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Imaging of a two-point object in apodized optical system. Theory and analog examination *

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The influence of chosen types of amplitude apodizers on the two-point object imaging in one- and two-dimensional optics systems of axial symmetry has been examined. An analog method of examination of apodization properties by simulating the convolution in noncoherent optical processor has been proposed.

1. Introduction

The present paper is a continuation of the earlier works concerning the application of amplitude apodizers in optical system. In paper [1] numerical examinations of light intensity in diffraction pattern produced by an object consisting of a system of N slits have been performed for partially coherent light at the presence of amplitude apodizers. It has been stated that the diffraction field may be modified, the more the closer is the used irradiation to the incoherent one.

The modification of the diffraction light field, for instance, by broadening or narrowing the principal maxima, enhancing or lowering the secondary maxima or displacing the position of the maxima by using the properly designed amplitude apodizers may find an application in determination of distribution moments: of the first order (from the transform slope at the point $x' = 0$), and of the second order (from the transform curvature at the point $x' = 0$). The moments may be used as classification parameters for fringe chromosomes [2]. In the paper [3] the application of amplitude apodizers to modification of the point spread function of one- or two-dimensional systems of rotational symmetry has been examined. The apodization was there realized in noncoherent optical processor by a diaphragm (of the profile correspond-

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ing to the apodizer transmission) and a cylindric lens.

In the present paper the results obtained by employing the amplitude apodizers to imaging of a two-point object in optical system is presented. The convolution operation was simulated in a noncoherent optical processor.

2. Image of a two-point object

A linear stationary optical system with incoherent illumination may be described by a convolution of the intensity distribution in the object $I(x)$ and spread function $S(x)$, i.e.

$$I(x') = \int_{-\infty}^{\infty} I(x)\, S(x'-x)\,dx, \tag{1}$$

where x' - normalized image coordinate, $x = kX'/f$.

The relation between the point spread function $S(x, y)$ and a pupil function $T(\xi,\eta)$ is described by a two-dimensional Fourier transform. In the cases discussed below, the relation may be simplified for the one-dimensional system and an even-function apodizer to take the form of a square modulus of a one-dimensional Fourier transform

$$S(x) = \left|\int_{0}^{\infty} T(\xi)\exp\left(\frac{ik\xi x}{f}\right)d\xi\right|^2 = \left|2\int_{0}^{\infty} T(\xi)\cos\left(\frac{kx\xi}{f}\right)d\xi\right|^2 \tag{2a}$$

and, if the rotational symmetry is added, to the form of a square modulus of a Hankel transform

$$S(r) = \left|\int_{0}^{\infty} T(\rho)\, J_0\left(\frac{k\rho}{f}\, r\right)\rho\, d\rho\right|^2, \tag{2b}$$

where r' - normalized image coordinate, $r' = kR'/f$.

The following types of apodizers have been considered (Fig. 1):

I $\quad T(\xi) = \begin{cases} 1 & |\xi| \leqslant 1 \\ 0 & |\xi| > 1, \end{cases}$ nonapodized system

II $\quad T(\xi) = \begin{cases} 1/2(1+\xi^2) & |\xi| \leqslant 1, \\ 0 & |\xi| > 1, \end{cases}$

$$\text{III} \quad T(\xi) = \begin{cases} \xi^2 & |\xi| \leqslant 1, \\ 0 & |\xi| > 1. \end{cases}$$

$$\text{IV} \quad T(\xi) = \begin{cases} 1 - \xi^2 & |\xi| \leqslant 1, \\ 0 & |\xi| > 1. \end{cases} \tag{3}$$

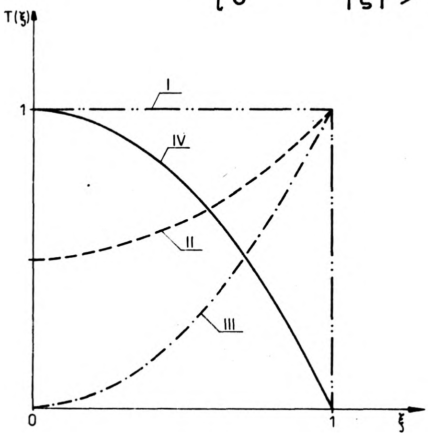


Fig. 1. Transmittance of the apodizers: I – $T(\xi) = 1$, II – $T(\xi) = 1/2(1+\xi^2)$, III – $T(\xi) = \xi^2$, IV – $T(\xi) = 1 - \xi^2$

Apodizers in the system of rotational symmetry are described by the functions $T(r)$.

