Optica Applicata, Vol. XII, No. 2, 1982

Elastooptic effect in a bent slab waveguide

Henryk Marciniak

Institute of Physics, Technical University of Warsaw, ul. Koszykowa 75, Warsaw, Poland.

The elastic strain imposed on the slab waveguide by physical deformation is shown to have a significant influence on a wave propagation. Due to the existing stress the dielectric medium changes into the anisotropic biaxial one. In result the bending loss in the slab waveguide is reduced.

1. Introduction

The physical deformation of a slab waveguide from its undisturbed flat shape resulting, for example, in a bending, is known as inducing mode coupling. Due to this bending the fundamental mode power is coupled to higher order modes and to radiation field, so that the fundamental mode is attenuated. Most of several theories on curvature loss that have been proposed assume [1–3] that the fields at the bend can be approximated by those of a slab waveguide. In practice, the radius of curvature of the bent waveguide is everywhere much greater than its thickness, so that the assumption is satisfied. This makes plausible the assumption that the refractive index profile remains undisturbed as the waveguide is bent. However, in order to be accurate, even to first order in small curvature, the elastooptic effect should be taken into consideration in analysis of a bent waveguide. In the paper [4] an isotropic change of the refractive index due to the density variation of the dielectric material of the waveguide is considered. Here, we consider a bent slab waveguide in which account is taken of the elastooptic effect caused by an elastic strain.

2. Optical anisotropy of a bent slab waveguide

Let us consider a bent slab waveguide which is unlimited in the direction of the y-axis and has the thickness $2d$ along the x-axis. The radius of curvature is R. The structure geometry is shown in Fig. 1.

The elastooptic (or piezooptic) effect can be expressed by the components of the elastic strain tensor or by the components of the elastic stress tensor. In the local rectangular coordinate system the components of the strain tensor of a bent slab can be obtained easily from the theory of the bent elastic slab

$$
\begin{aligned}
u_x &= -\frac{\nu}{1-\nu}\frac{x}{R},\\
u_y &= 0, \qquad\qquad\qquad (1)\\
u_z &= \frac{x}{R},
\end{aligned}
$$

where ν is Poisson's ratio. The remaining components are equal to zero.

In a dielectric medium the optic properties (the square of the refractive index) depend upon the direction of the mechanical stress and the direction of the light wave and are described by a second rank tensor having six independent parameters. In the general case the index ellipsoid referring to the main

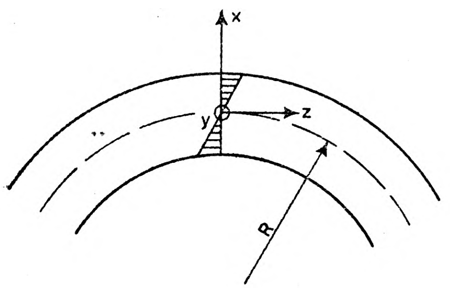

Fig. 1. Geometry of a bent slab

coordinate system becomes [5]

$$a_1 x^2 + a_2 y^2 + a_3 z^2 + 2a_4 yz + 2a_5 zx + 2a_6 xy = 1. \tag{2}$$

The change Δa_i of the six coefficients of the index ellipsoid a_i caused by elastooptic effect can be expressed in form of equations

$$\Delta a_i = p_{ij} u_j \ . \tag{3}$$

These equations contain 36 elastooptic constants, in general. For the isotropic dielectric medium there exist two independent elastooptic constants, that is a longitudinal elastooptic constant $p_1 = p_{11} = p_{22} = p_{33}$ and a lateral elastooptic constant $p_2 = p_{12} = p_{21} = p_{23} = p_{32} = p_{31} = p_{13}$. All the remaining elastooptic constants vanish.

Assuming

$$a_i = \frac{1}{n_i^2}, \tag{4}$$

the following relation can be written

$$\Delta n_i^2 = -n_i^4 \Delta a_i. \tag{5}$$

Taking advantage of the eqs. (1) and (2), the change of the main refractive indices for the case of the bent slab can be written as follows

$$\Delta n_x^2 = -n^4 \left(p_2 - p_1 \frac{\nu}{1-\nu}\right) \frac{x}{R}, \tag{5a}$$

$$\Delta n_y^2 = -n^4 \frac{1-2\nu}{1-\nu} p_1 \frac{x}{R}, \tag{5b}$$

$$\Delta n_z^2 = -n^4 \left(p_1 - p_2 \frac{\nu}{1-\nu}\right) \frac{x}{R}.$$

Hence, the change of refractive indices in the directions of the respective axes is different. Due to the existing strain the isotropic slab waveguide becomes an anisotropic (biaxial) waveguide. The grade of anisotropy changes linearly in the direction perpendicular to the boundary of the waveguide.

