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More about the interference colours

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In the physics textbook or professional literature many controversary opinions may be found about the possibility of measuring the optical path differences with help of interference or pseudointerference polariscope colours. These controversies are mainly due to the acceptance of some incorrect assumptions. In this paper a possibility is discussed of applying the interference and polariscope colours to the real conditions of measurement as a measure of optical path difference.

1. Introduction

Interference colours are the subject of many publications being discussed in almost every textbook on physics as well as in every textbook on optical metrology and optics of anisotropic media. However, the respective information is usually simplified. In particular, the readers get an impression that the Newtonian interference colours in the reflected or transmitted light are identical with those appearing in the chromatic polarization (see [1], for instance) and that the respective colours are associated with the definite differences of optical path. The optics people, metrologists and mineralogists may quote many publications with tables in which the colours are associated with the optical path differences. MÖNCH cites in [2] also the chromaticity graphs with colour lines in reflected light for thin collodium layers, taken from work [3] by Wichert. He suggests that by measuring the colour of the light reflected from a thin film it is possible to read out the thickness of the latter from the chromaticity graph. All the publications cited are based on certain simplifying assumptions which, unfortunately, have not been specified. These simplifying assumptions as well as the possibility of exploiting the interference colours in metrology are the subject of considerations in this paper.

2. Intensity of light reflected and transmitted through the isotrope plane-parallel plates

Let a heterochromatic light of amplitude $A_0(\lambda)$ which corresponds to the light intensity $I_0(\lambda)$ fall on an isotropic plate of thickness d (see Fig. 1). The light reflected from its first surface is of amplitude $K_1(\lambda)A_0(\lambda)$, while that reflected from the other surface has the amplitude $K_2(\lambda)A_0(\lambda)$. Both the beams are shifted by the optical path difference $R_0(\lambda) = 2dn(\lambda) + \lambda/2$ with respect to each other. In the transmitted beam the respective amplitudes are $K_3(\lambda)A_0(\lambda)$ and $K_4(\lambda)A_0(\lambda)$ shifted with respect to each other by $R_p(\lambda) = 2dn(\lambda)$, where $n(\lambda)$ is a relative light refractive index of the plate.

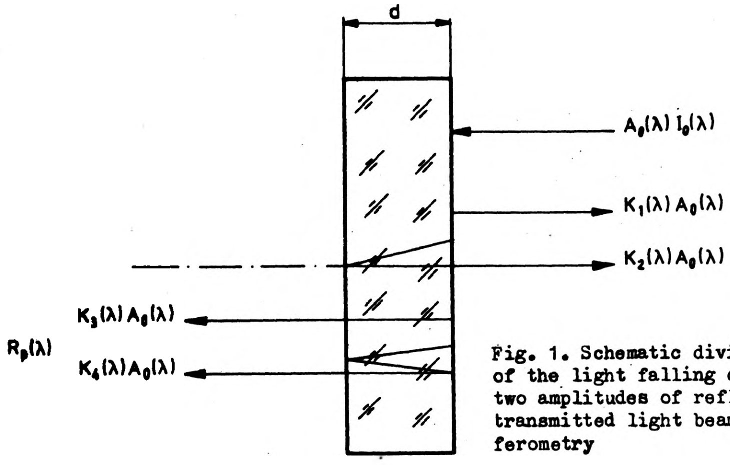


Fig. 1. Schematic division of amplitude A_0 of the light falling on a thin plate into two amplitudes of reflected and two of trans-transmitted light beams in two-beam interferometry

From the elementary considerations the intensities of the light due to interface of the respective beam pairs are:

- for the light reflected

$$I(\lambda) = I_0(\lambda)\left[(K_1(\lambda) + K_2(\lambda))^2 - 4K_1(\lambda)K_2(\lambda)\sin^2\pi\frac{R_0(\lambda)}{\lambda}\right], \quad (1)$$

- for the light transmitted

$$I(\lambda) = I_0(\lambda)\left[(K_3(\lambda) + K_4(\lambda))^2 - 4K_3(\lambda)K_4(\lambda)\sin^2\pi\frac{R_p(\lambda)}{\lambda}\right]. \quad (2)$$

3. Intensity of the light transmitted through a polariscope system

A linear dichroic plane-parallel plate O of the azimuth 45° is located between the polarizer P of azimuth 90° and analyzer A of azimuth 0° or 90°.

A simplifying assumption is accepted that the polarizer P is perfect, i.e., that the light of the amplitude $A_0(\lambda)$ incident on the plate is vibrating in vertical plane (Fig. 2a). In the entrance surface the light is decomposed into two beams polarized linearly, as shown in Fig. 2b. For one of them the coefficient of amplitude transmission in the plate is $K_1(\lambda)$, while for the other – $K_2(\lambda)$.

In the exit surface both the amplitudes diminish nonuniformly to the values presented in Fig. 2c. The respective amplitude transmission coefficients of the analyser in the direction consistent with the vector $A_0(\lambda)$ and perpendicular to the latter are $t_0(\lambda)$ and $t_{90}(\lambda)$.