The respective intensity spread functions have the forms:

– for a slit pupil

$$S_I(x) = \left[\frac{2 \sin x}{x}\right]^2, \tag{4a}$$

$$S_{II}(x) = \left[\frac{2 \sin x}{x} + \frac{2 \cos x}{x^2} - \frac{2 \sin x}{x^3}\right]^2 \tag{4b}$$

$$S_{III}(x) = \left[\frac{2 \sin x}{x} + \frac{4 \cos x}{x^2} - \frac{4 \sin x}{x^3}\right]^2, \tag{4c}$$

$$S_{IV}(x) = \left[\frac{4 \sin x}{x^3} - \frac{4 \cos x}{x^2}\right]^2, \tag{4d}$$

– for a circular pupil

$$S_I(r) = \left[\frac{2J_1(r)}{r}\right]^2, \tag{5a}$$

$$S_{II}(r) = \left\{J_1(r)\left[\frac{1}{r} - \frac{4}{r^3}\right] + \frac{2}{r^2} J_0(r)\right\}^2, \tag{5b}$$

$$S_{III}(r) = \left\{J_1(r)\left[\frac{2}{r} - \frac{8}{r^3}\right] + \frac{4}{r^2} J_0(r)\right\}^2, \tag{5c}$$

$$S_{IV}(r) = \left[\frac{8J_1(r)}{r^3} - \frac{4J_0(r)}{r^2}\right] . \tag{5d}$$

The intensity distribution in the image of two identical point objects equally distant from the optical axis and illuminated incoherently is equal to

$$I(x') = I_0\left[S(x' - \Delta x') + S(x' + \Delta x')\right], \tag{6}$$

where $2\Delta x'$ – normed distance between image points.

The application of the Rayleigh cryterion leads to the following limiting distance between the two points:

- in one-dimensional system

$$\begin{array}{ll} \text{I} & 2\Delta x' = 3.14 \\ \text{II} & 2\Delta x' = 2.8 \\ \text{III} & 2\Delta x' = 2.0 \\ \text{IV} & 2\Delta x' = 4.4 \end{array} \tag{7a}$$

- in two-dimensional system

$$\begin{array}{ll} \text{I} & 2\Delta r' = 3.83 \\ \text{II} & 2\Delta r' = 3.4 \\ \text{III} & 2\Delta r' = 3.0 \\ \text{IV} & 2\Delta r' = 5.2 \end{array} \tag{7b}$$

The intensity distribution normed to unity at the middle of the images for the Rayleigh distances between the two points for the one-dimensional and two-dimensional systems are presented in Figs. 2a–d and 3a–e, respectively.

From the results obtained it follows that for the apodizers of $1/2(1 + x^2)$, x^2 type resolution is better than for respective nonapodized system (Fig. 2b,d). The apodizer of $1-x^2$-type behaves oppositely (Fig. 2c). The optical systems of rotational symmetry behave similarly (Figs. 3b,d and 3 c).

Additionally, for the limiting Rayleigh distance of two points the value of intensity at the middle point of the distance is for the apo-

