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Recognition of coloured objects using thick holographic multiplexed filter (THMF)*

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A thick holographic multiplexed filter described in this paper operates as a Fourier hologram and volume hologram simultaneously. This novel filter was applied, in our studies, to the recognition of transparent coloured objects by using the nominal double diffraction arrangement illuminated with polychromatic light.

1. Introduction

The volume hologram displays three dimensional diffraction characteristics being described earlier by the well known Bragg relation

$$2d\sin\Theta' = \lambda$$

where d is the spacing between reflacting planes, Θ' is the angle of incidence, and λ is the wavelength.

The use of thick media for making thick (volume) holograms was first proposed by DENISYUK [1], then STROKE et al. [2] extended the technique to record reflection holograms in thick emulsions, and reconstructed monochromatic images using white light.

We remember that Vander-Lugt filter [3] is usually considered as the base in construction of any holographic filter. Recently, CASE [4] has described a holographic multiplexed filter using polychromatic light. In this paper a thick holographic multiplexed filter (THMF) is described from the experimental and theoretical points of view. It was employed in optical correlation for the recognition of coloured shapes using polychromatic light for the illumination.

2. Analysis

The transmission amplitude function of a coloured transparent shape shown in Fig. 1 can be represented by

$$g(x, y, \lambda) = \sum_{i=B,G}^{R} g_i(x, y, \lambda_i) \tag{1}$$

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where: λ_i is the polychromatic wavelength of light, $g_{i=B}$ is the spectral blue component of the coloured shape, etc. The amplitude transmitted from the object, given by formula (1), is Fourier transformed by means of a converging lens L in order to obtain the following equation in the holographic zone

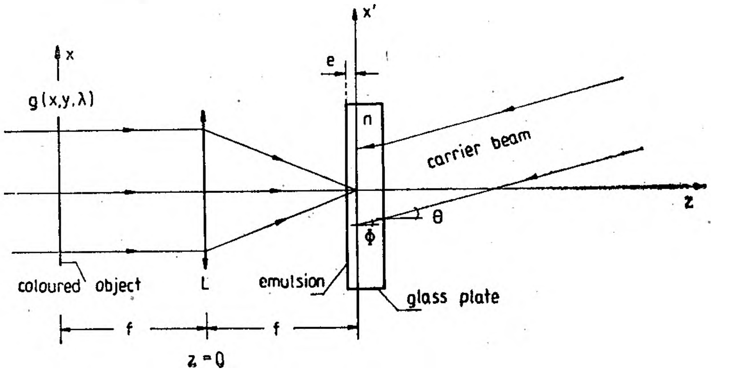


Fig. 1. Schematic diagram for the construction of thick holographic multiplexed filter: $g(x, y, \lambda)$ – coloured object in the plane x, y; L – Fourier transformed lens of focal length f; $H(x', y')$ – holographic plane; n – refractive index of the glass plate; e – emulsion thickness

$$\mathscr{F}\{g(, x, y, \lambda)\} = \sum_i \beta_i \tilde{g}_i(u, v, \lambda_i) \exp(jk_i r^2/2z) \tag{2}$$

($r = \sqrt{x'^2+y'^2}$, $k_i = 2\pi/\lambda_i$ – the wave number) where the exponential which appears in Eq. (2) indicates that we deal with a spatial variable plane inside the emulsion. $u = x'/\lambda f$ and $v = y'/\lambda f$ are the reduced coordinates, the coefficient β_i depends mainly on the spectral intensity curve of the emulsion, $\sim$ denotes the performed transformation.

An inclined plane wave, making an angle Θ with the Z-axis is incident on the opposite side of the holographic plate represented as follows:

$$\sum_l A_l \exp(-jk_l x' \sin\Theta)$$

where A_l are the complex amplitudes corresponding to each spatial coloured component $k_l = 2\pi/\lambda_l$ is the wave number corresponding to each wavelength of light illumination.

