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A study of geometrical factor in optical particle counters

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Variation of geometrical factor for various geometries of illumination and collection optical system in optical particle counters has been studied.

1. Introduction

Light scattering single particle Optical Particle Counters (OPC) have found a widespread use in clean room monitoring, pollution research, laboratory aerosol research, inhalation studies of aerosols and many other important fields. In the recent years the complexity of OPCs has increased and several types are now available commercially. The operation of OPCs is based on the principle that when aerosol flows through an intensely illuminated view volume, light flash scattered by a single particle into a particular solid angle is sensed photo-electrically, and the response pulses are classified according to their magnitude. Various authors [1–5] have studied the response function (R) of OPCs which can be calculated with the help of Mie scattering theory. The latter assumes that the particles are spherical and are illuminated by light of unit flux per unit beam cross-sectional area, and the scattered intensity is calculated as a function of scattering angle θ, measured with respect to the incident ray, particle refractive index m, and particle size parameter $a = \pi D/\lambda$, where D is the diameter of the particle and λ is the wavelength of the illumination light. The results of calculations based on Mie theory [6–8] are usually presented in form of angular intensity functions $i_1(a, m, \theta)$ and $i_2(a, m, \theta)$ which are the components of scattered light polarized in and normal to the plane containing the directions of illumination and observation. The scattering angle θ is measured with respect to the incident ray which, in general, is inclined at an angle Φ to the axis of collection aperture.

Therefore, the calculation of response function for each particular geometry of the optical system includes a geometrical factor [9], $f(\theta, \Phi)$ and, moreover, an additional factor $f(\lambda)$ which accounts for the spectral emissivity of the light source and the spectral response of the photo-sensitive detector. Thus, the total flux or counter response R of spherical particles, in the absence of coincidence and cross-sensitivity, is given by

$$R = \int\int\int \frac{\lambda^2}{8\pi 2} (i_1 + i_2) f(\theta, \Phi) f(\lambda)\, d\theta\, d\Phi\, d\lambda. \tag{1}$$

The calculation of the flux collected for various geometries have been discussed by HODKINSON and GREENFIELD [10], although the functional form of geometrical factor is not explicitely stated.

QUENZEL [1] in his response calculations carried out for Royco 220 and Bausch and Lomb Counters did not include appropriate geometrical factor. Even in a revised paper written to answer criticism by MARTENS and DOONAN [11], he [2] utilized a geometrical factor which appears to differ from that of HODKINSON and GREENFIELD [10]. Recently, in a paper on optimization of response function, HEYDER and GEBHART [4] have calculated the geometrical factor only implicitely, through the use of matrix formulation and rotation of co-ordinate system, the analysis being not limited to circular apertures. In addition, it may be seen that WILLEKE and LIU [9] and COOKE and KERKER [3] have wrongly termed Φ an angle made by the incident ray with the axis illumination. In fact, Φ is the angle made by the incident ray with the collection axis [10].

The verification of the reported results on response function might be therefore difficult, especially when the formulation of geometrical factor is ambiguous. The purpose of this communication is to present the explicit functional form of the geometrical factor for various geometries of the optical system of commonly used OPCs.

2. Geometrical factor: definition

Let the particle be illuminated by parallel rays making an angle Φ with the axis of collection aperture, carrying unit energy flux, per unit transverse area. Let the intensity scattered by the particle in the direction θ be $I(\theta)$ unit of flux per unit solid angle per unit transverse area of the particle. Then, the total flux F scattered through the angles θ and $\theta + d\theta$, collected by the collection system for any geometry, may be written in a general form as

$$F = f(\theta, \Phi)\, I(\theta)\, d\theta\, d\Phi, \tag{2a}$$

or if $\Phi = 0$

$$F = f(\theta, \Phi) I(\theta) d\theta, \tag{2b}$$

where $F(\theta, \Phi)$ may be defined as the geometrical factors for this geometry of illumination and collection system. Thus, $f(\theta, \Phi)$ represents in a way the collection efficiency of the differentially scattered flux through θ and $\theta + d\theta$. This definition constitutes the basis for calculation of $f(\theta, \Phi)$ for all the instrumental arrangements considered in this analysis.