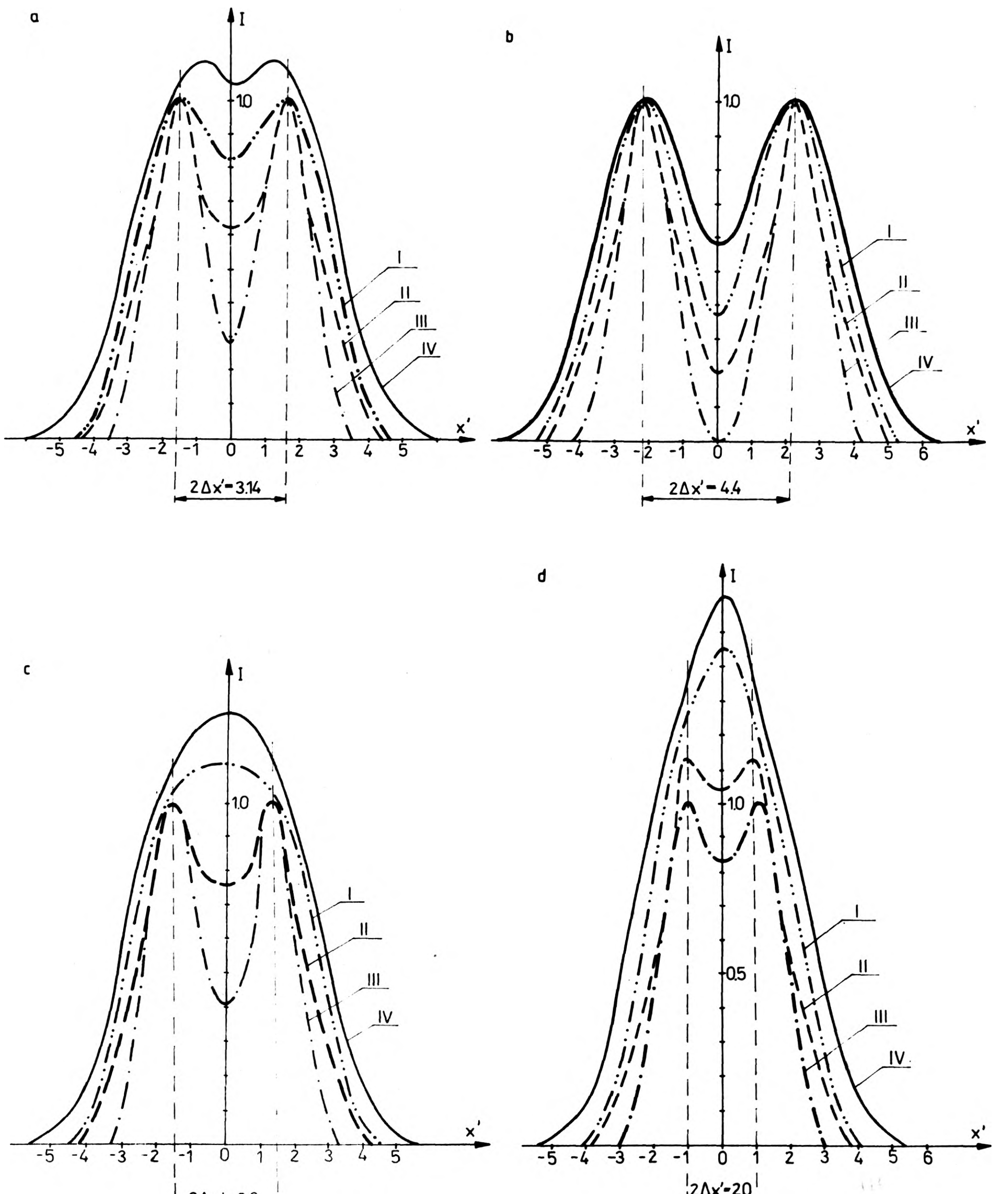


Fig. 2. Resultant intensity distribution in a one-dimensional image of the two-point object composed of noncoherent points positioned at the distance: a. $2x = 3.14$ (Rayleigh limit for an apodization - free system), b. $2x = 4.4$ (limit for apodization $1 - x^2$), c. $2x = 2.8$ (limit for apodization $1/2(1 + x^2)$)

