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Optical pseudocolour encoding of spatial frequency information

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1. Theory

An application of nonmonochromatic light to spatial filtering in optical system allows to produce an additional effect, which would be impossible when the radiation of single wavelength is used [1–7]. Namely, it enables the transformation of black-white (monochromatic) plane object to a colour image, the colours of which depend upon the local spatial frequencies in the object. This method of optical filtration is called pseudocolour encoding. The analysis of optical system used to pseudocolour encoding will be here repeated after BESCÓS and STRAND [1]. Such a system is presented in fig. 1. Here, a spatially incoherent and plane light source Σ of finite size emits a radiation spectrum consisting of three wavelengths λ_j ($j = B, G, R$) corresponding to blue, green and red parts of spectrum.

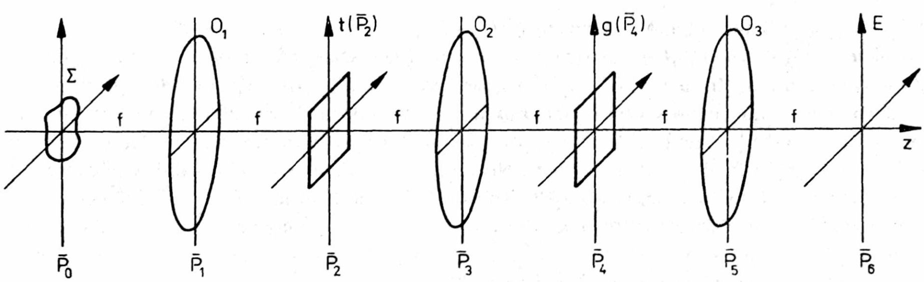


Fig. 1. Optical system for pseudocolour encoding of spatial frequency information

The light source luminance for each of the three wavelengths is described by $E_j(\overline{P}_0)$.

The mutual coherence function in the source may be written for each λ_j in the form:

$$\Gamma_{0j}(\overline{P}_{01}, \overline{P}_{02}) = \begin{cases} E_j(\overline{P}_{01})\delta(\overline{P}_{01}-\overline{P}_{02}), & \text{when } \overline{P}_{01} \in \Sigma \\ 0, & \text{when } \overline{P}_{p1} \notin \Sigma. \end{cases} \tag{1}$$

Here $\overline{P}_{ij} = (x_{ij}, y_{ij})$ denotes the j-th point on i-th plane.

Let a sinusoidal amplitude grating of spatial frequency $\overline{\xi} = (\xi_x, \xi_y)$ be the object of imaging. Its (real) amplitude transmittance is:

$$t(\overline{P}_2) = A_0 + A_1 \cos(2\pi\overline{\xi}\overline{P}_2) \tag{2}$$

where $A_0 \geqslant A_1$, and $\overline{\xi}\cdot\overline{P}_2$ denotes $\xi_x x_2 + \xi_y y_2$. $t(\overline{P}_2)$ does not depend upon the wavelength. Due to the square-law detection the intensity transmittance is important, i.e.

$$I(\overline{P}_2) = a + b\cos(2\pi\overline{\xi}\overline{P}_2) + c\cos(4\pi\overline{\xi}\overline{P}_2), \tag{3}$$

where $a = A_0^2+A_1^2/2$, $b = 2A_0A_1$, $c = A_1^2/2$.

In the Fourier plane ($\bar{P}_4$) of the system a colour filter is inserted, the amplitude transmittance of which $g(\lambda, \bar{P}_4)$, depends upon the light wavelength. It may be considered as being composed of three different monochromatic spatial filters for each of the wavelengths

$$g(\lambda, \bar{P}_4) = \sum_{j=B}^{G,R} g_j(\bar{P}_4). \tag{4}$$

Let us assume that this filter is real and symmetric with respect to the axis of the system. Under such conditions in the plane $\bar{P}_6$ the image produced is of light distribution given by

$$I(\bar{P}_6) = \sum_{j=B}^{G,R} [a_j(\bar{\xi})+\beta_j(\bar{\xi})\cos(2\pi\bar{\xi}\bar{P}_6)+\gamma_j(\bar{\xi})\cos(4\pi\bar{\xi}\bar{P}_6) = \sum_{j=B}^{G,R} a_j(\bar{\xi})+\sum_{j=B}^{G,R} \beta_j(\bar{\xi})\cos(2\pi\bar{\xi}\bar{P}_6)$$
$$+\sum_{j=B}^{G,R} \gamma_j(\bar{\xi})\cos(4\pi\bar{\xi}P_6), \tag{5}$$

where

$$a_j(\bar{\xi}) = A_0^2 \int E_j(\bar{P}_0)g_j(\bar{P}_0)dP_0+\frac{A_1^2}{2}\int E_j(\bar{P}_0-\lambda f\bar{\xi})g_j^2(\bar{P}_0)d\bar{P}_0. \tag{6a}$$

$$\beta_j(\bar{\xi}) = 2A_0A_1 \int g_j(\bar{P}_0)E_j(\bar{P}_0)g_j(\lambda_j f\bar{\xi}-\bar{P}_0)d\bar{P}_0, \tag{6b}$$

$$\gamma_j(\bar{\xi}) = \frac{A_1^2}{2}\int g_j(\bar{P}_0)E_j(\lambda_j f\bar{\xi}-\bar{P}_0)g_j(2\lambda_j f\bar{\xi}-\bar{P}_0)d\bar{P}_0, \tag{6c}$$

Here, $d\bar{P}_0 = dx_0dy_0$ and the integrals occuring in (6a)–(6c) are the surface integrals.