3. Discussion: geometries of illumination and collection

In what follows the geometrical factor has been calculated and its plot discussed for various gemetries in the order of their complexity. Such curves illustrating variations of geometrical factor have not yet been reported.

3.1. Instruments with collimated illumination along the axis of collection aperture

In this type of instruments laser light is normally employed for illumination in order to achieve highly intense and collimated beam. Figure 1 shows the geometry of illumination and collection for this arrangement. AB is the circular collection aperture which subtends an angle 2β at O, where the scattering parti-

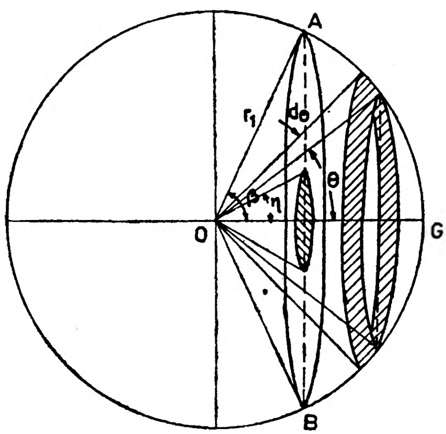

Fig. 1. A scheme of the instruments with collimated illumination along axis of collection aperture. AB is the circular collection aperture, and OG is the direction of the illumination beam, η is the half angle of the light trap, O is the illuminated particle

cle is situated. OG represents the direction of illuminating beam. The light directed into the collection aperture is eliminated by a light trap of circular aperture which subtends an angle 2η at O. The whole light scattered through the angles η to β is collected.

The light scattered through angles θ to $\theta+d\theta$ passes through a circular strip of radius $r_1 \sin\theta$ and width $r_1 d\theta$ to the surface of a sphere of radius r_1 centered at O. The area of this strip is $2\pi r_1^2 \sin\theta d\theta$ which subtends at O solid angle

$$w_0(\theta) = 2\pi \sin\theta d\theta. \tag{3}$$

Therefore, the flux F collected by the collection system is given by

$$F_1 = w_0(\theta) I(\theta) = 2\pi \sin\theta d\theta I(\theta). \tag{4}$$

Comparing eqs. (2) and (4), we can write

$$f_1(\theta, \Phi) = 2\pi \sin\theta, \text{ where } \eta < \theta < \beta, \text{ and } \Phi = 0, \tag{5}$$

where $f_1(\theta, \Phi)$ is the geometrical factor for this simple geometry. Figure 2 shows the plot of $f_1(\theta, \Phi)$ which is a simple sine curve bounded by half angles of light trap and collection aperture. It is easy to see that $f_1(\theta, \Phi)$ is maximum for $\theta = 90°$ if $\beta \geqslant 90°$. If $\beta < 90°$, $f_1(\theta, \Phi)$ is maximum when θ just approaches β.

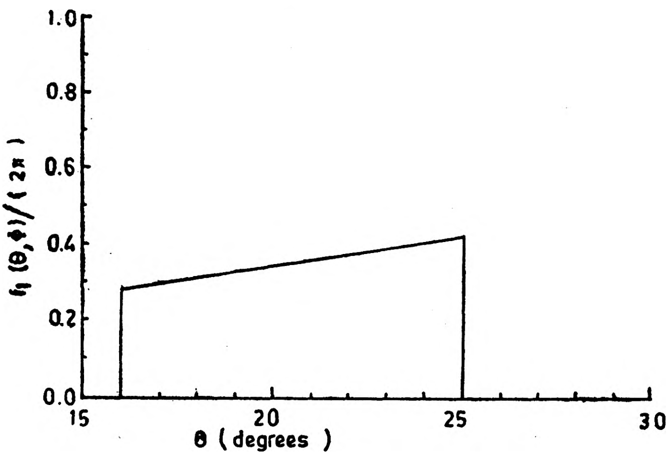

Fig. 2. Variation of $f_1(\theta, \Phi)$ against scattering angle θ for $\eta = 16°$ and $\beta = 25°$