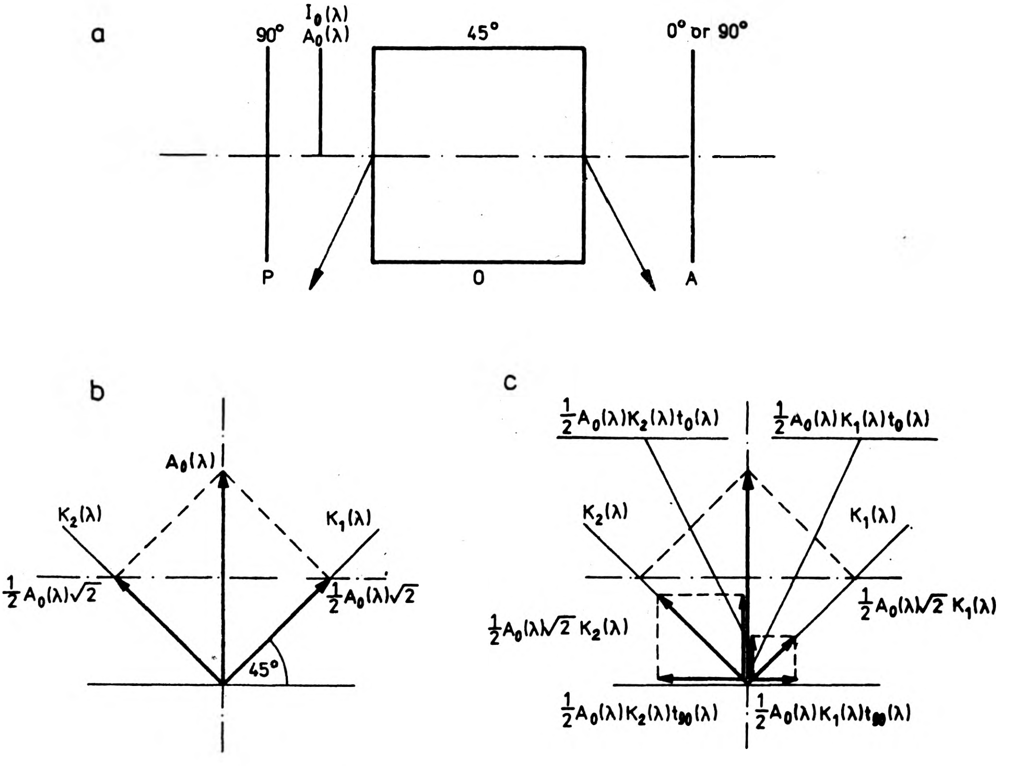


Fig. 2. Schematic division of the amplitude A_0 falling on a birefringent object in the polariscope into two amplitudes of waves passing through the analyser

The light intensity emerging from the polariscope and calculated from these data is

$$I(\lambda) = I_0(\lambda)\left\{t_{90}^2(\lambda)\left[\frac{(K_1(\lambda) - K_2(\lambda))^2}{4} + K_1(\lambda)\,K_2(\lambda)\,\sin^2\pi\frac{R(\lambda)}{\lambda}\right] + t_0^2(\lambda)\left[\frac{(K_1(\lambda) + K_2(\lambda))^2}{4} - K_1(\lambda)\,K_2(\lambda)\,\sin^2\pi\frac{R(\lambda)}{\lambda}\right]\right\}, \tag{3}$$

where $R(\lambda)$ is the optical path difference produced in the birefringent plate.

When knowing the spectral density of the light intensity determined by the formulae (1), (2), and (3) its chromaticity coordinates may be determined from the known formulae:

$$x = \frac{\int_{380}^{780} I(\lambda)\,\bar{x}(\lambda)d\lambda}{\int_{380}^{780} I(\lambda)\,\bar{x}(\lambda)d\lambda + \int_{380}^{780} I(\lambda)\,\bar{y}(\lambda)d\lambda + \int_{380}^{780} I(\lambda)\,\bar{z}(\lambda)d\lambda},$$

$$y = \frac{\int_{380}^{780} I(\lambda)\,\bar{y}(\lambda)d\lambda}{\int_{380}^{780} I(\lambda)\,\bar{x}(\lambda)d\lambda + \int_{380}^{780} I(\lambda)\,\bar{y}(\lambda)d\lambda + \int_{380}^{780} I(\lambda)\,\bar{z}(\lambda)d\lambda}, \tag{4}$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ are CIE 1931 tristimulus values.

The denominators of both the expressions are normed so that:

$$\int_{380}^{780} I_0(\lambda)\,\bar{y}(\lambda)d\lambda = 100.$$

In this case interval $\int_{380}^{780} I(\lambda)\,\bar{y}(\lambda)\,d\lambda$ expresses the percentage coefficient of light reflection or transmission.

From the formulae (4) it follows that in the general case the interference and polarization colours for the same value R of the optical path difference and for the same wavelength are not equal to one another. In the opposite case the formulae (1), (2) and (3) should be of identical form. Also, in a particular case when formula (3) simplified to a nondichroic crystal (i.e., when $K_1(\lambda) = K_2(\lambda)$) is not consistent with the formula (1) or (2), even when the interfering beams are of equal energies (i.e., $K_1(\lambda) = K_2(\lambda)$ and $K_3(\lambda) = K_4(\lambda)$). In practice such assumptions are difficult to be fulfilled with a sufficient accuracy within the whole spectral range.

Let us consider a purely academic case that everything is perfect. Under the assumption that both the interfering beams reflected from (or transmitted through) the plane parallel plate are equi-energetic $K_1(\lambda) = K_2(\lambda) = K(\lambda)$, the formulae (1) and (2) have the forms:

- for the reflected light

$$I(\lambda) = 4I_0(\lambda)\, K^2(\lambda)\, \sin^2\pi \frac{2dn(\lambda)}{\lambda}, \tag{5}$$

- for the transmitted light

$$I(\lambda) = 4I_0(\lambda)\, K^2(\lambda)\, \cos^2\pi \frac{2dn(\lambda)}{\lambda}. \tag{6}$$

Assuming for the polariscope that from the birefringent medium there emerge to equi-energetic beams (which is possible only approximately) we have $K_1(\lambda) = K_2(\lambda) = K(\lambda)$. If also the analyzer is perfect then for crossed polarizers $t_{90}(\lambda) = 1$, $t_0(\lambda) = 0$, while for parallel polarizers - $t_{90}(\lambda) = 0$, $t_0(\lambda) = 1$. This leads to new forms of the above formulae:

- for the crossed polarizers

$$I(\lambda) = I_0(\lambda)\, K^2(\lambda)\, \sin^2\pi \frac{R(\lambda)}{\lambda}, \tag{7}$$

- for the parallel polarizers

$$I(\lambda) = I_0(\lambda)\, K^2(\lambda)\, \cos^2\pi \frac{R(\lambda)}{\lambda}. \tag{8}$$

From the similarity of the formulae it may be concluded that in such an abstract case the interference colours in the reflected light are identical with the polariscope colours obtained for crossed polarizers and the interference colours in the light transmitted are equal to those in polariscope for parallel polarizers. However, this is true only when the dispersion of the refractive index $n(\lambda)$ is equal to the birefringence dispersion $R(\lambda)$. The interference colours appearing in an air wedge as well as polariscope colours appearing in the quartz wedge may serve as an example, obviously, bearing in mind all the assumptions made earlier.