According to Snell's law of refraction and using Fig. 1 we get

$$\sin\Theta = n \sin\Phi$$

where n is the refractive index of the glass plate. Hence, the incident carried plane wave that makes an angle Φ is written as follows:

$$\sum_i A_l \exp(-jk_l x' \sin\Theta/n). \tag{3}$$

From Equations (2) and (3), the intensity recorded inside the holographic emulsion is

$$I(x', y', z, \lambda) = \Big|\sum_i \beta_i \tilde{g}_i(u, v, \lambda_i)\exp(jk_i r^2/2z) + \sum_l A_l \exp(-jk_l x' \sin\Phi)\Big|^2. \tag{4}$$

Assuming that the polychromatic light components are mutually incoherent and that of the coloured information is separated spatially, we are able to write Eq. (4) as follows:

$$I(x', y', z, \lambda) = \Big|\sum_i \beta_i \tilde{g}_i(u, v, \lambda_i)\Big|^2 + \sum_l |A_l|^2 + \\ + 2\sum_{i=l} \beta_i A_i \tilde{g}_i(u, v, \lambda_i)\cos\left[k_i\left(x'\sin\varphi + \frac{r^2}{2z}\right)\right]. \tag{5}$$

It is to be noted that the hologram must be recorded in the focal plane of the converging lens in order to be sure that we have a Fourier hologram. This condition implies (Fig. 2)

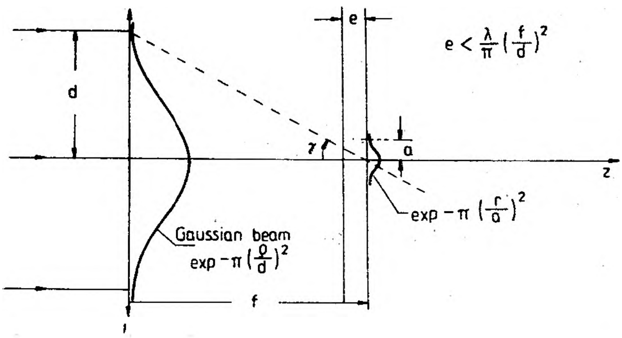


Fig. 2. Schematic diagram illustrating the necessary condition to construct a Fourier hologram within an emulsion of thickness e; d – transversal height of the Gaussian beam in the pupil plane, a – transversal height of the Gaussian beam in the plane (x', y') f – focal length of the lens L

$$f \gg (a^2/\lambda)\pi, \quad r = a = \sqrt{x'^2 + y'^2}$$

where a is the transversal height of the Gaussian spot in the plane (x', y'). From the characteristics of the Gaussian laser beam it follows that the radius of the laser convergent beam is

$$a = \frac{\lambda}{\pi}\left(\frac{f}{d}\right)$$

where the contraction angle $\gamma = \lambda/\pi a$, and $d = f\gamma$. Hence, in order to record Fourier hologram inside the emulsion of thickness e the following condition must be satisfied:

$$e < \frac{\lambda}{\pi}\left(\frac{f}{d}\right)^2,$$

e.g., if $\lambda = 0.63$ μm, $f = 50$ cm, and $d = 1$ cm, then $e < 500$ μm which is practically admitted.

Assuming the condition that the amplitude transmitted from the holographic plate after the development process is linearly proportional to the intensity recorded in the emulsion, i.e., that $t \simeq I$, from Eq. (5) we get

$$t(x', y', z, \lambda) \simeq \sum_i |\beta_i \tilde{g}_i(u, v, \lambda_i)|^2 + \sum_i |A_i|^2 + \\ +2 \sum_i \beta_i A_i \tilde{g}_i(u, v, \lambda_i) \cos\left[k_i \left(x' \sin\Phi + \frac{r^2}{2z}\right)\right]. \tag{6}$$

Consequently, the reconstructed image can be calculated by taking the Fourier transform of Eq. (6)

$$I_r(x, y, \lambda) = \mathscr{F}\{t(x', y', z, \lambda)\}, \quad r \to 0. \tag{7}$$

Now, we analyze the correlator shown in Fig. 6. In order the amplitude t transmitted from the plate multiplied by the Fourier transform of the amplitude transmitted from the object to be in the imaging plane it should be Fourier transformed again

$$A(x'', y'', \lambda) = \mathscr{F}\{t(x', y', z, \lambda) \times \mathscr{F}[g(x, y, \lambda)]\} = \mathscr{F}[t] * g(x'', y'', \lambda) \tag{8}$$

where $*$ denotes the convolution operation.