3.2. Instruments with collimated illumination not co-axial with collection aperture

In this type of instruments the collimation direction makes the angle Φ with the axis of collection aperture so that the collimated beam is not intercepted by the collection aperture. Figure 3 shows the scheme of geometrical arrangement for such instruments. *KCBE* represents the collection aperture of semi-

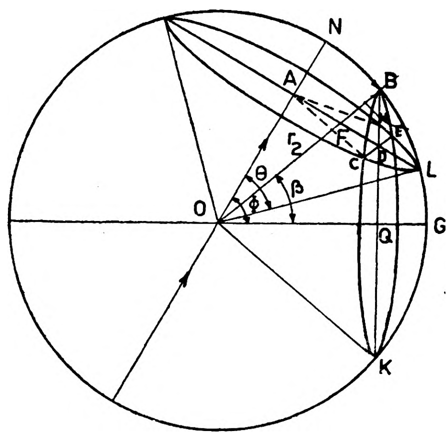

Fig. 3. A scheme of the instruments with collimated illumination not co-axial with collection aperture: Φ is the angle between direction of collimation and axis of collection aperture

angle β. The particle at O is illuminated by a pencil of rays travelling to the axis of collection aperture in the direction OB at an angle $\Phi > \beta$. The locus of rays scattered through the angle θ is a cone of semi-angle θ about ON. CLE is the intersection of this cone with the spherical surface centered at O, and the collection aperture. All the scattered rays passing through CLE are collected. If the radius of the sphere is r_2, the angle $DAC = \cos^{-1}[(\cos\beta - \cos\theta - \cos\Phi)/\sin\theta\sin\Phi]$, so that arc length $CLE = 2AC \times$ angle DAC. The increase of angle θ by $d\theta$ widens this arc to a strip of the width $r_2 d\theta$ which subtends at O a solid angle

$$w_1(\theta, \Phi) = 2\sin\theta\, d\theta \cos^{-1}[(\cos\beta - \cos\theta\cos\Phi)/\sin\theta\sin\Phi]. \quad (6)$$

It is easily seen that if $\theta \leqslant \Phi - \beta$ or $\theta \geqslant \Phi + \beta$ no flux is collected. As required, $w_1 = \theta$, when $\theta = \Phi - \beta$ or $\theta = \Phi + \beta$.

Equation (6) holds for the light scattered through an angle θ from all illuminating rays inclined at the angle Φ to the axis of collection aperture. Thus, the total flux scattered into the collection system for this geometry can be written as

$$F_2 = w_1(\theta, \Phi) I(\theta).$$

From eq. (2), it follows that for $\Phi - \beta < \theta < \Phi + \beta$

$$f_2(\theta, \Phi) = 2\sin\theta \cos^{-1}[(\cos\beta - \cos\theta\cos\Phi)/\sin\theta\sin\Phi]. \quad (7)$$

A plot of $f_2(\theta, \Phi)$ against θ for $\beta = 15°$ and $\Phi = 23°$, 33° and 48° is shown in Fig. 4; $f_2(\theta, \Phi)$ is a smooth curve for all the cases. It is noticed that $f_2(\theta\Phi)$ attains the maximum value for the value of θ slightly greater than Φ.