3. Transformation of the bent anisotropic slab waveguide into an equivalent straight slab waveguide

In the coordinate system of the main axes of the dielectric permittivity tensor (if a refractive index depends only on x coordinate) the electromagnetic fields of an anisotropic slab waveguide can be classified as TE and TM modes. Therefore, the TE and TM modes will be considered separately.

Moreover, the bent slab waveguide (Fig. 2) with a main refractive index $n = n_i(x)$ (i indicates the x-, y-, z-axes of the coordinate system) may be transformed into an equivalent straight waveguide with effective refractive index profile [6]

$$n_{i,\mathrm{ef}}^2 = n_i^2(x) + \Delta n_i^2(x, z). \tag{6}$$

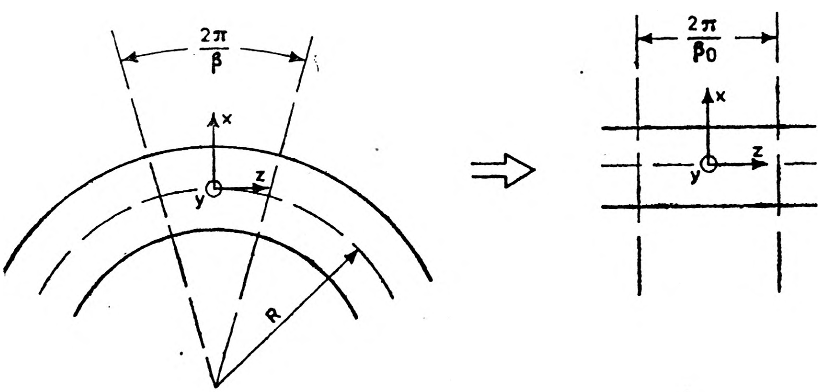

Fig. 2. Transformation of a bent into an equivalent straight waveguide [6]

The subscript i which distinguishes one of the three main refractive indices is different for different directions of wave propagation. $\Delta n_i^2(x, z)$ represents the influence of curvature of the radius R as a function of the z-coordinates.

The perturbation $\Delta n_i^2(x, z)$ is obtained from the condition that the dependence of the transverse field on x in the bent and the equivalent straight waveguides must be equal.

For the case of the TE mode, an anisotropic slab waveguide has the same properties as the isotropic one with the refractive index profile $n^2 = n_y^2(x)$. The transverse wavenumber u in the bent slab equals

$$u^2 = k^2 n_y^2(x) - \beta^2, \tag{7}$$

where k denotes the free space wavenumber. β in eq. (7) denotes the local propagation constant perpendicular to the x, y-plane. In contrast to propagation constant β_0 of equivalent slab waveguide β depends on x. According to Fig. 2 the local propagation constant in the bent slab depends on x as

$$\beta = \beta_0 \frac{R}{R+x}, \tag{8}$$

where β_0 denotes the propagation constant on the z-axis. By assuming that the radius of curvature R is large, thus that $R \gg x$ holds for all values of x up to which the fundamental mode field extends, and $\beta_0 \approx kn_1$ (n_1 is a refractive index on the z-axis) eq. (7) may be written in form

$$u^2 = k^2 n_{y,\mathrm{ef}}^2 - \beta_0^2 \tag{9}$$

with

$$n_{y,\mathrm{ef}}^2 = n_y^2(x) + 2n_1^2 \frac{x}{R}. \tag{10}$$

A slab waveguide with refractive index $n_{y,\mathrm{ef}}^2$ has the same properties as a bent slab waveguide with curvature radius R. Since due to elastic strain (eq. (5b)) the actual refractive index profile of the bent slab differs from $n^2(x)$, the total change of refractive index profile of the equivalent slab waveguides equals.

$$\Delta n_y^2 = 2n_1^2 \frac{x}{R} - n_i^4 \frac{1-2\nu}{1-\nu} p_{1,i} \frac{x}{R}. \tag{11}$$

Here, the subscript $i = 1, 2, 3$ denotes core region, substrate and region above the core, respectively. The first component of this equation results solely due to the change of geometrical shape of the deformed waveguide, whereas the second component – to the elastooptic effect. Eq. (11) may be written in form

$$\Delta n_y^2 = 2n_1^2 \frac{x}{R} (1 - p_{y,i}) \tag{12}$$

with

$$p_{y,i} = \frac{1}{2} \frac{n_i^4}{n_1^2} \frac{1-2\nu}{1-\nu} p_{1,i}. \tag{13}$$