Since in the analysis we are interested in the modulation term of interference, we get from Eq. (8)

$$A(x'', y'', \lambda) \simeq \mathscr{F}\left\{\sum_i \beta_i A_i \tilde{g}_i(u, v, \lambda_i) \cos\left[k_i \left(x' \sin\Phi + \frac{r^2}{2z}\right)\right]\right\} * g(x'', y'', \lambda) \tag{9}$$

assuming that $r^2 \ll (\lambda/\pi)z$, and $A_i = 1$. Then, after calculating the Fourier transformation, Eq. (9) will give the correlation amplitudes as follows:

$$A_c(x'', y'', \lambda) \simeq g(x'', y'', \lambda) * g(x'' + f \sin\Phi, y'', \lambda). \tag{10}$$

The correlation intensity detected in the imaging plane is calculated from Eq. (10) to give

$$I_c(x'', y'', \lambda) = |A_c(x, y, \lambda)|^2, \tag{11}$$

e.g., for two coloured (red, green) spatially separated objects, the correlation intensity is expressed by

$$I_c = |g_R * g_R|^2 + |g_G * g_G|^2.$$

There is no cross-correlation term between the green and the red components since they are spatially separated and because of the function of the thick holographic multiplexed filter.

3. Experiment

Firstly, we describe experimental arrangement used for construction of thick holographic multiplexed filter shown in Fig. 3, where the holographic grating results from the interference of the Fourier spectral components of the coloured

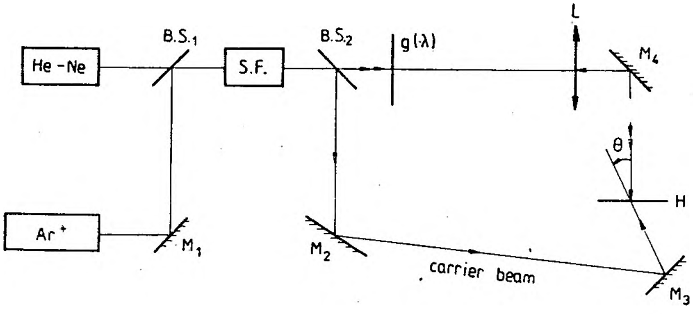


Fig. 3. Experimental arrangement for THMF construction: *S. F.* – spatial filter, $g(\lambda)$ – coloured object, *B.* S. – beam splitter, *M* – mirror, *H* – holographic domain, *f* – focal length of the lens *L* which is the distance between *L* and *H* throught M_4

signal, obtained by means of a converging lens *L* and inclined carrier plane wave incident on the opposite side of the holographic plate. We reconstruct simultaneously both the shape of the object and its colour utilising set-up in transmission Fig. 4, or at reflection as in Fig. 5.

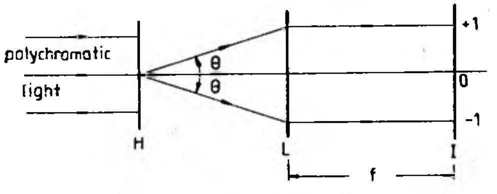


Fig. 4. Reconstruction of the coloured image at transmission: *H* – processed hologram, *L* – Fourier transformed lens of focal length *f*, *I* – imaging plane

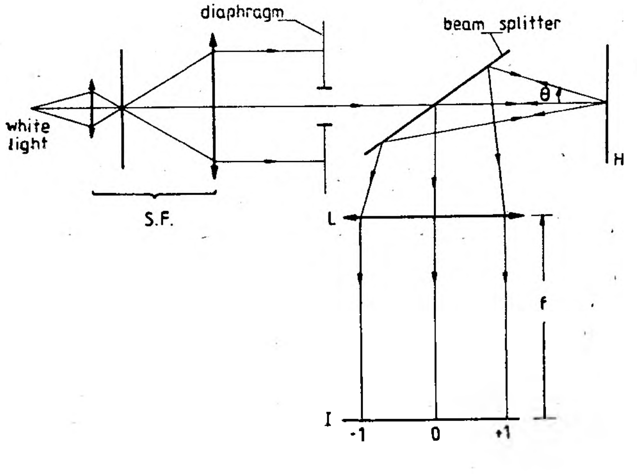


Fig. 5. Reconstruction system at reflection: *S. F.* – spatial filter, *H*, *L*, *I* – as defined in Fig. 4

