Figs. 3a, b and 4a, b illustrate the situation. Actually, we work on a construction of a filter representing the three-dimensional kernel of Hilbert transform and on its application to produce the quadratures of aperture functions in an incoherent processor.

Translated by Ireneusz Wilk

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Single-wavelength coding of colour in one-step rainbow holography

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1. Introduction

The basic tricks used in the rainbow holography consist in restricting the object beam with a narrow slit and next recording this beam after its passage through an imaging system. Thus, the wavefronts generated by the examined object are recorded together with the light beam diffracted by the slit on a rainbow hologram. Consequently, in the reconstruction step the image of the object is reconstructed together with the image of the slit. In monochromatic light the reconstructed image is visible from the position where the slit image is reconstructed. When white light is used to reconstruction, the image of the slit becomes spectrally diffused, but from the given point of this image a monochromatic image of the object may be seen. By locating the eye in another point of the slit image the image of the object will appear in another colour.

2. Colour coding in single-step rainbow holography

In order to obtain a rainbow hologram of Benton-type [1] the wavefronts reconstructed from a conventional hologram must be recorded. During the rainbow hologram recording the conventional hologram is diaphragmed by a screen with a slit. A simpler method is the so-called single-step method [2] in which the recorded image is produced by an optical system. The slit is located in the object space while the plane of the recording material is placed close to the image plane of the system. Such a system may be used to produce a hologram which, during the reconstruction with a white light beam, may generate an image of the priorly coded colours. This method is called a pseudocolour method. The pseudocolour image reconstructed from the rainbow holograms produced by a two-step method has been described by TAMURA [3]. The two-step method is very laborious. The way proposed below seems to be much simpler.

It is well known that the reference wave which interferes with an object wave generates an interference pattern of spatial frequency depending, among others, on the light wavelength used. Thus, the blue colour, for instance, is recorded with higher frequency than the green colour, while the latter – with higher frequency than the red one. The change in the spatial frequency may thus be caused by the respective change in the wavelength and also by a change of the incidence angle in the reference wave. The system in which the reference wave falls into the recording material plane under different angles is shown in Fig. 1.

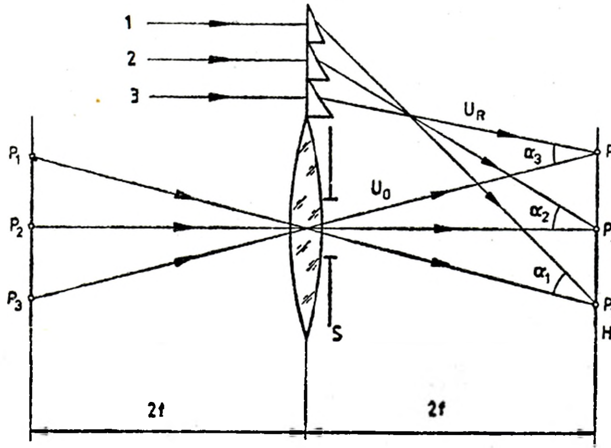


Fig. 1. Optical system for production of the rainbow holograms by using three different reference beams (1, 2, 3)

A plane monochromatic wave falls on a zone plane and is split into three beams each of which hits, in turn, the hologram under a different angle. In Figure 1 all of the three beams are represented by their principal rays. A two-dimensional image is of three-colour structure and the points P_1, P_2 and P_3 are the centres of the coloured strips. The lens produces the image of magnification 1:1. The recording medium plane overlaps exactly the image plane of the lens.