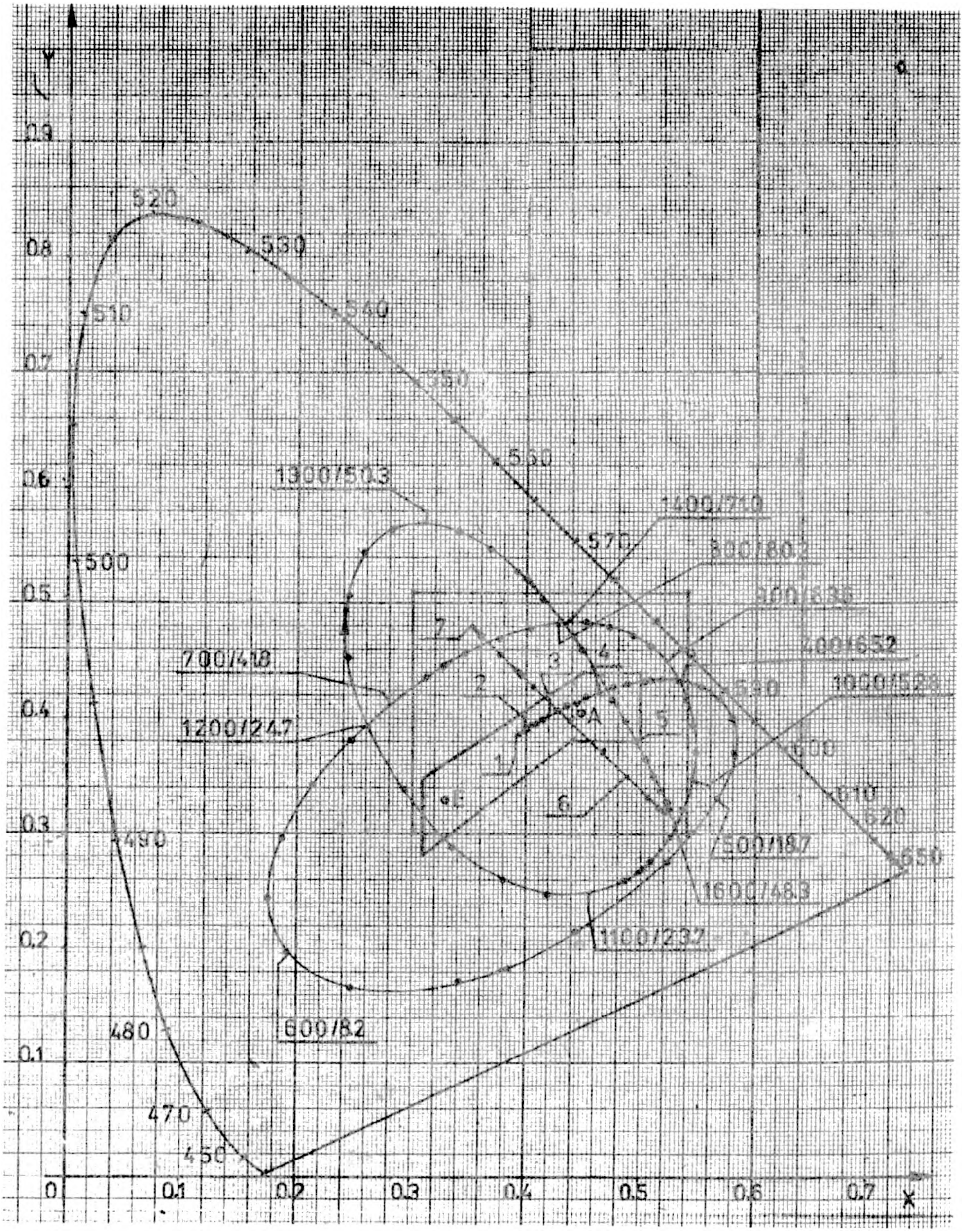


Fig. 3 a. Graphs of polariscope colours for illuminant A as functions of optical path differences for calcite and the wavelength λ = 536 nm. The colours are observed for crossed polarizers. In the denotations the optical difference R(536) is given which is followed by the transmission coefficient. 1 - 20/1.2, 2 - 100/27.2, 3 - 200/78.6, 4 - 300/97.7, 5 - 1500/67.7, 6 - 1700/34.6, 7 - 1800/38.1