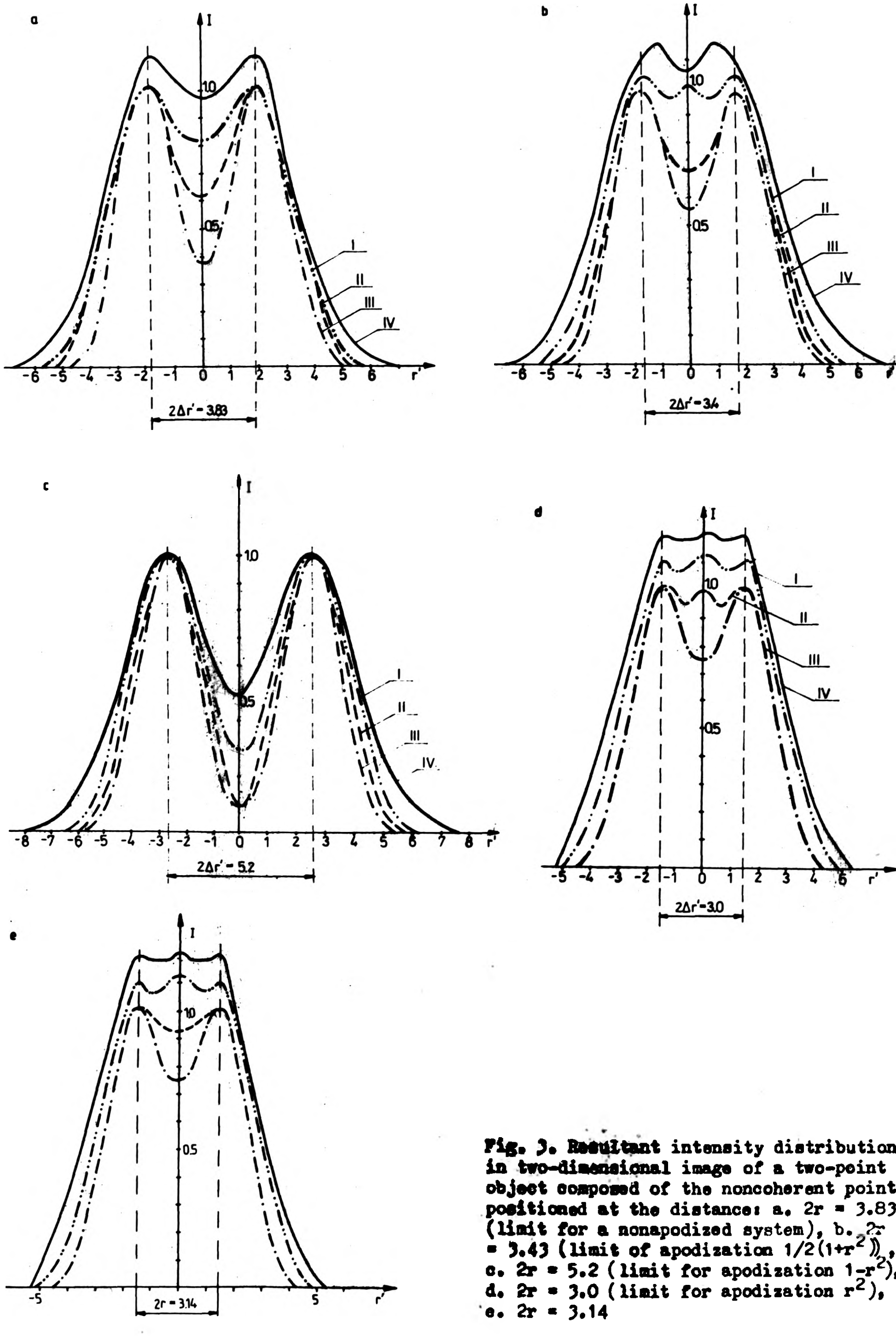


Fig. 3. Resultant intensity distribution in two-dimensional image of a two-point object composed of the noncoherent points positioned at the distance: a. 2r = 3.83 (limit for a nonapodized system), b. 2r = 3.43 (limit of apodization $1/2(1+r^2)$), c. 2r = 5.2 (limit for apodization $1-r^2$), d. 2r = 3.0 (limit for apodization r^2), e. 2r = 3.14

dizers $1/2(1 + x^2)$ and x^2 less than for the nonapodized system ($I'/I_0 = 0.82$), or equivalently, if it is assumed that the two points are resolved if the intensity of the middle point between the images is equal to $0.82\ I_0$ (see Figs. 2b,c and 3b,c), the limiting distance between them may be smaller.

The results obtained have been illustrated experimentally by simulating the convolution in a noncoherent optical processor [4].

3. Simulation of the convolution in a noncoherent optical processor

3.1. Noncoherent case

In a noncoherent optical processor a convolution of two one-dimensional nonnegative functions represented by the transmissions of the respective filter may be performed.

From the commutativity property of convolution in a linear and stationary system, instead of the formula (1) the following operation may be realized

$$I(x') = \int_{-\infty}^{\infty} S(x)I(x' - x)dx. \tag{8}$$

The scheme of the system, in which the convolution simulation is performed, is shown in Fig. 4. The input plane of the system was irradiated with a uniformly scattered light. A screen with an aperture of changing width which was placed at the input modified the spread function (S). The averaging operation of the lens (CL_1) gave in front of the filter plane (MASK) the light distribution $S(y_p)$ independent of the coordinate x. The one-dimensional object is given in the form of a filter of transmission $t(x_p, y_p) = I(x_p - y_p)$. Behind the filter the light intensity was proportional to the product $S(y_p)I(x_p - y_p)$. The lens CL_2 averaged this distribution, with respect to the variable y_p and, consequently, the light distribution obtained at the output plane was $I(x_p') = \alpha \int_{-a}^{a} S(y_p)I(x_p - y_p)dy_p$, where numbers $-a$, a define the borders of the filter. The coefficient α depends upon the light intensity at the input and upon the geometrical properties of the system. An important role in the system was performed by the spherical lens SL,

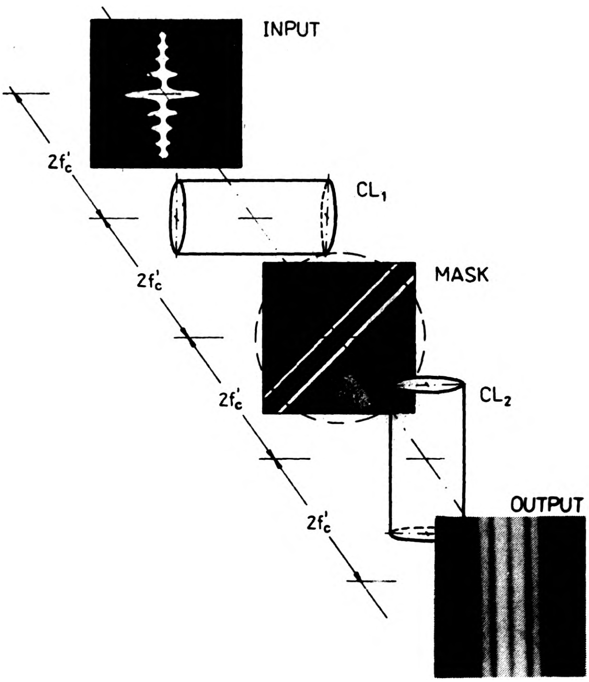


Fig. 4. Scheme of the measurement system (simulation of the convolution)

(marked with a broken line in the figure) of focal length chosen so that all the rays passing through the lens CL_1 hit the lens pupil CL_2 $(f' = (4/3)f_c')$.