The slit is located immediately behind the lens. Each of the image points P_1', P_2' and P_3' is recorded on a hologram by using another reference beam (inclined under a different angle). The principal rays of the reference beam create the respective angles α_1, α_2 and α_3 with the principal rays of the image beams. If

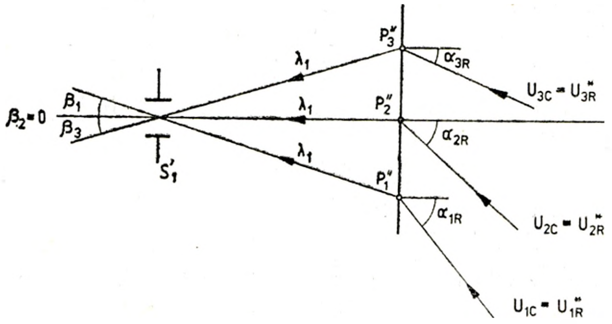


Fig. 2. The reconstruction of the rainbow hologram by using beams conjugated with the respective reference beams

the complex amplitude of the object wave is denoted by $U_{n0}(x, y)$, where n denotes the number of object zones, and the complex amplitude of the reference wave corresponding to the given object wave is denoted by $U_{nR}(x, y)$, the intensity distribution of the recorded interference structures is expressed by the formula

$$I(x, y) = \sum_{n=1}^{3} |U_{nO} + U_{nR}|^2 = \sum_{n=1}^{3} (|U_{nO}|^2 + |U_{nR}|^2 + U_{nO}U_{nR}^* + U_{nO}^*U_{nR}) \tag{1}$$

where the spatial frequencies corresponding to the particular points are the following:

$$\nu_1 = \frac{\sin\alpha_{1R} - \sin\alpha_{1O}}{\lambda_1} \quad \text{for the point } P_1', \tag{2}$$

$$\nu_2 = \frac{\sin\alpha_2}{\lambda_1} \quad \text{for the point } P_2', \tag{3}$$

$$\nu_3 = \frac{\sin\alpha_{3R} + \sin\alpha_{3O}}{\lambda_1} \quad \text{for the point } P_3'. \tag{4}$$

Here, α_{1R}, α_{3R}, α_{1O}, α_{3O} denote the angles created by the respective reference and object beams with the normal to the recording medium. It is evident that

$$\alpha_1 = \alpha_{1R} - \alpha_{1O}, \quad \alpha_2 = \alpha_{2R}, \quad \alpha_3 = \alpha_{3R} + \alpha_{3O}. \tag{5}$$

The hologram presented above may be treated as a set of three independent image-plane holograms, of which the first located in the region P_1' is the hologram of the object P_1 and its surrounding, while the second and the third ones, placed in the regions P_2' and P_3' are the respective holograms of the object elements located in the surroundings of the points P_2 and P_3. Each of these holograms is produced by using another reference wave of the suitable slope and the amplitude U_{nR}.

3. Reconstruction of the wavefronts from the hologram of coded colour structure

Consider the reconstruction of our hologram with three plane monochromatic waves conjugate with respect to the respective recording waves (Fig. 2). It may be seen that for such geometry each reconstructing wave reconstructs the same image of the slit located at the same distance from the hologram plane. The position and the magnitude of the slit image will be the same as those during recording. However, let us imagine that our hologram is reconstructed with three parallel beams of white light of the same inclination. This inclination corresponds to a wave being conjugate to the first reference beam. In Figure 3 there are shown only three principal rays of the monochromatic components λ_1, λ_2 and λ_3 which correspond to the colour distribution in the image.

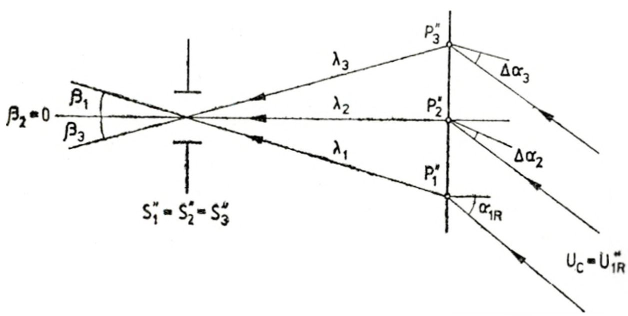


Fig. 3. The reconstruction of the rainbow hologram using collimated beam of white light