The coefficients $a_j(\bar{\xi})$, $\beta_j(\bar{\xi})$, $\gamma_j(\bar{\xi})$ describe, respectively, the transfer of the background, fundamental frequency and first harmonic frequency of the amplitude distribution for the light of colour j. These coefficients are determined by the equations (6a)–(6c), only in the case of the objects of sinusoidal transmittance distribution, but due to the fact that the light detection is quadratic they cannot be considered as characterizing the classical transfer function of the system. However, since there exists a possibility of representing any object in the form of Fourier series these coefficients describe the transfer of the particular components of the object spatial frequency spectrum through the system. Hence, they may be used to describe the action of the system on any object. The coefficients a_j, β_j and γ_j may be treated as an analogon of the transfer function under assumption of square-law detection [8].

As it may be seen from the formula (5) the filtered image of the sinusoidal grating is also of sinusoidal structure of the same spatial frequency (compare the formula (3)), but this time the coefficients describing the intensities of the background, fundamental frequency and first harmonics, i.e.

$$a(\bar{\xi}) = \sum_{j=B}^{G,R} a_j(\bar{\xi}), \tag{7a}$$

$$\beta(\bar{\xi}) = \sum_{j=B}^{G,R} \beta_j(\bar{\xi}), \tag{7b}$$

$$\gamma(\bar{\xi}) = \sum_{j=B}^{G,R} \gamma_j(\bar{\xi}), \tag{7c}$$

depend upon the characteristics of the light source $E_j(\bar{P}_0)$ and the filters $g_j(\bar{P}_2)$, and, of course, upon the spatial frequency of the sinusoidal object.

In the coloured image $I(\bar{P}_6)$ these coefficients play the role of weighting factors in additive summing of three monochromatic images for $\lambda_j = \lambda_B, \lambda_G, \lambda_R$.

Thus, the resultant colour of the sinusoidal grating image depends upon the spatial frequency of this grating (for given characteristic of the source and filters). In this way a correspondence appears, which atributes the colours in the image to the respective spatial frequencies.

2. The influence of the source width on the pseudocolour encoding effect

In this Section (similarly as was the case in the work [1]) we restrict our attention to the one-dimensional case.

For each of the three colours let the space filters located in the Fourier plane have the following transmittances:

$$g_B(x_4) = \mathrm{rect}\left(\frac{x_4}{2x_c}\right), \tag{8a}$$

$$g_G(x_4) = \mathrm{rect}\left(\frac{x_4}{4x_c}\right)\left[1-\mathrm{rect}\left(\frac{x_4}{2x_c}\right)\right], \tag{8b}$$

$$g_R(x_4) = \mathrm{rect}\left(\frac{x_4}{6x_c}\right)\left[1-\mathrm{rect}\left(\frac{x_4}{4x_c}\right)\right], \tag{8c}$$

where

$$\mathrm{rect}\left(\frac{x}{2a}\right) = \begin{cases} 1 \text{ for } |x| < a \\ 0 \text{ for } |x| > a \end{cases}$$

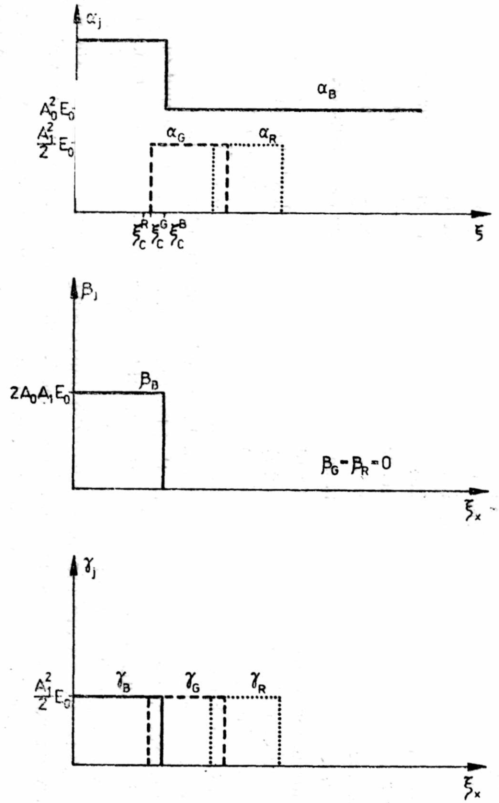
Let us assume further that the light source is a uniformly shining slit of the width $2x_s$:

$$E_j(x_0) = E_0 \mathrm{rect}\left(\frac{x_0}{2x_s}\right), \; j = B, G, R \tag{9}$$