For the case of the TM mode the field of an anisotropic slab waveguide differs considerably from the identical one in the isotropic slab waveguide. Now, the transverse dependence of a field component can be expressed by the wave equation (assuming that the gradient of the refractive index is sufficiently small)

$$\frac{1}{\varepsilon_z} \frac{d^2 E_z}{dx^2} + \left(k^2 - \frac{\beta^2}{\varepsilon_z}\right) E_z = 0. \tag{14}$$

The transverse wavenumber u in the bent slab waveguide is obtained as

$$\frac{u^2}{n_z^2}+\frac{\beta^2}{n_x^2}=k^2. \tag{15}$$

Transformation of this equation leads to the effective refractive indices of the equivalent waveguides (by the virtue of the same assumption as in the case of TE mode)

$$n_{x,\mathrm{ef}}^2=n_x^2(x)+2n_1^2\frac{x}{R}, \tag{16}$$

$$n_{z,\mathrm{ef}}^2=n_z^2(x)+2n_1^2\frac{x}{R}. \tag{17}$$

The total change of the refractive indices due to an elastooptic effect (eqs. (5a), (5c)) is

$$\Delta n_x^2=2n_1^2\frac{x}{R}(1-p_{x,i}), \tag{18}$$

$$\Delta n_z^2=2n_1^2\frac{x}{R}(1-p_{z,i}), \tag{19}$$

where:

$$p_{x,i}=\frac{n_i^4}{2n_1^2}\left(p_{2,i}-p_{1,i}\frac{\nu}{1-\nu}\right), \tag{20}$$

$$p_{z,i}=\frac{n_i^4}{2n_1^2}\left(p_{1,i}-p_{2,i}\frac{\nu}{1-\nu}\right) \tag{21}$$

depends only on parameters of the layer with subscript i.

4. Transition loss in bent slab waveguide

As already shown, a slab waveguide with an effective refractive index has the same properties as a bent slab. The curvature radius of a bent slab and the change of a refractive index profile can be the functions of waveguide length. The mode coupling theory [7] can be applied to the case of a small change in the refractive index profile.

The transverse electromagnetic fields components of a slab waveguide having the effective refractive index can be expressed by a series expansion in terms of guided and radiation modes

$$\mathbf{E}=\sum_{\nu}(a_\nu(z)+b_\nu(z)\mathbf{E}_\nu(x), \tag{22}$$

$$\mathbf{H}=\sum_{\nu}(a_\nu(z)-b_\nu(z)\mathbf{H}_\nu(x), \tag{23}$$

where $\mathbf{E}_\nu$, $\mathbf{H}_\nu$ are the transverse modal fields and $a_\nu(z)$, $b_\nu(z)$ are their z-dependent mode amplitudes. For simplicity of notation the single summation symbol is used to indicate both the sum over the guided modes and the sum and integral over the radiation modes. Substitution of eqs. (21) and (22) into Maxwell's equations and the use of the orthogonality conditions

$$2\int_{-\infty}^{+\infty} \mathbf{E}_\nu \times \mathbf{H}_\mu^* dx = \delta_{\nu\mu} \tag{24}$$

fead to the system of coupled equations. It is assumed here that there exists at the beginning of a bend only the fundamental mode and that the curvature radius R is not too small, thus that there is only a weak coupling between the lundamental mode and the other modes. By introducing slowly varying mode amplitudes

$$\begin{aligned} A_\mu &= a_\mu e^{+i\beta_\mu z}, \\ B_\mu &= b_\mu e^{-i\beta_\mu z}, \end{aligned} \tag{25}$$

the system of coupled equations can be solved by perturbation theory. A complete set of forward and backward waves can thus be described by

$$\begin{aligned} A_\varrho(L) &= A_0 K(\beta) \int_0^L \frac{1}{R(z)} e^{-i(\beta_0-\beta)z} dz, \\ B_\varrho(0) &= A_0 K(\beta) \int_0^L \frac{1}{R(z)} e^{-i(\beta_0+\beta)z} dz, \end{aligned} \tag{26}$$

where β_0 is the propagation constant of the fundamental mode, β and ϱ are the propagation constant and the transverse wavenumber in the substrate of the radiation mode, respectively.