The reconstructing wave may be, thus, represented in the form

$$U_C = \sum_{m=1}^{3} U_{mC}, \tag{6}$$

for the three colours of interest, where U_{mC} are the complex amplitudes of the wavelengths λ_1, λ_2, λ_3, respectively. The corresponding wavefronts reconstructed from such a hologram may be described by the formula

$$U_I = I \sum_{m=1}^{3} U_{mC} = \sum_{m=1}^{m=3} \sum_{n=1}^{n=3} U_{mC} |U_{nO} + U_{nR}|^2 \tag{7}$$

where the primary and secondary images are respresented respectively by the expressions:

$$U_{I'} = \sum_{m=1}^{m=3} \sum_{n=1}^{n=3} U_{mC} U_{nO} U_{nR}^*,$$
$$U_{I''} = \sum_{m=1}^{m=3} \sum_{n=1}^{n=3} U_{mC} U_{nO}^* U_{nR}. \tag{8}$$

By comparing the Figures 2 and 3 it may be seen that the direction of incidence of the wave in the second and third beams of the white light differs from the direction of incidence of the monochromatic beams by the following magnitudes

$$\begin{aligned} \Delta\alpha_2 &= \alpha_{1R} - \alpha_{2R}, \\ \Delta\alpha_3 &= \alpha_{1R} - \alpha_{3R}. \end{aligned} \tag{9}$$

Therefore, the waves of the wavelengths λ_1, λ_2, λ_3 are diffracted by the holograms under the angles β_1, β_2 and β_3, respectively. These three waves cause that there appears only one image of the slit. Moreover, each of them reconstructs this part of the image that corresponds to the contribution of the given colour in the original object. During the hologram reconstruction with a monochromatic ligth beam of the wavelength λ_1 (Fig. 2) the following relations are valid:

$$d_1(\sin\alpha_{1R} - \sin\beta_1) = \lambda_1, \tag{10}$$

$$d_2\sin\alpha_{2R} = \lambda_1, \tag{11}$$

$$d_3(\sin\alpha_{3R} + \sin\beta_3) = \lambda_1, \tag{12}$$

where β_1, β_3 are the diffraction angles of the reconstructing waves falling under the angles α_{1R}, α_{3R} on the hologram, while the diffraction angle of the wave falling under an angle α_{2R} is equal to zero. If the recorded hologram is considered as a set of three diffraction gratings, then d_1, d_2, d_3 denote the respective grating constants. If the reconstruction is performed by using a collimated white light beam for the wavelengths corresponding to the colour distribution in the object, the relations (10)–(12) take the forms:

$$d_1(\sin\alpha_{1R} - \sin\beta_1) = \lambda_1, \tag{10a}$$

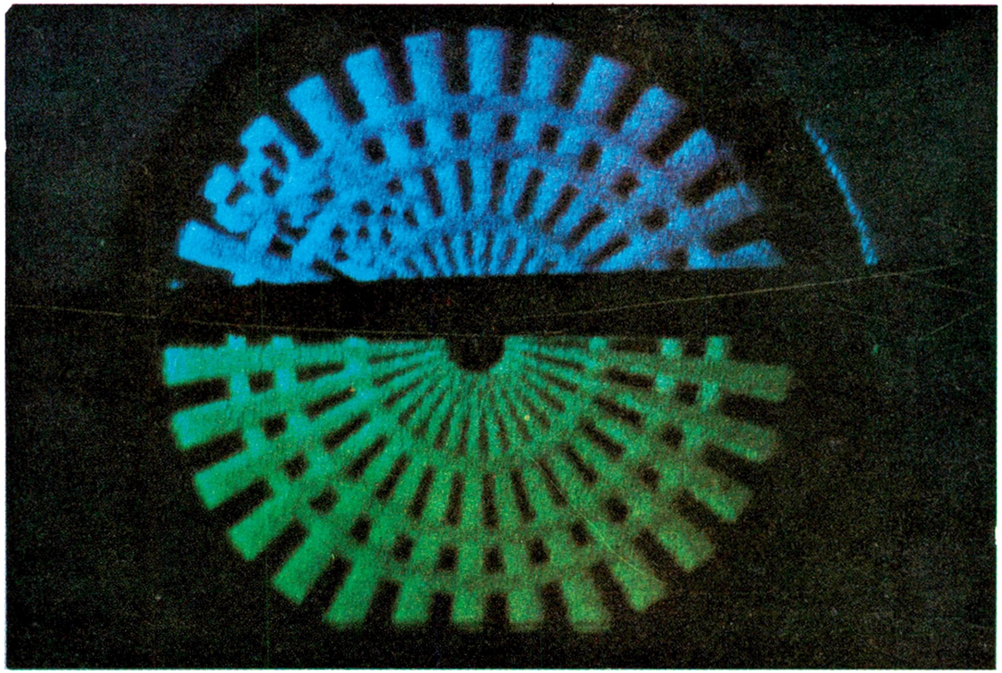
$$d_2\sin\alpha_{1R} = \lambda_2, \tag{11a}$$