Secondly, we describe schematic arrangement of double diffraction using polychromatic light emitted from He-Ne laser and Ar ion laser which works in transmission as in Fig. 6, and at reflection as in Fig. 7. In both the optical

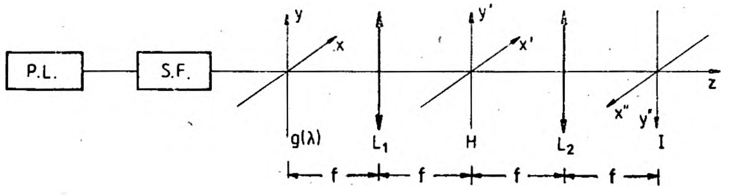


Fig. 6. Optical polychromatic correlator at transmission: $P.\ L.$ – polychromatic light, L_1, L_2 – Fourier transformed lenses each of focal length $f = 40$ cm, $g(\lambda)$ – coloured object, H – processed holographic plate, I – imaging plane

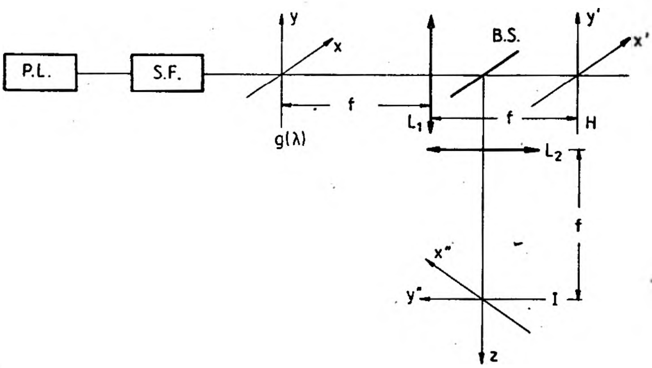


Fig. 7. Optical polychromatic correlator at reflection (we used the same notation as in Fig. 6)

systems, the development holographic plate is found in the back focal plane of the lens L. The optical system working at reflection has an amplitude reflected from the developed holographic plate given by $r = \sqrt{1-|t|^2}$, assuming that the absorption loss is negligeable and t is the transmitted amplitude. In the imaging plane (x'', y''), the coloured correlation peaks are detected.

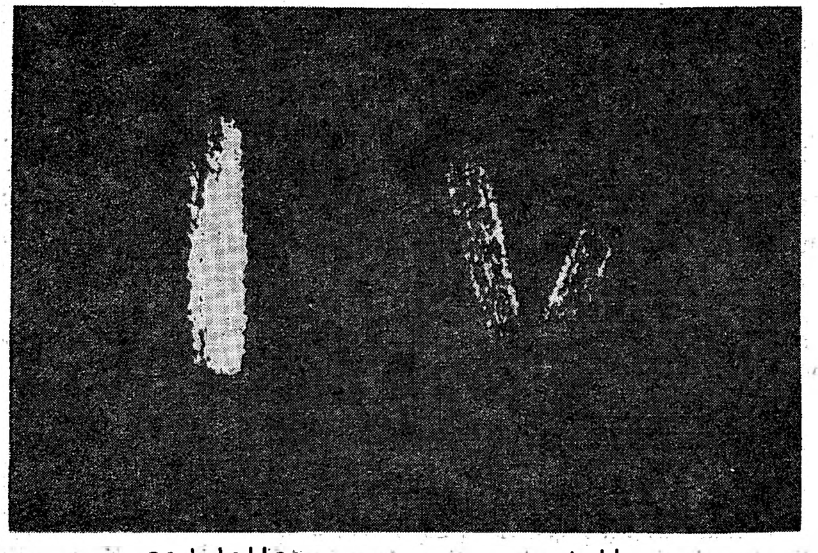
4. Results and discussion

Two coloured letters (I is red, and V is green) are recorded on a thick emulsion, and the reconstructed image corresponding to these coloured shapes is shown in Fig. 8. This image reconstructed in transmission on both its sides has parasite images which are attributed to difficulty connected with a simultaneous adjustment of the shape and colour.