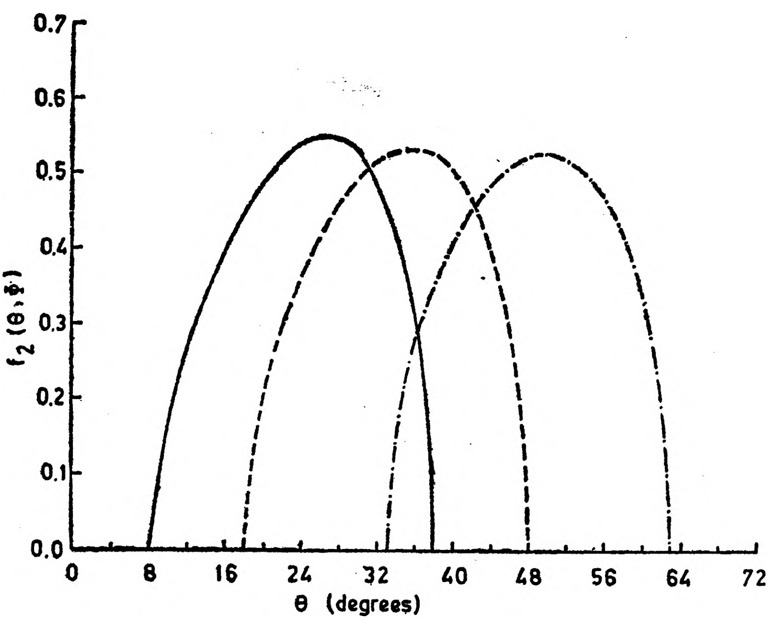

Fig. 4. Variation of $f_2(\theta, \Phi)$ against scattering angle θ for $\beta = 15^0$, $\Phi = 23^0$ (—), $\beta = 15^0$, $\Phi = 33^0$ (— — — —), $\beta = 15^0$, $\Phi = 48^0$ (—.—.—.—)

This is consistent with the observation made by PINNICK et al. [12]. Moreover, as Φ increases, the maximum value of $f_2(\theta, \Phi)$ remains nearly the same, i.e., the collection efficiency is nearly constant.

3.3. Instruments with convergent illumination not co-axial with collecting aperture

In order to increase the level of illumination of this type of instruments the illuminating rays should converge on the view volume. Here, the axis of collection aperture is inclined to the axis of illumination at an angle Ψ such that no light from the illumination cone is intercepted by the collection aperture.

Let the illumination be provided by a lens of circular aperture $IXJW$ (Fig. 5) which subtends an angle 2γ at O where the light converges at the particle,

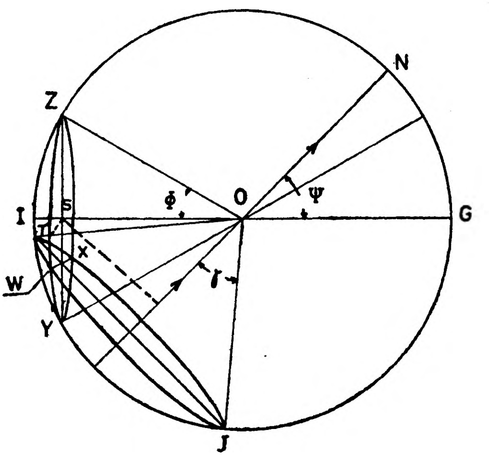

Fig. 5. A scheme of the instruments with convergent illumination not co-axial with collection aperture: Ψ is the angle between axis of illumination and collection aperture, γ is the half angle of the illumination cone

and where the axis is inclined at an angle Ψ to the axis of collection aperture. The illuminating rays making an angle Φ with the collection axis OG will appear to originate from the circular arc WYX, which is the intersection of the conical surface of semi-angle Φ about collection axis, with the surface of the sphere. Thus, illuminating rays inclined to the collection axis OG at the angles Φ and $\Phi + d\Phi$ originate from a band of the length WYX which subtends at O a solid angles

$$w_2(\theta, \Psi) = 2 \sin\theta \, d\theta \cos^{-1}\left(\frac{\cos\gamma - \cos\Phi\cos\Psi}{\sin\Psi\sin\Phi}\right). \tag{8}$$

Thus, for this geometry the scattered flux received from the particle will be

$$F_3 = w_1(\theta) w_2(\theta) I(\theta). \tag{9}$$

So that,

$$F_3(\theta, \Phi) = 4\sin\theta\sin\Phi\left[\cos^{-1}\frac{(\cos\beta-\cos\theta\cos\Phi)}{\sin\theta\sin\Phi}\right]$$
$$\otimes\cos^{-1}\left(\frac{\cos\gamma-\cos\theta\cos\Psi}{\sin\Psi\sin\Phi)}\right), \tag{10a}$$

$$\Phi-\beta\leqslant\theta\leqslant\Phi+\beta \quad \Psi-\gamma\leqslant\Phi\Psi+\gamma. \tag{10b}$$

A variation of $f_3(\theta, \Phi)$ against θ, for different values of Φ at $\beta = 25°$ $\Psi = 90°$ and $\gamma = 5°$ (Royco PC 220) is shown in Fig. 6. It may be seen that as Φ increases, the maximum value of $f_3(\theta, \Phi)$ increases too, and the curves are nearly symmetrical about $\theta = \Phi$.