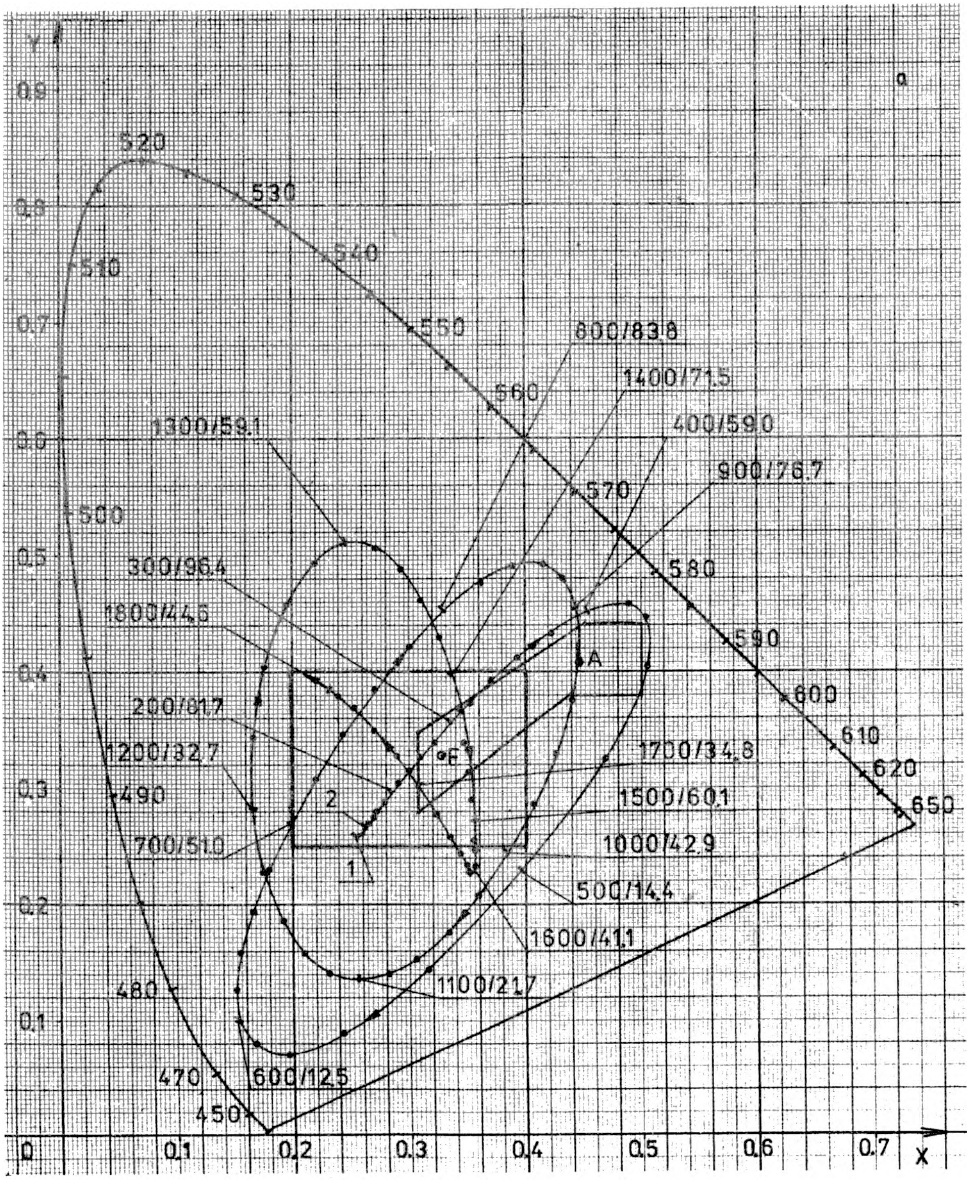


Fig. 4 **a.** Graphs of the polariscope colours for illuminant C observed for crossed polarizers. The other explanations like in Fig. 3. 1 - 20/1.2, 2 - 100/28.7

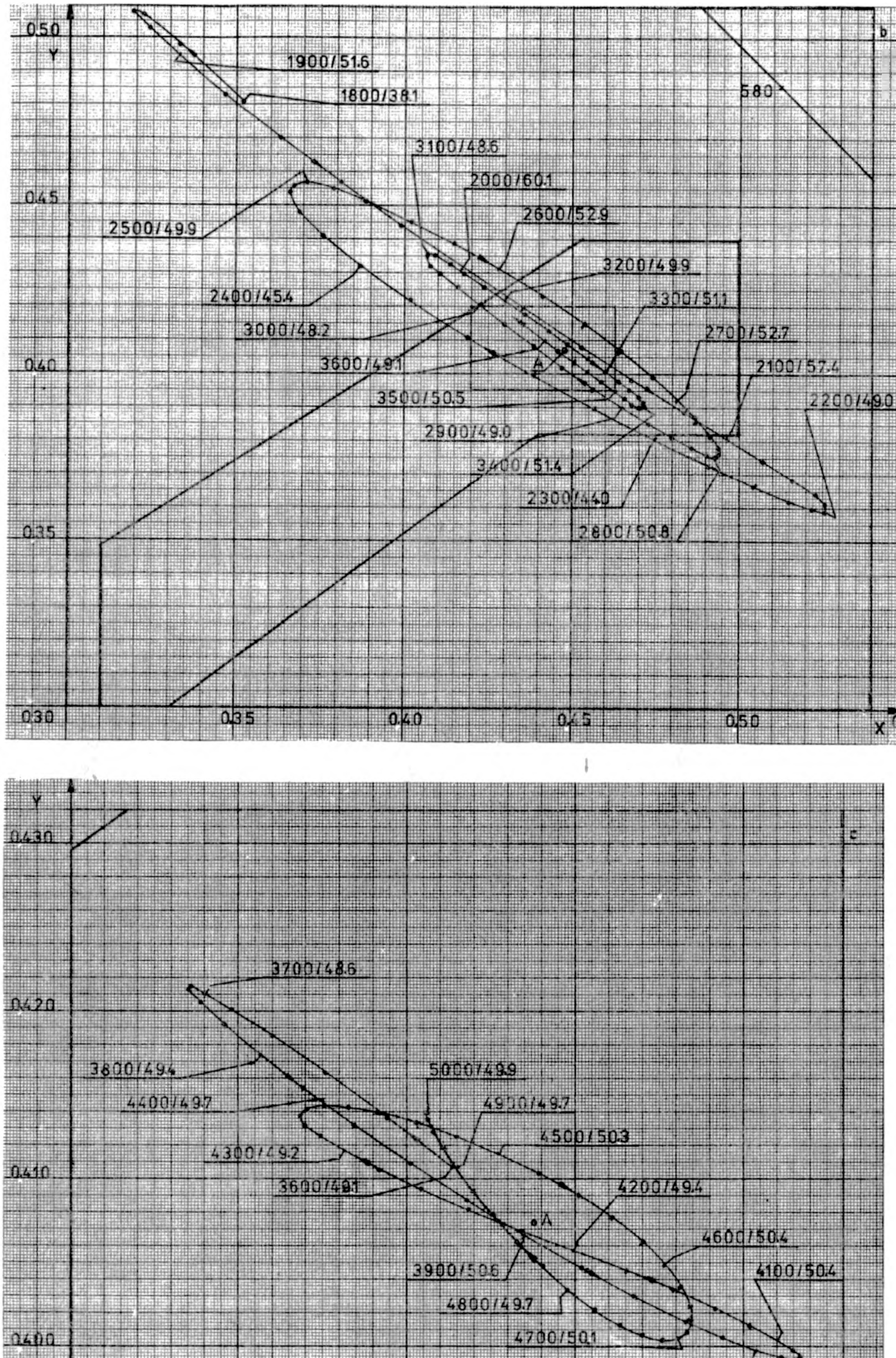
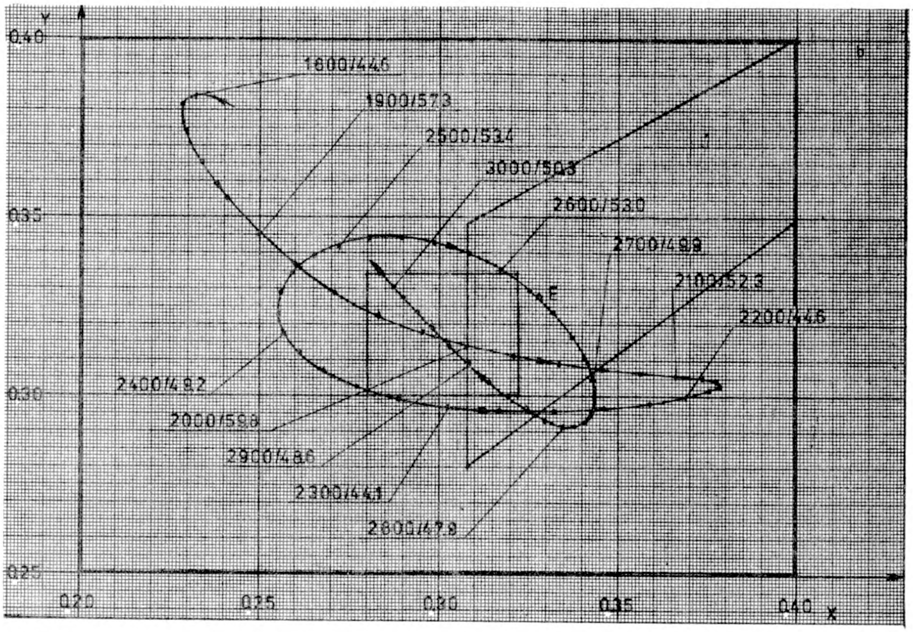


