For illustration two examples of interference patterns appearing in holographic and speckling interferometry, respectively, are given in Fig. 4. The holographic interferogram containes 19 fringes which indicates the bending of order of $4 \mu \mathrm{~m}$, while the speckling interferogram containing 6 fringes indicates the bending of about $27 \mu \mathrm{~m}$. When using the holographic method the last bending would produce the number of fringes which, being at the limit of resolving power of the eye, would be difficult to analyse.

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# Imaging performance of angular and apodized aperture* 

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## 1. Introduction

The complex degree of coherence is defined, according to Van Zittert [1], by the normed Fourier transform of the source intensity function. In this case the source is represented by the diaphragm, the dimensions of which and optical field distribution are identical with those of the source. The intensity distribution in the source plane may be modified by introducing a central coverage. This leads to a one-dimensional change of the diffraction image of the source, i.e., an increase of the central converage degree lowers the contrast and improves the resolution [2,3]; this is the case in visualization of the objects in microscopy or in objective interferometry, for instance.

It is possible, moreover, to apply a programmed modification of the optical field by using an amplitude apodizer, i.e., by replacing the diaphragm-source by a profiled diaphragm and a cylindric lens [4]. The optical field of the source may be then modified by changing only the profile of the diaphragm. In this work the results obtained by applying both these methods are compared.

## 2. Coherence of the apodized source and the ring source

An extended circular and noncoherent source located in the ( $X_{0}, Y_{0}$ ) plane and equivalent to the circle diaphragm of the radius ( $r$ ) produces in the ( $X, Y$ ) plane the partial coherence described by its complex degree $\mu_{12}$ :

[^0]

Fig. 1. Light source-diffraction plane

$$
\begin{equation*}
\mu_{12}=\frac{\exp (i \Psi) \iint_{S} I\left(X_{0}, Y_{0}\right) \exp \left[\frac{-2 \pi i}{\lambda}\left(p X_{0}+q Y_{0}\right)\right] d X_{0} d Y_{0}}{\iint_{S} I\left(X_{0}, Y_{0}\right) d X_{0} d Y_{0}}, \tag{1}
\end{equation*}
$$

where ( $X_{0}, Y_{0}$ ) - coordinates of a point in the source plane (Fig. 1),
$I\left(X_{0}, Y_{0}\right)$ - light source intensity,

$$
p=\frac{X_{1}-X_{2}}{R}, q=\frac{Y_{1}-Y_{2}}{R},
$$

( $X_{i}, Y_{i}$ ) - coordinates of the point in the diffraction image plane,

$$
\Psi=\frac{2 \pi}{\lambda}\left[\frac{\left(X_{1}^{2}+Y_{1}^{2}\right)-\left(X_{2}^{2}+Y_{2}^{2}\right)}{2 R}\right] .
$$

For a uniform circular source of the radius $(r)$ and transmittance $T(r)=1$, under the condition $O P_{1}$ $-O P_{2} \ll \lambda$ (Fig. 1), the formulae for the degree of partial coherence in the image plane may be written as

$$
\begin{equation*}
\mu_{12}(v)=\frac{2 J_{1}(v)}{v}, \tag{2}
\end{equation*}
$$

where $v=\frac{2 \pi r}{\lambda} \sqrt{p^{2}+q^{2}}-$ transversal shift.
The apodizer of transmittance $T(r)=1-r^{2}$ produces in the image plane the partial coherence described by the formula

$$
\begin{equation*}
\mu_{12}(v)=\frac{8}{v^{2}}\left[\frac{2 J_{1}(v)}{v}-J_{0}(v)\right] . \tag{3}
\end{equation*}
$$