In the system described some analog measurements of the image intensity distribution generated by the noncoherent two-point object have been performed and compared with the corresponding image for coherence degrees $\gamma = 1$ and $\gamma = -1$. In these measurements, performed for a pair of points positioned at a constant distance from each other $2\Delta x' = 2\Delta r' = 3.14$, the models of the point spread function were changed.

Theoretically, the two-point object should be determined by the transmission

$$t(x, y) = \delta(x - y + \Delta x) + \delta(x - y - \Delta x), \tag{9a}$$

However, the δ-function is physically unrealizable and, therefore, the function composed of two narrow straight line segments

$$t(x, y) = \sqcap\left(\frac{x - y}{b} + \Delta x\right) + \sqcap\left(\frac{x - y}{b} - \Delta x\right), \tag{9b}$$

was used instead, where b is the slit width.

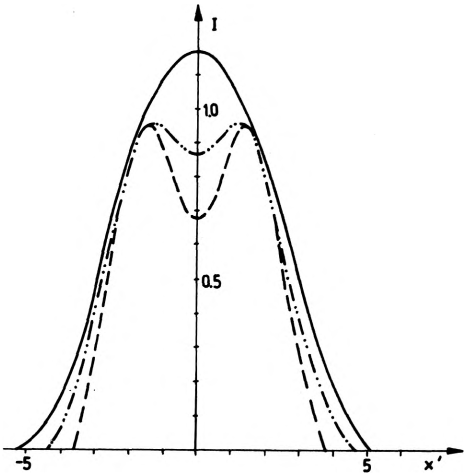


Fig. 5. Numerical results of the intensity distribution in the image of two slits of the width b = 1.5, and the distance of the middle points 2x = 3.14 in one-dimensional optical systems apodized by the functions I (-..-..-..-), II (- - - -), IV (———)

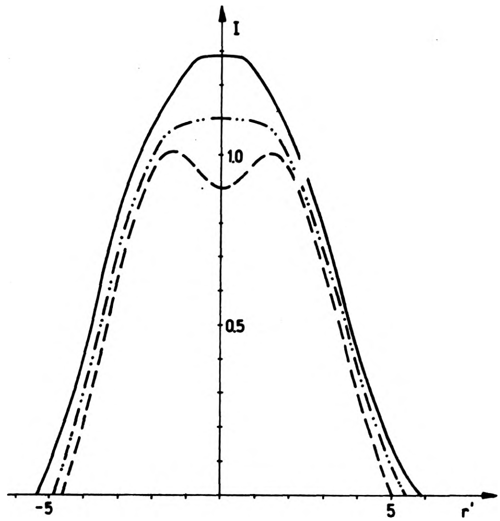


Fig. 6. Numerical results of the intensity distribution in the image of two slits of the width b = 1.5, and the distance of the middle points 2r = 3.14 in two-dimensional optical system apodized by the functions: I (-..-..-), II (- - - -), IV (———)

The width of the slits b = 1.5 (accepted after respective numerical analysis) assures their good approximation (Figs. 2a and 5, and 3e and 6) real slit to ideal two-point object.

An XBO-101 Xenon lamp with the diffuser was used as a source, a photomultiplier employed as a detector. The recorder was an X-Y plotter. The results obtained for the convolution are illustrated in Figs. 7a,b, while the properties of the diffuser, detector and amplifier are described in paper [3].