Fig. 3 b, c. Enlarged sections of the previous Figure 3a



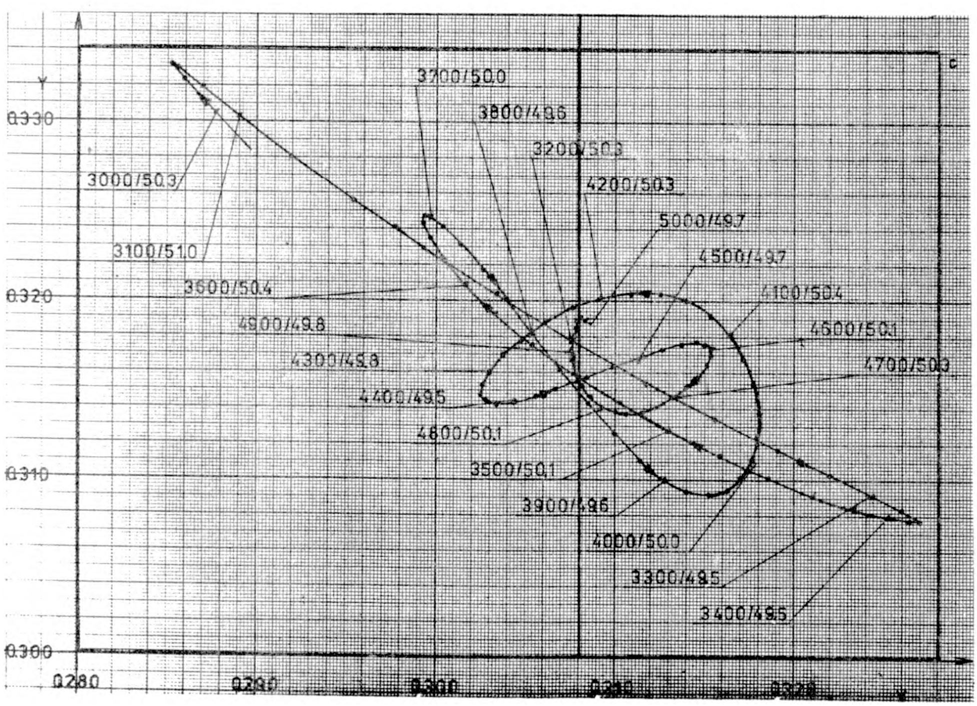


Fig. 4 b, c. Enlarged sections of the previous Figure 4a

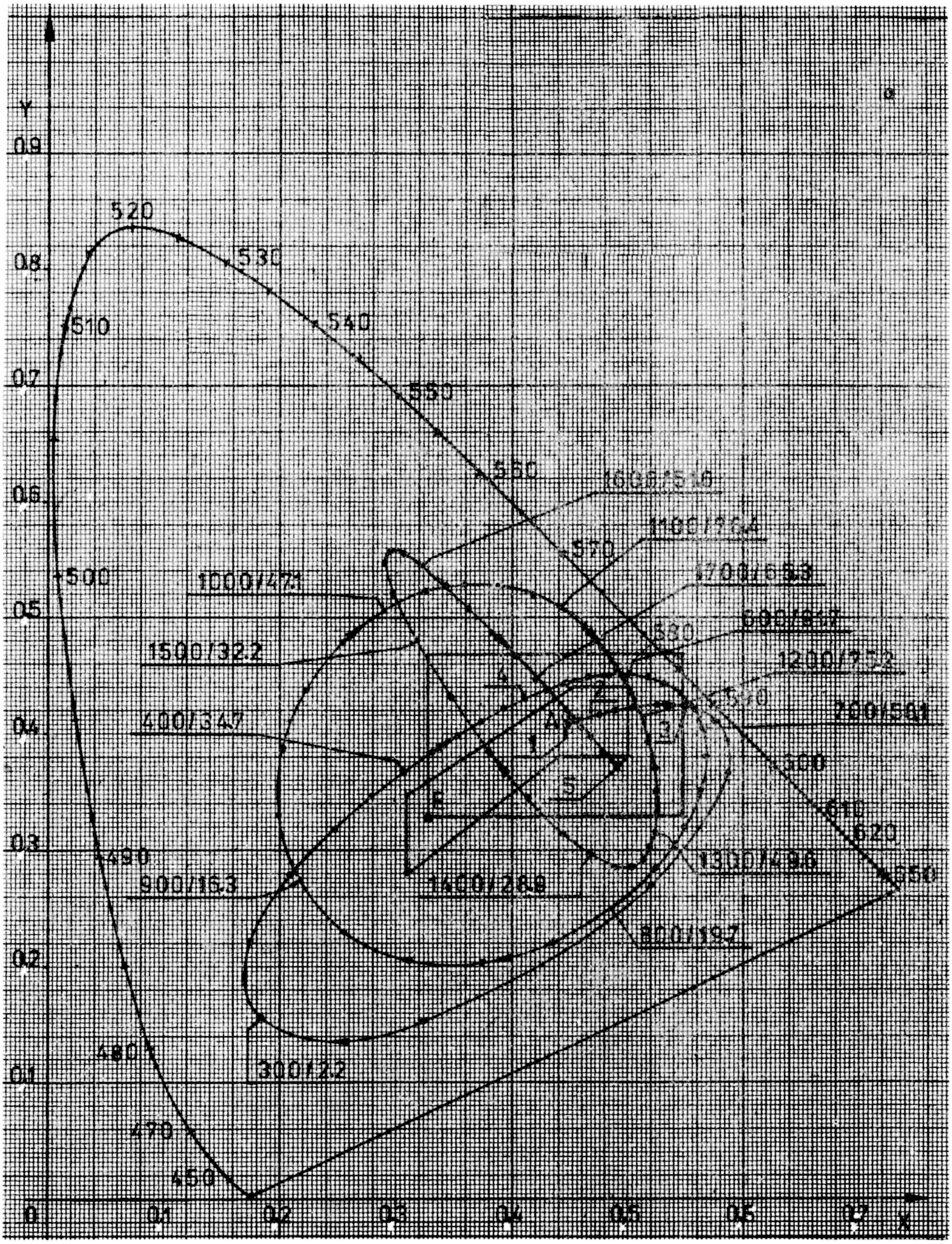


Fig. 5 a. Graphs of the polariscope colours for illuminant A observed for parallel polarisers. The other explanations like in Fig. 3. 1 - 20/98.7, 2 - 100/72.2, 3 - 200/21.3, 4 - 500/81.2, 5 - 1800/61.8