Correspondingly, the apodizer of transmittance $T(r)=\frac{1}{2}\left(1+r^{2}\right)$ produces the degree of partial coherence given by:

$$
\begin{equation*}
\mu_{12}(v)=\frac{8}{3 v}\left[J_{1}(v)-\frac{2}{v^{2}} J_{1}(v)+\frac{1}{v} J_{0}(v)\right], \tag{4}
\end{equation*}
$$

while the apodizer of transmittance $T(r)=r^{2}$

$$
\begin{equation*}
\mu_{12}(v)=\frac{4}{v}\left[J_{1}(v)-\frac{4}{v^{2}} J_{1}(v)+\frac{2}{v} J_{0}(v)\right] . \tag{5}
\end{equation*}
$$

The results are shown in Fig. 2. The respective distributions of the light intensities in the diffraction images of the sources are shown in Fig. 3. The part of energy ( $E$ ) which is contained in the increasing regions ( $\boldsymbol{v}$ ) of the diffraction images, the centrum of which is the geometrical centre is shown in Fig. 4.


Fig. 2. Coherence of the apodized source of transmittance $T(r)$ and the uniform source of central coverage: a - radius of circular coverage normed by the source radius. I-non-apodized, 11-1-r $r^{2}, 111-\frac{1}{1}\left(r+r^{2}\right), I V-r^{2}$



Fig. 3. The intensity distribution in the diffraction image of the apodized source and the ring source. I-non-apodized, II-1-r2, III - $\frac{1}{\mathbf{2}}\left(1+r^{2}\right)$, IV $-r^{2}$


Fig. 4. The energy distribution in the increasing regions examined

Table. Coherent areas $P_{1} P_{2}=\frac{\nu \lambda R}{2 \pi r}$ for the apodized sources of transmittance $T(r)$ and for uniform sources covered centrally (a degree of coverage)

| Source | $1-r^{2}$ | 1 | $a=0.2$ | $\frac{1}{2}\left(1+r^{2}\right)$ | $a=0.4$ | $r^{2}$ | $a=0.6$ | $a=0.8$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $v$ | 1.22 | 1 | 0.95 | 0.93 | 0.90 | 0.80 | 0.78 | 0.73 |
| $\left(P_{1} P_{2}\right)$ | 0.194 | 0.159 | 0.151 | 0.148 | 0.143 | 0.127 | 0.124 | 0.116 |

Greater area of the object is coherent
Smaller area of the object is coherent

By assuming that for a uniform diaphragm of transmittance $T(r)=1$ and of angular size $a=r / R$ the region illuminated coherently has the sizes $P_{1} P_{2}=\frac{0.159 R}{r}$, [1], the coherent areas have been determined for the remaining sources. The results are given in Table.

From these results it follows that the sources of transmittance $T(r)=r^{2}$ and $T(r)=\frac{1}{2}\left(1+r^{2}\right)$ act in the same directions as the uniform sources covered centrally, i.e., they cause a decrease of the coherence area as compared to those produced by the source of transmittance $T(r)=1$. On the other hand, the apodizer of transmittance $T(r)=1-r^{2}$ produces higher coherence than a uniform source.

## 3. Two-point resolution of Sparrow-type. Energy contrast

The result obtained has been verified by calculating numerically the Sparrow resolution for a two-point object and the energy contrast for a periodic object. In Fig. 5 the critical values ( $\delta_{0}$ ) of the distance between two object points have been shown as a function of the coherence degree. In Fig. 6 the energy contrast


Fig. 5. Critical Sparrow distance ( $\delta_{0}$ ) between two-point objects for different source types


Fig. 6. Energy contrast of a cosinusoidal test of modulation depth equal to 0.5 . I non-apodized, II $-1-r^{\mathbf{2}}, \mathrm{III}-\frac{1}{\mathbf{2}}\left(1+r^{\mathbf{2}}\right)$
is shown for a periodic test of the modulation depth equal to 0.5 . From the results obtained it may be seen that when the apodizers are used a stimulated modulation of the optical field is possible, which requires a solution of the respective optimization problem. An example of apodizing diaphragm profile determination for the required resolution is given in [5].

Similarly, by introducing an apodizer the contrast may be changed depending on the modulation depth in the object. This topic will be widely discussed in the work [6] prepared for publication.

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