3.2. Coherent case, $\gamma = 1$

In the case of coherent object the imaging is linear with respect to the complex amplitude. If the object is described by the complete function $A(x)$ and the system is characterized by an amplitude spread function $h(x)$, the complex amplitude in the image is described also by the convolution

$$A'(x') = \int_{-\infty}^{\infty} A(x)\, h(x' - x)dx = \int_{-\infty}^{\infty} h(x)\, A(x' - x)dx. \tag{10}$$

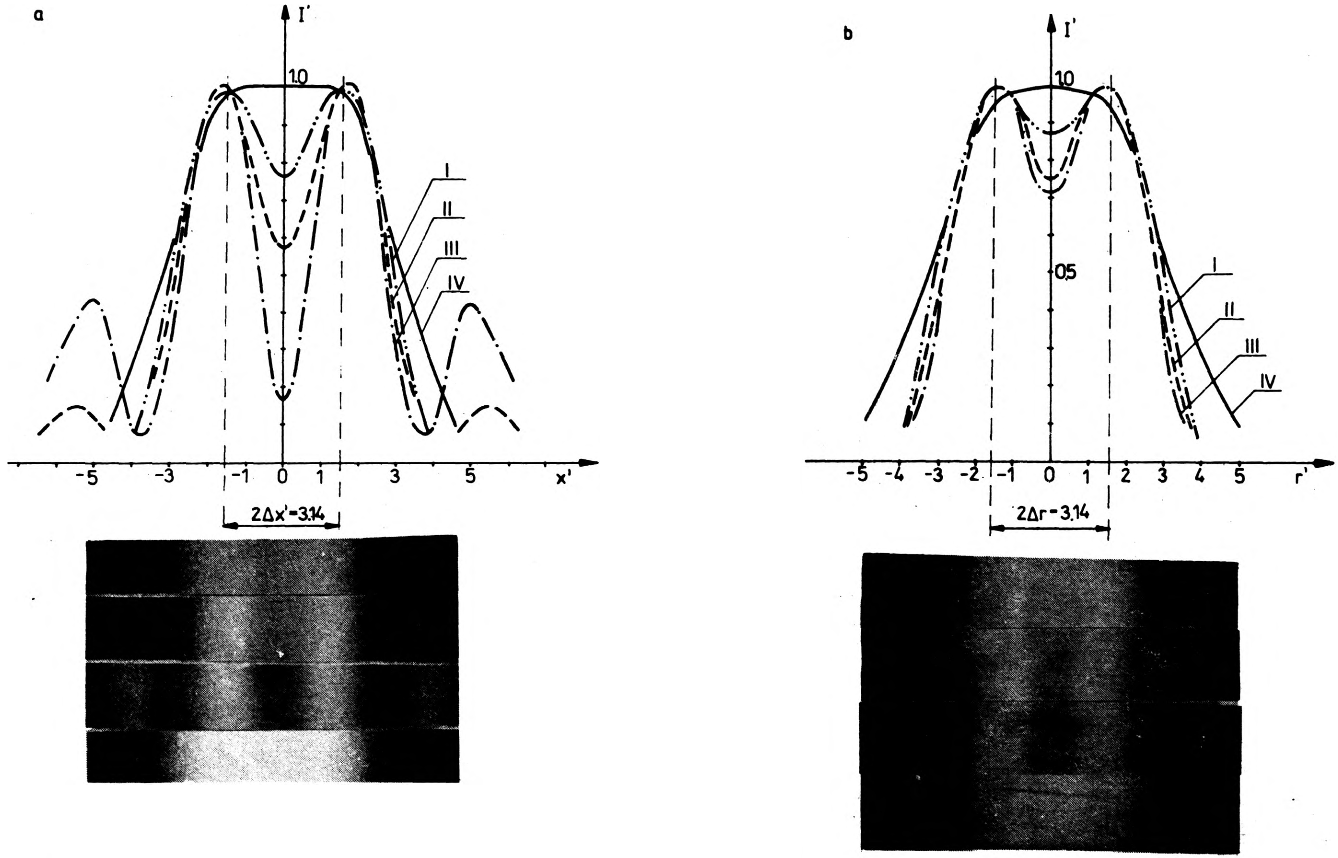


Fig. 7. The measurement results for intensity distribution in the image of two noncoherent points at the distance $2\Delta x = 2\Delta r = 3.14$, for: a. one-dimensional system, b. two-dimensional system of rotational symmetry