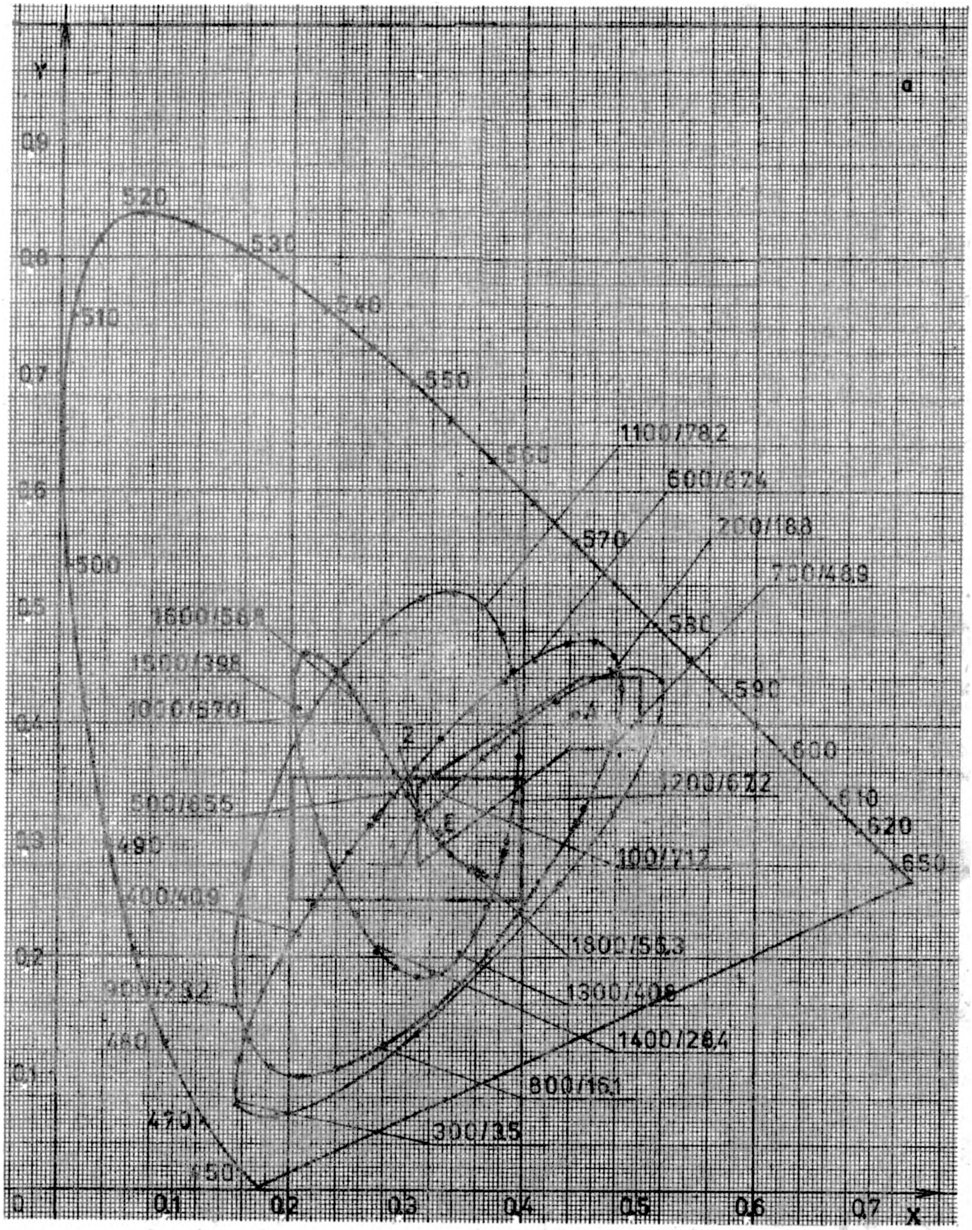


Fig. 6 a. Graphs of the polariscope colours for illuminant C observed for parallel polarizers. The other explanation like in Fig 3. 1 - 20/98.7, 2 - 1700/65.1