The coupling coefficient $K(\beta)$ between the fundamental mode and the radiation mode of order ϱ is given by

$$K(\beta) = i\omega\varepsilon_0 \int_{-\infty}^{+\infty} R(z)(\Delta n_x^2 E_{x0} E_{x\varrho} + \Delta n_y^2 E_{y0} E_{y\varrho} + \Delta n_z^2 E_{z0} E_{z\varrho}) dx. \tag{27}$$

Finally, the power contained in the radiation mode at the end of the bend is

$$P_r = |A_0|^2 \int_{-n_2 k}^{+n_2 k} |K(\beta)|^2 F_R(\beta, L) \frac{\beta}{\varrho} d\beta, \tag{28}$$

with

$$F_R(\beta, L) = \int_0^L \frac{1}{R(z)} e^{-i(\beta_0-\beta)z} dz. \tag{29}$$

The power density for the radiation mode is given by

$$\frac{dP_r}{d\beta} = |A_0|^2 |K(\beta)|^2 F_R(\beta, L) \frac{\beta}{\varrho}. \tag{30}$$

Thus, the power loss coefficient of the bent slab waveguide is proportional to the square of the coupling coefficient. It is interesting to know to what extent the elastooptic effect in a bent waveguide changes the bending loss if compared with the expression which ignores the bending strain effect. The coupling coefficient may be written in the following form

$$K(\beta) = K_k(\beta)\big(1 - s(\beta)\big), \tag{31}$$

with

$$s(\beta) = \frac{K_e(\beta)}{K_k(\beta)}, \tag{32}$$

where K_k denotes coupling coefficient describing the coupling caused by a change of the geometrical shape of the waveguide alone and K_e is an additional coupling due to elastooptic effect (compare eqs. (12), (18), and (19)). Hence, the elastooptic effect causes a $\big(1-s(\beta)\big)^2$-fold change of a bending loss. Because the coefficient $s(\beta)$ is positive the loss of a bent slab waveguide is less than in the waveguide which has „no" elastic strain. Therefore, the elastooptic effect leads to the reduction of bending loss by the factor $(1-s)^2$.

For the guided TE mode the coefficient $s(\beta)$ is obtained, as

$$s(\beta) = \frac{p_{y,1}\int_{-d}^{d} xE_{y0}E_{y\varrho}\,dx + p_{y,2}\int_{-\infty}^{-d} xE_{y0}E_{y\varrho}\,dx + p_{y,3}\int_{d}^{+\infty} xE_{y0}E_{y\varrho}\,dx}{\int_{-\infty}^{+\infty} xE_{y0}E_{y\varrho}\,dx}, \tag{33}$$

and for the guided TM mode, as

$$s(\beta) = \frac{1}{\int_{-\infty}^{+\infty} x(E_{x0}E_{x\varrho} + E_{z0}E_{z\varrho})\,dx}\left\{\int_{-d}^{d} (p_{x,1}xE_{x0}E_{x\varrho} + p_{z,1}xE_{z0}E_{z\varrho})\,dx\right.$$
$$\left. + \int_{-\infty}^{-d} (p_{x,2}xE_{z0}E_{z\varrho} + p_{z,2}xE_{z0}E_{z\varrho})\,dx + \int_{d}^{+\infty} (p_{x,3}xE_{x0}E_{x\varrho} + p_{z,3}xE_{z0}E_{z\varrho})\,dx\right\}. \tag{34}$$

From the value $s = 0.18$ obtained for fused silica it can be seen that by taking account of bending strain a significant modification of the result can be obtained for mode coupling.

Acknowledgement — The author is grateful to Doc. Dr. Jan Petykiewicz for stimulating discussion.

References

[1] Lewin L., IEEE Trans. MTT-22 (1974), 718–727
[2] Marcuse D., J. Opt. Soc. Am. **66** (1976), 311–320.
[3] Gambling W. A., Matsumura H., Ragdale C. M., Opt. Quant. Elect. **11** (1979), 43–49.
[4] Hannay J. H., Elect. Lett. **12** (1976), 173–174.
[5] Born M., Wolf E., *Principles of Optics*, Pergamon Press, New York 1965.
[6] Petermann K., Elect. Lett. **12** (1976), 107–109.
[7] Marcuse D., *Theory of Dielectric Optical Waveguides*, Academic Press, New York 1974.

Received June 16, 1981

Эластооптический эффект в дифрагированном планарном волноводе

В работе обсуждается влияние напряжений, существующее в дифрагированном планарном волноводе, на связь мод и связанную с этим потерю мощности проводимых мод. С точки зрения теории упругости планарный волновод является плитой. Распределение напряжений в дифрагированной плите вызывает то, что первично изотропная среда после дифракции становится двухосной анизотропной средой. Величина анизотропии не является постоянной, но изменяется линейно в направлении перпендикулярном к слою. Градиент коэффициента преломления вызывает то, что распространяющееся поле дифрагируется в направлении, согласном с волноводом. Из анализа потери мощности на излучение проводимой моды, вызванной дифракцией волновода, следует, что эластооптический эффект в значительной степени уменьшает дифракционные потери.