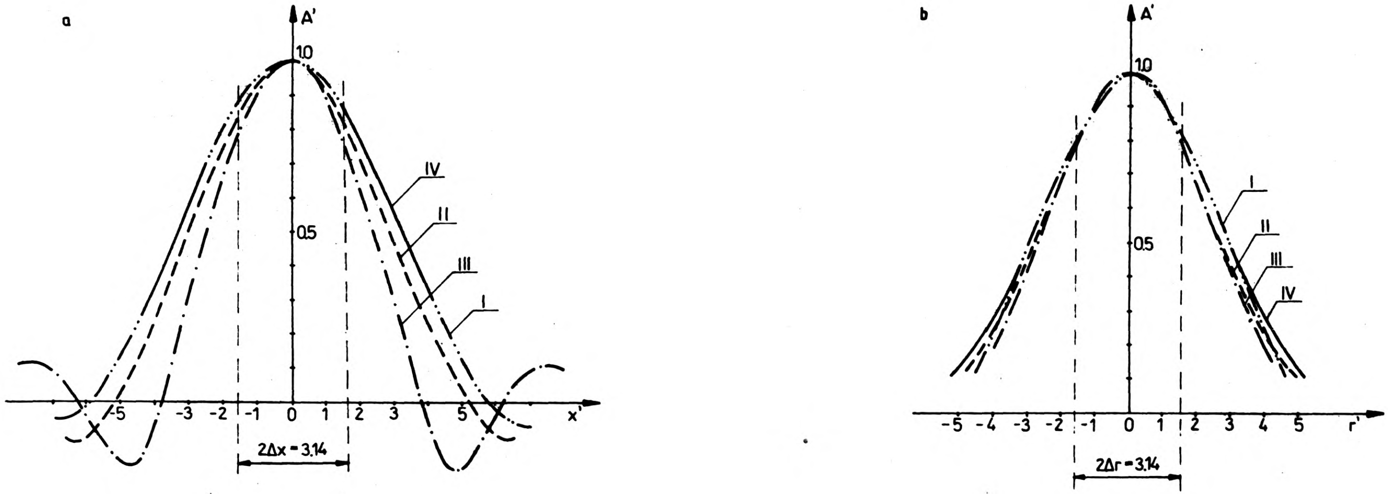
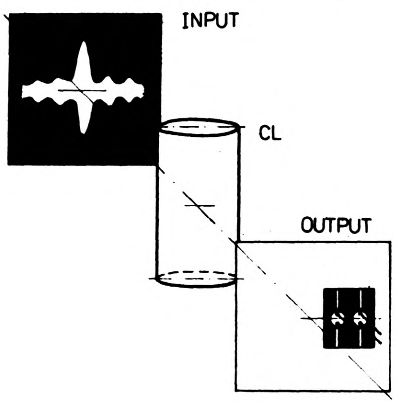


Fig. 8. The measurement results for the resultant amplitude distribution in the image for two coherent points at the distance $2\Delta x = 2\Delta r = 3.14$ for a. one-dimensional system, b. two-dimensional system of rotational symmetry



◄ Fig. 9. Scheme of the system to measure the intensity distribution in the image of two antiphase object points

The amplitude spread functions for apodizers I, II, III and IV are determined by the formulae (4) and (5), where second order terms have been neglected. Thus, it is possible to examine also the model values of the amplitude $A'(x')$ by employing the processor mentioned above, provided that both the image and the point spread functions are real. Since the amplitude spread functions take also negative values some positive constant $S \ll \max h(x)$ may be added so that the whole function be non-negative. Then, at the processor output it is obtained

$$I(x) = \alpha \int_{-a}^{a} [h(y) + s]\left[\prod \frac{x-y}{b} + \Delta x + \prod\left(\frac{x-y}{b} - \Delta x\right)\right] dy$$
$$= \alpha h(x) \otimes \left[\prod\left(\frac{x}{b} - \Delta x\right) + \prod\left(\frac{x}{b} + \Delta x\right)\right] + 2as\alpha. \tag{11}$$

Beside the true signal of convolution a d.c low-level signal, weaker than the effective signal level is present, its value may be determined outside the centre of the exit plane, since the value of the convolution for typical narrow spread functions $h(x)$ approaches rapidly zero.

Under the conditions described the measurements of separability of two narrow objects $\Delta x = \Delta r = 3.14$ have been carried out. The results are shown in Figs. 8a,b.

3.3. Coherent case, $\gamma = -1$

In the case of coherence degree $\gamma = -1$ the antiphase two-point object should have the form

$$f(x) = \delta(x + \Delta x) - \delta(x - \Delta x)$$

Similarly as it was in the case $\gamma = 1$, the addition of a constant allows to realize physically the function

$$t(x, y) = A(x - y) = 0.5\left[\prod\left(\frac{x-y}{b} + \Delta x\right) - \prod\left(\frac{x-y}{b} - \Delta x\right)\right] + 0.5, \tag{12a}$$

then

$$I(x') = 0.5\alpha\left[\int_{-a}^{a} h(y)dy + \int_{(x-\Delta x)-b/2}^{(x-\Delta x)+b/2} h(y)dy - \int_{(x+\Delta x)-b/2}^{(x+\Delta x)+b/2} h(y)dy\right]. \tag{12b}$$