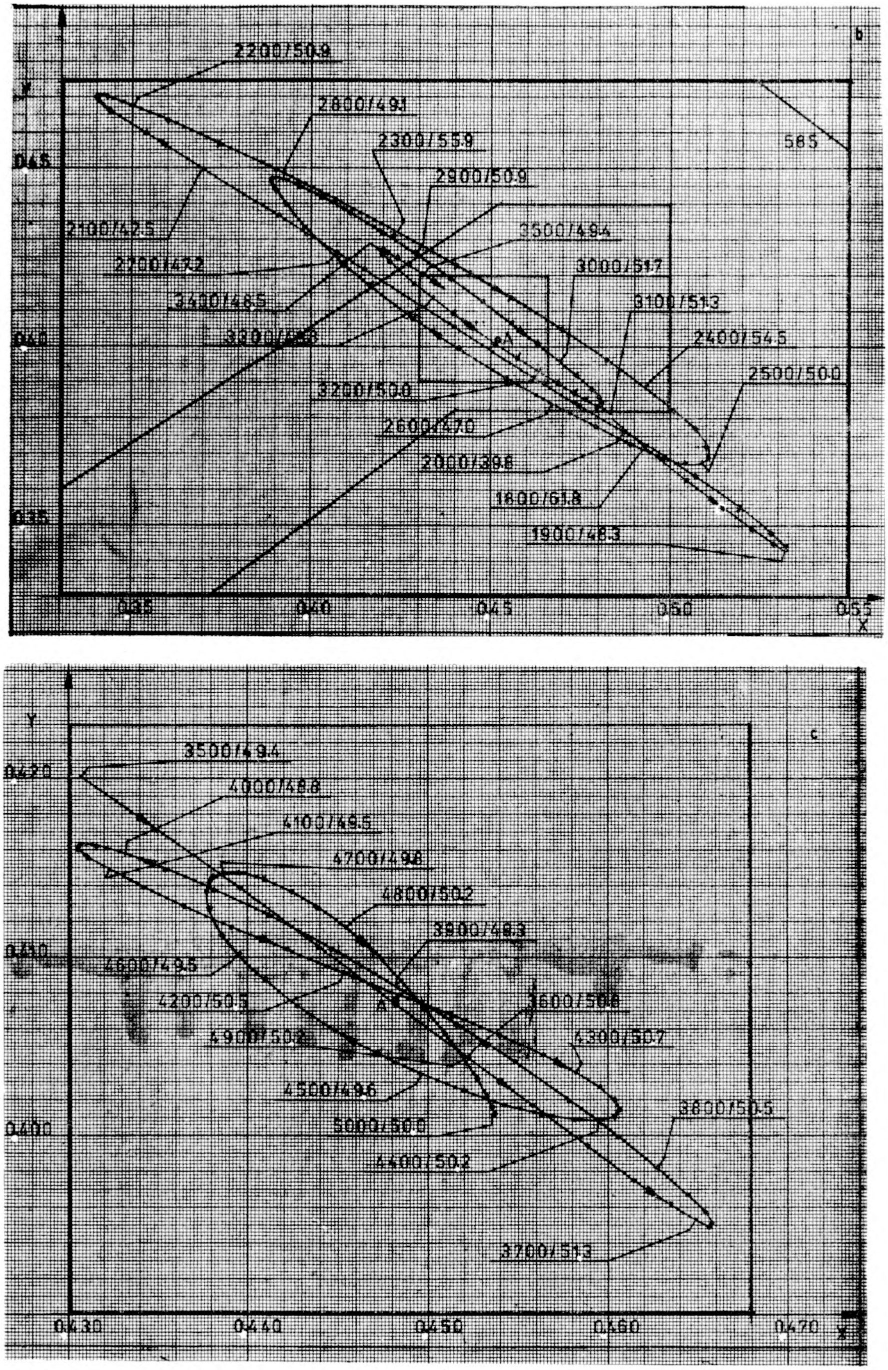


Fig. 5 b, c. Enlarged sections of the previous Figure 5 a

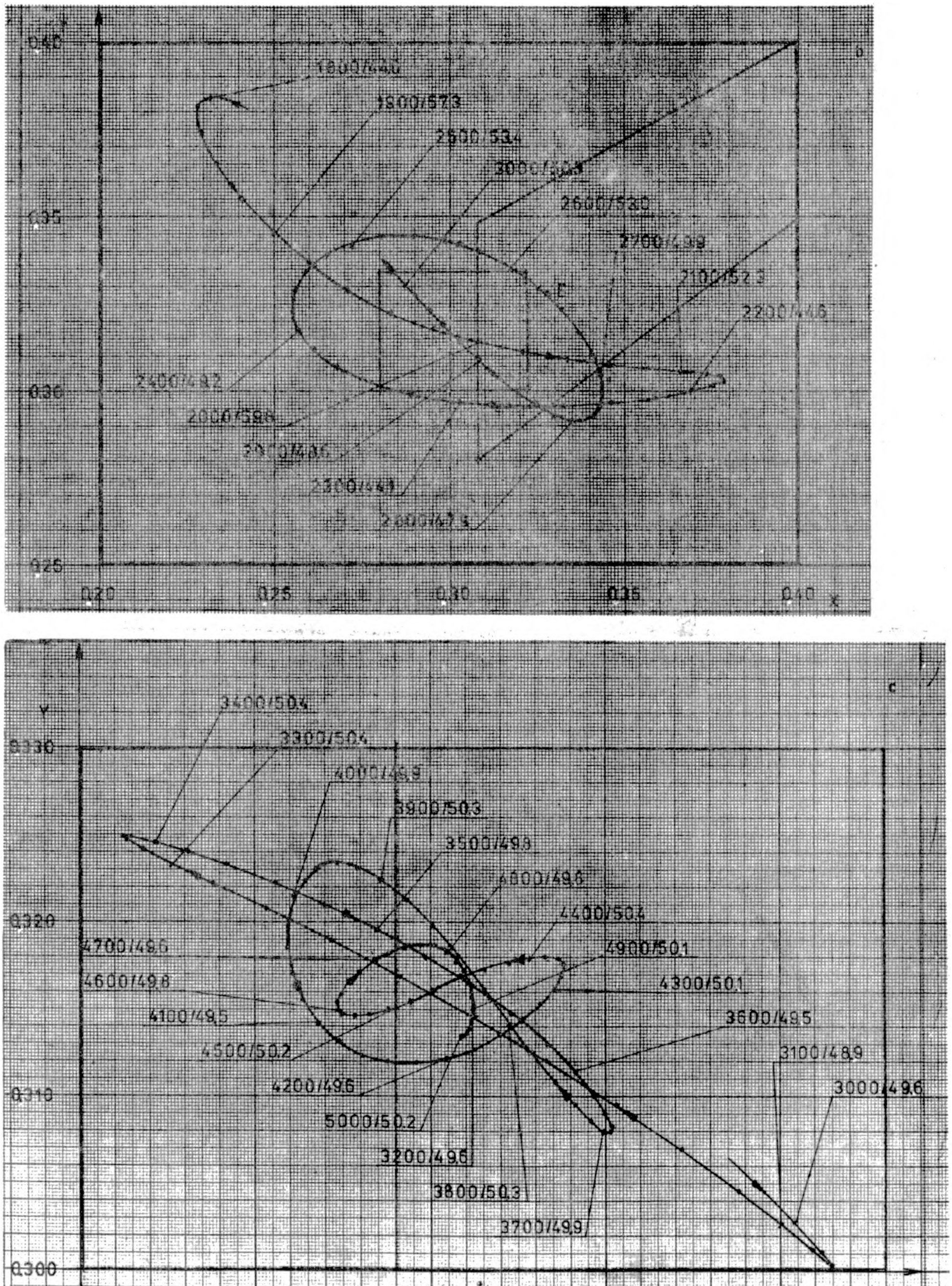


Fig. 6 b, c. Enlarged sections of the previous Figure 6a

Generally, it may be said that in real cases the interference colours differ from those appearing in the polariscope.

Under such circumstances a question arises whether it is possible to measure both the optical path differences $R(\lambda)$ in the birefringent media (with help of the polariscope colours) and the thickness d (with help of interference colours). In order to get an idea about it the ideal polariscope colours have been calculated from the formulae (7) and (8) for calcite wedge by neglecting the losses in light due to its absorption dissipation and reflection occurring within it and on its surfaces ($K = 1$). The calculations have been made for two standard light sources: A and C for parallel and crossed polarizers. The results are illustrated by 4 series of graphs presented in Figs. 3–6.

Figures 3 and 4 present the polariscope colour spiral for calcite wedge observed for crossed polarizers. In the even page there are presented colours for the light emitted by the CIE illuminant A, while those for the CIA illuminant C are given in the odd page. The colours of the same wedge observed for parallel polarizers are presented in Fig. 5 and 6, according to the same convention. A scale for R(536 nm) of elementary degree 20 nm is associated to the spirals. In the references the transmission coefficients defined by the formula

$$\int_{380}^{780} I(\lambda)\mathfrak{Z}(\lambda)d\lambda$$ are given.

4. Final remarks

The following conclusions result from the graphs:

1. The commonly cited tables of interference colours are of no practical value from the metrological viewpoint, the more that, as a rule, the illuminant for which they are valid, is not specified. It would be so even if this were done since the authors usually describe the colours in a subjective way and thus not identical in meaning for the user.