$$d_3(\sin\alpha_{1R} - \sin\beta_3) = \lambda_3. \tag{12a}$$

Basing on the formulae (10)–(12) and (10a)–(12a) it may be shown that a change in the incidence angle of the reconstructing wave by $\Delta\alpha_n$ (which causes a change in colour in the image reconstructed by the while light), corresponding to the change $\Delta\lambda$ in the wavelength, is described by the following relation

$$\Delta\alpha_n = \frac{\Delta\lambda}{\lambda}\tan\alpha_n. \tag{13}$$

Thus, the choice of the suitable angles of incidence for the reference beams (formula (9)) allows us to obtain during reconstruction the image the colours of which were characteristic of the object.



Phot. An image of a two-colour image reconstructed with a white light from the rainbow hologram

4. Experimental verification

Basic role in colour coding in rainbow holograms (performed by using the method suggested) is played by the change in reconstructing beam direction with respect to those of reference beams used in recording of particular zones of the object in the hologram. As indicated above, it is only for the first zone in the vicinity of the point P_1' that the wavefront reconstruction is performed by using the beam conjugate with respect to the respective reference beam. In the other zones of the hologram the reconstructing beams are inclined by the angles $\Delta\alpha_2$ and $\Delta\alpha_3$ with respect to the corresponding reference beams. Thus, if the hologram is reconstructed by a collimated beam of white light inclined under the angle α_{1R} then in the case of a three-colour object recorded the image of the slit at the position S_1' will be reconstructed independently by the light of the wavelengths λ_1, λ_2 and λ_3, respectively (as shown in Fig. 3).

The experiment carried out consisted in producing a hologram of a plane object of green and blue colour by using a monochromatic light of the wavelength $\lambda = 514.5$ nm and reconstruction of the image with a white light beam in the setup shown in Fig. 3, equipped with an objective of $f = 400$ mm. A two-colour object of radial structure and 20 mm in diameter is located at the object plane symmetrically with respect to the optical axis. The collimated reference beams, recording both the green and blue halves of the object were inclined under the angles $\alpha_{2R} = 30°$, and $\alpha_{1R} = 32.2°$, respectively, with respect to the recording medium plane. The change in the reference beam inclination was determined from the formula (13) assuming that the wavelength at the transition from the green colour to the blue one the wavelength changes by $\Delta\alpha = 35$ nm. The hologram was produced on a Agfa Gevaert 10E56 plate. The colour-photo shows an image of the object reconstructed by a collimated white light beam inclined under the angle $\alpha = 30°$ with respect to the hologram normal.

5. Conclusions

The method suggested allows to reconstruct the image of an object of given colours, provided that the proper colour coding is made (see formulae (2)–(4) and (10)–(12)). If the conditions determined by these formulae are not fulfilled the colours in the object have no influence on the colour distribution in image, even if the suggested procedure is applied. In our consideration we have used the three-colour object of characteristic colour distribution only to better illustrate the method. The pseudocolouring method does not lead to exact reconstruction of natural object colour. It enables, however, to code and next to reconstruct the image in the chosen colours. If the object is two-dimensional and focussed by an optical system exactly in the recording medium plane, the reconstruction obtained is good, free of diffusion caused by dispersion, which is of particular importance in application of this method.

Translated by Ireneusz Wilk

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