The same recorded object is reconstructed using reflection set-up given in Fig. 9. We reconstruct exactly the shape without parasite images, but the colour is shifted towards the shorther wavelength due to the emulsion shrinkage

parasite actual image parasite

Fig. 8. Black and white photograph of the reconstruction at transmission for coloured image. In the center the actual image, while on both sides parasite images appear



red letter green letter

Fig. 9. Black and white photograph of the reconstruction at reflection for the same coloured image

during the chemical holographic processing. This problem of emulsion contraction was solved by HARIHARAN [5] by utilizing the chemical product D(–)sorbitol* during the holographic processing.

Figure 10 shows the coloured correlation peaks photographed in the imaging plane. We detect green peak of correlation which manifests the existence o

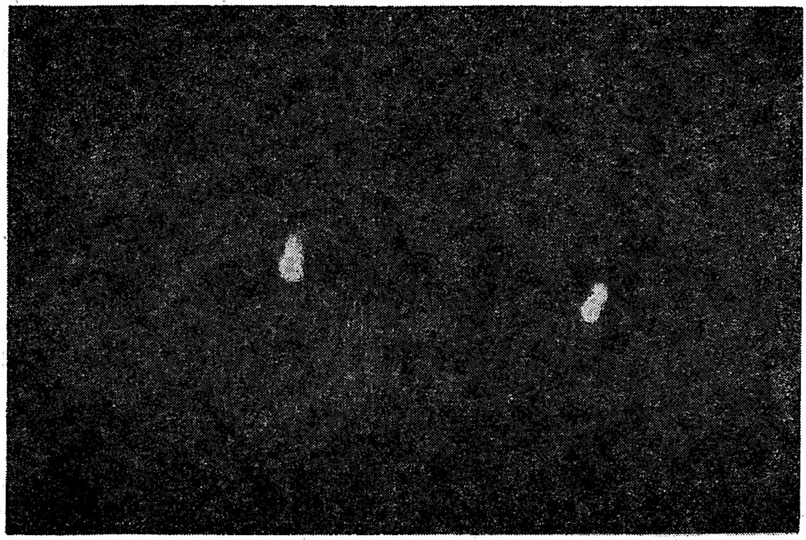


Fig. 10. Black and white photograph for the coloured peaks of correlation. In the *R.H.S.* green peak for the letter *V* as a green object, while in the *L.H.S.* red peak for the letter *I* as a red object

the green letter *V*, and we detect red peak of correlation corresponding to the letter *I* (red). If the colour or shape of the object is modified, the correlation peaks vanish and the detection fails. This confirms that a real recognition is possible by using polychromatic correlator provided with thick holographic multiplexed filter.

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References

[1] DENISYUK Yu. N., Doklady Akad. Nauk SSSR **144** (1962), 1275.
[2] STROKE G. W., LABEYRIE A. E., Phys. Lett. **20** (1966), 368.
[3] GOODMAN G. W., *Introduction to Fourier optics*, McGraw-Hill Book Co., New York 1968.

* D(–)sorbitol has the following chemical symbol: $CH_2OH(CHOH)_4CH_2OH$.

[4] CASE S. K., Appl. Opt. **18** (1979), 1890.
[5] HARIHARAN P., J. Opt. **11** (1980), 53.

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Распознавание цветных предметов с использованием толстых голографических мультиплексных фильтров

Описан толстый голографический мультиплексный фильтр, действующий как смесь голограммы Фурье и объемной голограммы. Этот новый голографический фильтр применялся в наших исследованиях для распознавания цветных прозрачных предметов путем использования номинальной двойной дифракции при освещении полихроматическим светом.

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