2. In all the chromaticity graphs (Figs. 3–6) the spirals cross each other many times. This means that although only one colour is attributed to the given optical path differences, there exist colours to which two values of R(536) may be attributed.

3. In many places the spiral windings are so close to one another, that the colours represented by the neighbouring points lying on different windings may be visually undistinguishable, i.e., they lie within the Stiles ellipse area. Simultaneously they lie within the measurement error range.

4. When comparing the spirals shown in chromaticity graphs for illuminants A and C a considerable influence of the illuminant on the interference and polariscope colours is visible. For instance, the colour of chromaticity coordinates (0.43, 0.48) in the A light corresponds to the path difference R(536) = 800 or 1400 nm, while that in the C light corresponds to the path difference R(536) = 880 nm.

Consequently, it may be stated that in the general case the interference colours are not a good measure of the thin film thickness or the optical path difference. It is possible, as it is suggested by WEICHERT [3], to compare the measured colours with those of the sample plate produced of the same material and examined under the same conditions. However, even then there exists a risk of ambiguity, because two different values for plate thickness or optical path difference may correspond to the same or similar colour.

References

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К ВОПРОСУ ОБ ИНТЕРФЕРЕНЦИОННЫХ ОКРАСКАХ

В учебниках физики, а также в специальной литературе встречается много спорных высказываний на тему возможности измерения разности оптических длин путей с помощью оценки интерференционных, а также псевдоинтерференционных (поляроскопических) окрасок. Они заключаются, главным образом, в принятии недопустимых упрощений. В работе обсуждается возможность применения интерференционных и поляроскопических окраоок в реальных условиях как измерителя разности оптических длин путей.