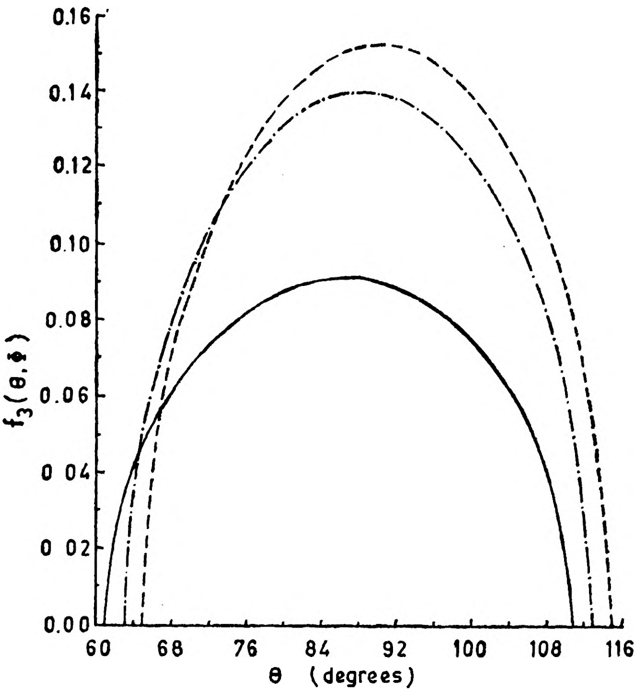

Fig. 6. Variation of $f_3(\theta, \Phi)$ against scattering angle θ, for $\beta = 25$, $\Psi = 90°$, $\gamma = 5°$ (Φ = 86°———, 90° — — — —, 88° —.—.—)

3.4. Instruments with hollow-cone illumination, co-axial with collection aperture inside the cone of darkness

As in the case of dark field microscopy, in this type of instruments illumination lens subtending angle 2ε (see Fig. 7) at the particle is co-axial with the collection lens and has a circular dark stop which subtends at the particle an angle 2δ greater than 2β subtended by the collections lens. Now, the illuminating rays inclined at an angle Φ to the common axis appear to originate from the entire circle YZ of the radius $r_4 \sin\Phi$ made by the intersection of the cone with semi angle Φ about this axis and the sphere of radius r_4 centered at O. Illuminating rays, from Φ to $\Phi + d\Phi$, thus appear to originate from a circular strip of width $r_4 d\Phi$. The solid angle subtended by this strip at O is given by

$$w_3(\Phi) = 2\pi \sin \Phi d\Phi. \tag{11}$$

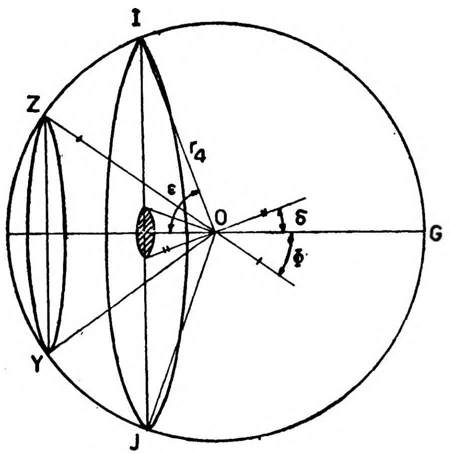

Fig. 7. A scheme of instruments with hollow cone illumination co-axial with collection aperture inside the cone of darkness: 2 ε, 2β and 2δ are the angles subtended by the illumination lens, collection aperture and circular dark stop respectively at the particle