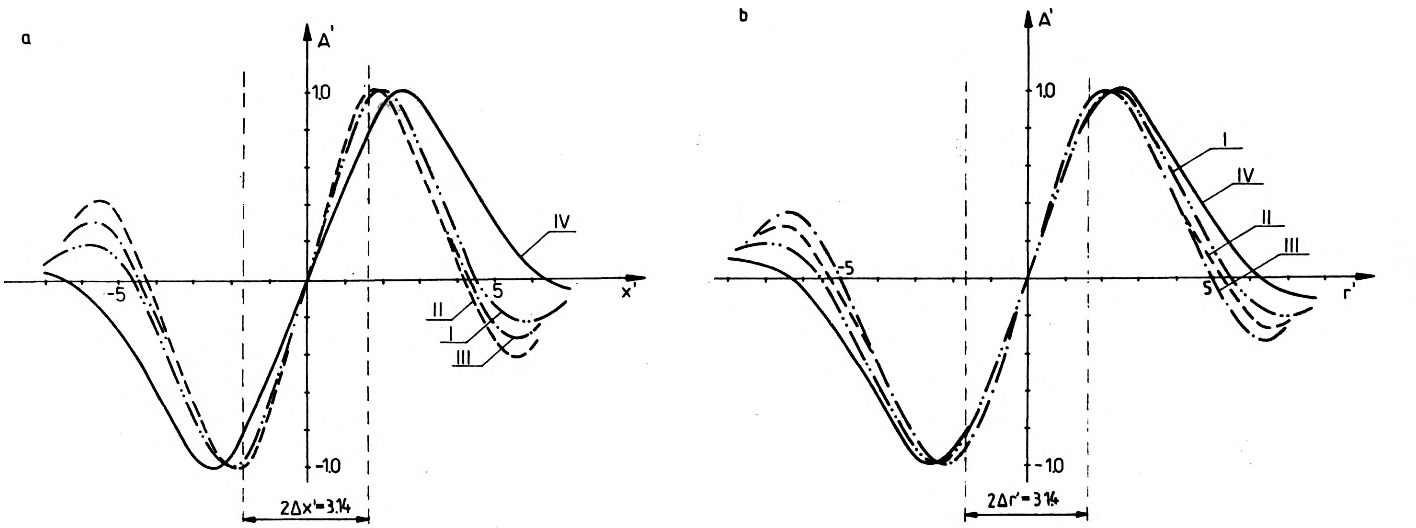


Fig. 10. The measurement results for intensity distribution in the image of two antiphase object points in the system:
a. one-dimensional, b. two-dimensional of rotational symmetry

but the rectangular function should be possibly narrow ($b \ll a$), resulting in a great value of the first component, when compared to the useful signal determined by difference of two next components. Therefore, the measurement of the amplitude distribution in the image of an antiphase object is based on linearity property of the system. Thanks to this property the real function

$$f(x) = \prod\left(\frac{x}{b} + \Delta x\right) - \prod\left(\frac{x}{b} - \Delta x\right) \tag{13a}$$

may be treated as a difference of two positive functions, which may be convoluted independently with the spread function. The amplitude corresponding to the function f(x) may be determined as a difference of convolutions

$$A'(x') = h(x') \otimes \prod\left(\frac{x}{b} + \Delta x\right) - h(x') \otimes \prod\left(\frac{x'}{b} - \Delta x\right). \tag{13b}$$

The subtraction may be performed on the stage of detection and amplification of the electric signal coming from a pair of detectors in the differential system. The measurements have been produced in the system presented in Fig. 9. A pair of detectors recording the light intensity in the regions restricted by two parallel slits have been located in the output plane. Two photodiodes connected differentially with an amplifier and an X-Y plotter were used in measurement. The results of the measurement are presented in Fig. 10a,b. Independently of the type of the apodizer used in either the one-dimensional system or two-dimensional system of rotational symmetry one obtains the resolution of the images with a full contrast (infinite resolution).

4. Final remarks

An application of the amplitude apodization enables a modification of diffraction-limited spread function. It may be also used to improve the imaging quality. The final result concerning the resolution or the contrast depends upon the degree of coherence. The amplitude apodization introduces the greatest changes in the image when noncoherent light is used. This is confirmed by the results obtained during the examinations of Sparrow resolution [5], contrast in a coherent [6] and noncoherent [7] systems. The simulation of the image (as far as resolution or contrast is concerned) in a noncoherent optical processor may be carried out by applying the proper filter of suitable apodization function.

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ИЗОБРАЖЕНИЕ ДВУХТОЧЕЧНОГО ПРЕДМЕТА В АПОДИЗИРОВАННОЙ ОПТИЧЕСКОЙ СИСТЕМЕ. ТЕОРИЯ И АНАЛОГОВЫЕ ИССЛЕДОВАНИЯ

Исследовано влияние избранных типов амплитудных аподизаторов на изображение двухточечного предмета в одно- и двухмерной оптической системе с центральной симметрией. Разработан аналоговый метод исследования свойства аподизации, при имитации сплетения в некогерентном оптическом процессоре.