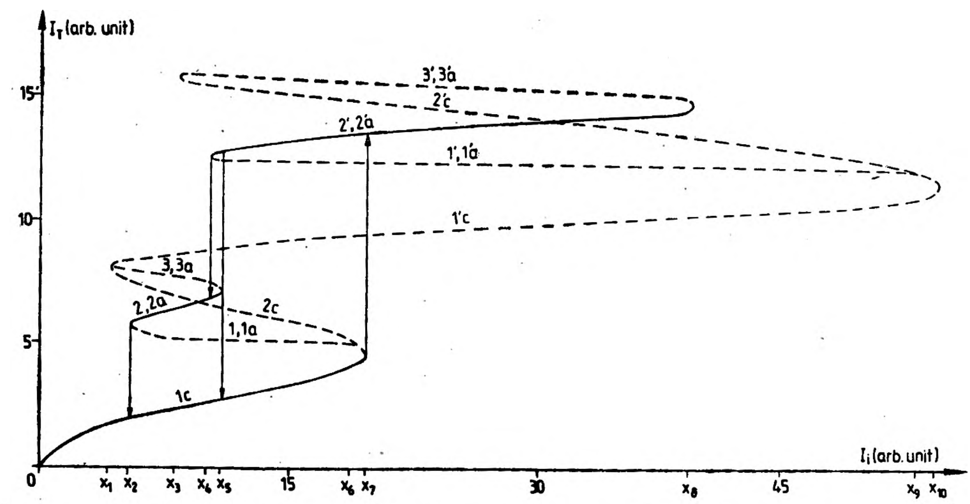

Fig. 4. Summary output intensity I_T vs. linearly polarized input

of light polarization instead of the scalar case, when intensity bistability takes place at $x_1 < I_i < x_7$ (the branches 1*c* and 1'*c*) we have:

i) intensity bistability and polarizational tristability at $x_2 < I_i < x_4$ and $x_5 < I_i < x_7$,

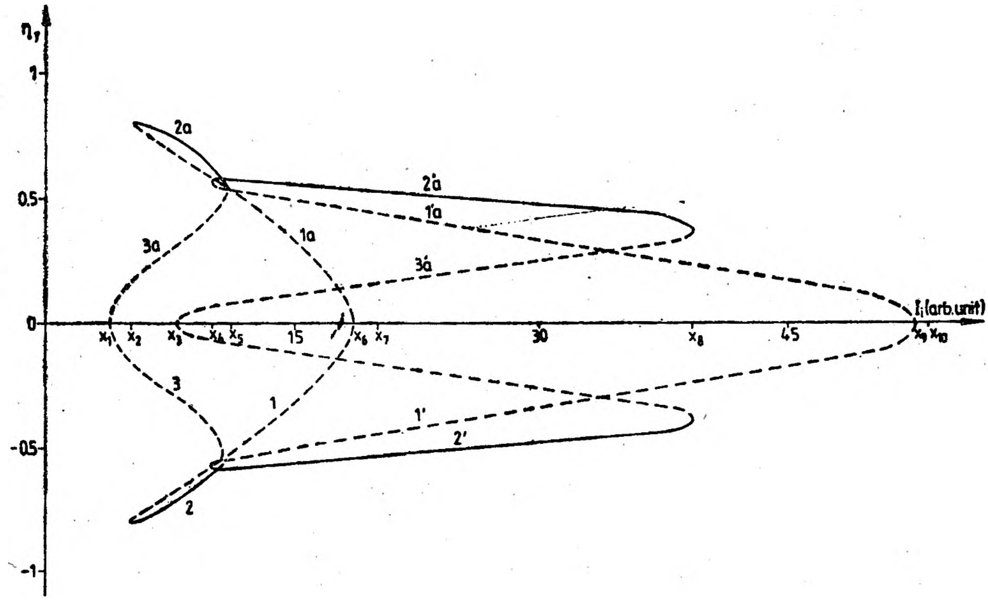

Fig. 5. The variation of the output ellipticity degree η_T as a function of linearly polarized input one

ii) intensity tristability and polarizational pentastability at $x_4 < I_i < x_5$.

It should be noted that the branch 1c is instable with respect to polarization degree (see [6], too). Hystereses of both intensity and ellipticity degree are possible (possible jumps are noted by arrows).

2.3. Elliptic polarization ($I_{i+} \neq I_{i-}$)

Let us consider the case of small ellipticity of the input wave

$$I_{i\pm} = I_{i0} + \xi_{i\pm} \text{ where } |\xi_{i\pm}| \leqslant I_{i0}$$

we try to solve the equation in the form

$$I_{T\pm} = I_{T0} + \xi_{T\pm} \text{ where } |\xi_{T\pm}| \leqslant I_{T0}.$$

For $\xi_{T+} - \xi_{T-}$ we obtain the following relation:

$$\xi_{T+} - \xi_{T-} = \frac{(\xi_{i+} - \xi_{i-})(1-R)^2}{1 + R^2 + 2R(a_1 - a_2) I_{i0} \sin[(a_1 + a_2) I_{i0} + \varphi_n] - 2R \cos[(a_1 + a_2) I_{i0} + \varphi_n]}. \tag{9}$$

From the obtained formula one can see that there are critical values of input intensities at which the denominator is equal to zero. At these values the output wave ellipticity increases dramatically for arbitrarily small input ellipticity. These critical points coincide with x_1, x_3, x_6 and x_9 (Figs. 3–5) at which asymmetric solutions arise from symmetric ones (in the case of linear polarization of the input wave). Since in real experimental situation there is some ellipticity in linearly polarized wave only one of two degenerated solutions is realized.

3. Conclusions

Thus, we have developed the polarizational theory of a Fabry-Pérot cavity filled with nonlinear resonant medium to obtain the initiation of specific polarization multistability. It should be stressed that this phenomenon appears not only in the case of elliptically polarized input light but also in the case of linearly polarized one.

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