Thus, the flux scattered through angles θ to $\theta + d\theta$ from circular strip of radius $r_4 d\Phi$, collected by the collection aperture is given by

$$F_4 = w_1(\theta) w_3(\Phi) = 2\pi \sin \Phi I(\theta) d\Phi. \tag{12}$$

Thus

$$f_4(\theta, \Phi) = 4\pi \sin \Phi \sin \theta \cos^{-1}\left[\frac{(\cos\beta - \cos\theta\cos\Phi)}{\sin\theta\sin\Phi}\right], \tag{13a}$$

$$\Phi - \beta < \theta < \Phi + \beta, \quad \delta < \Phi < .$$

Figure 8 shows a plot of $f_4(\theta, \Phi)$ against θ for $\beta = 10°$, $\delta = 10^5$, $\eta = 20°$ and $\Phi = 16°$, $17°$ and $18°$, only. Like $f_3(\theta, \Phi)$, $f_4(\theta, \Phi)$ is also smooth but slightly

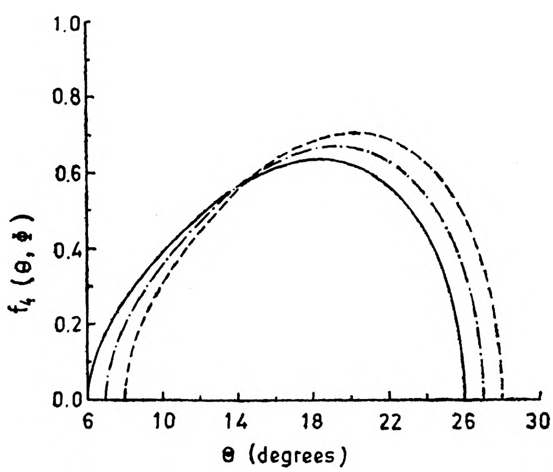

Fig. 8. Variation of $f_4(\theta, \Phi)$ against scattering angle θ, for $\beta = 10°$, $\delta = 15°$, $\eta = 20°$, $\Phi = 16°$ (——), and $\beta = 10°$, $\partial = 15°$, $\eta = 20°$, $\Phi = 18°$ (— — —)

biased towards higher values of θ. As Φ increases, the maximum value of $f_4(\theta, \Phi)$, as well as value of θ where this maximum occurs, increases too. It is interesting to note that for $\theta \sim 14.5°$, $f_4(\theta, \Phi)$ is nearly the same for all the values of Φ. This means that for all the rays inclined at an angle Φ to the common axis the same amount of flux is collected for $\theta = 14.5°$.

3.5. Instruments with convergent illumination, co-axial with collection aperture having a central dark stop

A schematic representation of the geometry of this type of instruments is shown in Fig. 9. Here, BK represents the aperture of the collection lens which subtends an angle 2β at O. IJ is the intersection of cone with semi-angle around

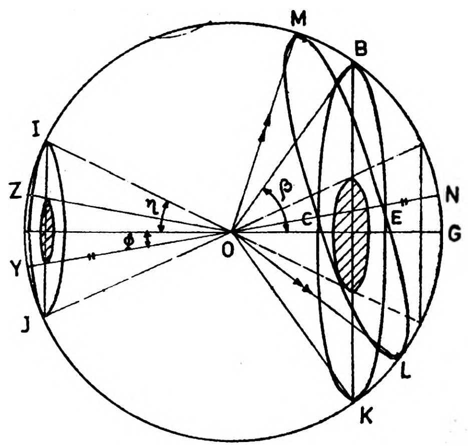

Fig. 9. A scheme of the instruments with convergent illumination co-axial with collection aperture having central dark stop: β, γ and η are the half angles subtended by collection aperture, illumination lens and light trap respectively at the particle

the common axis OG, and sphere of arbitrary centered at O. The formulation of geometrical factor is slightly complicated in this case. This is because the collection of the flux scattered through angles θ and $\theta + d\theta$ by illuminating rays inclined at an angle Φ to the axis of collection is not always complete, as a part of the flux is lost in the light trap. A complete collection of scattered flux can be achieved only when $\eta + \Phi < \theta < \beta - \Phi$. Under these conditions the total flux collected is

$$\begin{aligned} F_4 &= w_3(\Phi) w_0(\theta) I(\theta) \\ &= 4\pi^2 \sin\theta \sin\Phi I(\theta) d\theta d\Phi. \end{aligned} \tag{14}$$

In fact, flux is lost under the following circumstances:

i) $\theta \leqslant \theta \leqslant \eta - \Phi$ — no flux is collected, as the whole scattered flux falls into the trap.

ii) $\eta - \Phi \leqslant \theta \leqslant \eta + \Phi$ — only a part of the scattered light falls into the light trap, and the rest into the collection aperture.

iii) $\beta - \Phi \leqslant \theta \leqslant \beta + \Phi$ — part of flux falls outside the collection aperture and the rest falls into it.

iv) $\beta+\Phi \leqslant \theta \leqslant \pi$ — the whole scattered flux falls outside the collection aperture.

Under condition ii), the flux lost in the dark stop is calculated by putting $\eta = \beta$ in eq. (13a)

$$F'_4 = 4\pi \sin\theta \sin\Phi d\theta \cos^{-1}\left[\frac{(\cos\eta - \cos\theta\cos\Phi)}{\sin\theta\sin\Phi}\right]. \tag{15}$$

Similarly, based on the discussion in the previous section, the flux lost under the condition iii) is given by

$$F'_5 = 4\pi \sin\theta \sin\Phi d\theta d\Phi \left[\pi - \cos^{-1}\left\{\frac{\cos\beta - \cos\theta\cos\Phi}{\sin\theta\sin\Phi}\right\}\right]. \tag{16}$$

Thus, the total flux received by the collection aperture can be written as

$$F_5 = f_5(\theta, \Phi) I(\theta) d\theta d\Phi, \tag{17}$$

where

$$f_5(\theta, \Phi) = A + B + C. \tag{18a}$$

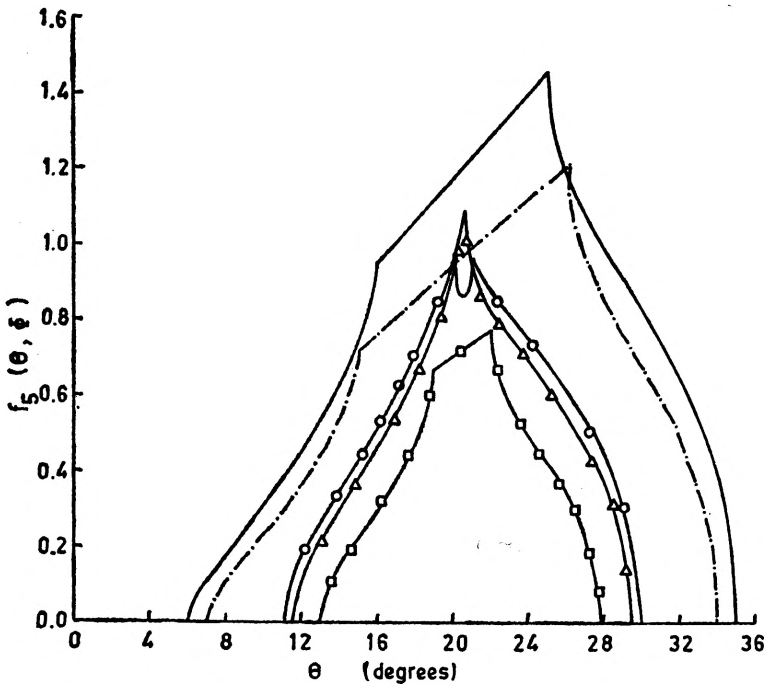

Fig. 10. Variation of $f_5(\theta, \Phi)$ against scattering angle θ, for $\beta = 30^0$, $\eta = 11^0$, $\gamma = 5^0$. $\Phi = 5^0$ (———), $\beta = 30^0$, $\eta = 11^0$, $\gamma = 5^0$, $\Phi = 4^0$ (—·—·—·), $\beta = 25^0$, $\eta = 16^0$, $\gamma = 5^0$, $\Phi = 5^0$ (○———○), $\beta = 25^0$, $\eta = 16^0$, $\gamma = 5^0$, $\Phi = 3^0$ (□———□), and $\beta = 25^0$, $\eta = 16^0$, $\gamma = 5^0$, $\Phi = 4.5^0$ (△———△)

In the latter equation, A represents the total flux that would have been collected if there were no loss of flux falling outside the collection aperture or into the light trap, B and C being the flux lost in the light trap and outside the collection aperture, respectively. The values of A, B and C are calculated according to the following formulae:

$$\left.\begin{aligned} A &= 4\pi^2 \sin\theta \sin\Phi, \qquad (\eta-\Phi \leqslant 0 \leqslant \beta+\Phi),\\ B &= -4\pi \sin\theta \sin\Phi \cos^{-1}\left(\frac{\cos\eta-\cos\theta\cos\Phi}{\sin\theta\sin\Phi}\right), \ (\eta-\Phi \leqslant \theta \leqslant \eta+\Phi),\\ C &= -4\pi \sin\Phi \sin\theta \left[\pi-\cos^{-1}\left(\frac{\cos\beta-\cos\theta\cos\Phi}{\sin\theta\sin\Phi}\right)\right],\\ &\quad (\beta-\Phi \leqslant \theta \leqslant \beta+\Phi). \end{aligned}\right\} \tag{18b}$$

Figure 10 shows the variation of $f_5(\theta, \Phi)$ against θ for $\beta = 25°$, $\eta = 16°$ and $\gamma = 5°$ (Royco 245) and $\beta = 30°$, $\eta = 11°$, $\gamma = 5°$ for different values of Φ.

1. It may be seen that as Φ increases, the range of θ for complete collection decreases, but the value of $f_5(\theta, \Phi)$ increases.

2. $f_5(\theta, \Phi)$ attains its maximum value when $\theta = \beta-\Phi$ and it varies sharply at $\theta = \eta+\Phi$ or $\theta = \beta-\Phi$. If $\beta-\Phi > \eta+\Phi$ the value of $f_5(\theta, \Phi)$ in the interval $\theta = \beta-\Phi$ and $\theta = \eta+\Phi$, rises smoothly. However, if $\beta-\Phi < \eta+\Phi$, $f_5(\theta, \Phi)$ for $\beta-\Phi \leqslant \theta \leqslant \eta+\Phi$ shows a dip in the middle. This is because for this range of θ, the collection of the scattered flux is never complete. Thers is always some part of flux lost in the light trap as well as outside the collection aperture.

3. If $\beta-\Phi = \eta+\Phi$, $f_5(\theta, \Phi)$ shows a sharp peak for $\theta_0 = \beta-\Phi = \eta+\Phi$. This value of $\theta = \theta_0$ divides the $f_5(\theta, \Phi)$ curve into two parts. For $\theta < \theta_0$ there is always a part of flux lost in the light trap. For $\theta > \theta_0$ there is always the same part of flux which falls outside the collection aperture.

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References

[1] Quenzel H., Appl. Opt. **8** (1969), 165.
[2] Quenzel H., Appl. Opt. **9** (1970), 1931.
[3] Cooke D.D., Kerker M., Appl. Opt. **14** (1975), 734.
[4] Heyder J., Gebhart J., Appl. Opt. **18** (1979), 705.
[5] Gucker F.T., Rose D.G., *Third National Air Pollut. Symp.* Pasadena, Ca., April 18–20, 1955.
[6] Hodkinson J.R., Greenleaves I., J. Opt. Soc. Am. **53** (1965), 577.
[7] Gucker F.T., Rowell R.L., Chui G., *First National Conf. on Aerosols*, Liblice, Czechoslovakia, October 8–13, 1962.
[8] Kerker M., *The Scattering of Light and other Electromagnetic Radiations*, Academic Press, New York 1969.
[9] Willeke K., Liu B.Y.H., *Fine Particles*, Ed. B.Y.H. Liu, Academic Press, New York 1976.

[10] HODKINSON J.R., GREENFIELD J.R., Appl. Opt. **4** (1965), 1463.
[11] MARTENS A.E., DOONAN D.D., Appl Opt. **9** (1970), 1930.
[12] PINNICK R.G., ROSEN J.M., HOFMANN D.T., Appl. Opt. **12** (1973), 37.

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Исследования геометрического фактора в оптическом счётчике частиц

Исследована роль геометрического фактора для оптических осветительной и собирающей систем различной геометрии в оптических счётчиках частиц.