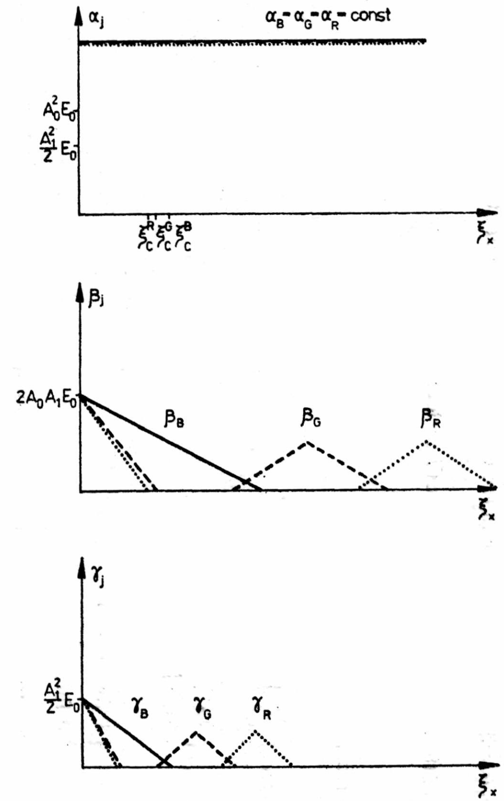
If the source width tends to zero the illuminating light becomes spatially coherent. The other limiting case of complete spatial incoherence occurs when the slit width tends to infinity, i.e. the source becomes infinitely spread. In the intermediate cases we have to do with the partially coherent light.

In accordance with the conclusions of the work [1] the effect of pseudocolour encoding depends very strongly upon the source width. This influence may be analysed by using fig. 2 showing the dependence of coefficients α_j, β_j, $\gamma_j (j = B, G, R)$ upon the spatial frequency ξ_x for different light source widths. In the first case $x_s \to 0$ (coherent illumination–fig. 2a) the image is of well saturated colours. This is confirmed by the values of coefficients α_j, β_j and γ_j which are essentially different from zero. Simultaneously, as these coefficients become constant within the wide intervals of spatial frequency ξ_x there appears nonuniqueness in the correspondence of the colour in the image to the spatial frequency in this image. Additionally, for this type of illuminaton a strong effect of speckling should be expected.

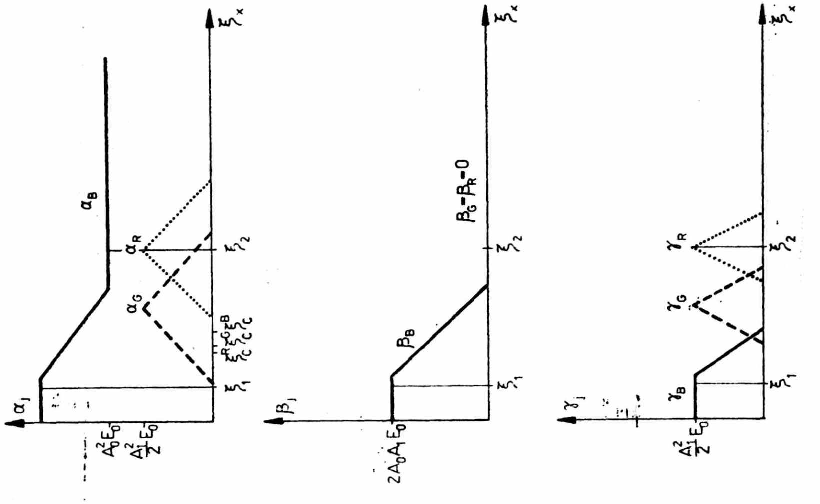
When the source width increases to infinity (fig. 2b), the case of completely incoherent illumination is obtained. This time there appears a strong background of white colour (great α_j for $j = B, G, R$), which makes the image brighter, but reduces the colour saturation of the image. Both these limiting cases are considered to be less advantageous. The figs. 2c–2g concern the partially coherent illumination obtained for gradually increased light source width. Hence, it follows that for the source width equal to a half of the blue filter width ($x_s = 0.5x_c$) a unique correspondence of the spatial frequency to a definite colour may be obtained. When the transversal magnification of the system between the planes x_0 and x_4 is different from 1 this relation concerns both the filter width and the width of the source image in the x_4 plane. For the source width chosen in this way a good colour saturation in the image is achieved, while the speckling effect is eliminated. The case $x_s = 1/2x_c$ is optimal one. Further increment of the source width causes intensification of the background brightness and consequently further reduction of the colour saturation. A rapid increase of brightness appears when the source width x_s exceeds the value of x_c. The image background becomes an additive mixture of blue (B) and green (G) colours. Another rapid increase of brightness occurs when x_s exceeds the value $2x_c$. Then, the background colour (being a sum of blue (B), green (G), and red (R)) becomes close to white colour.



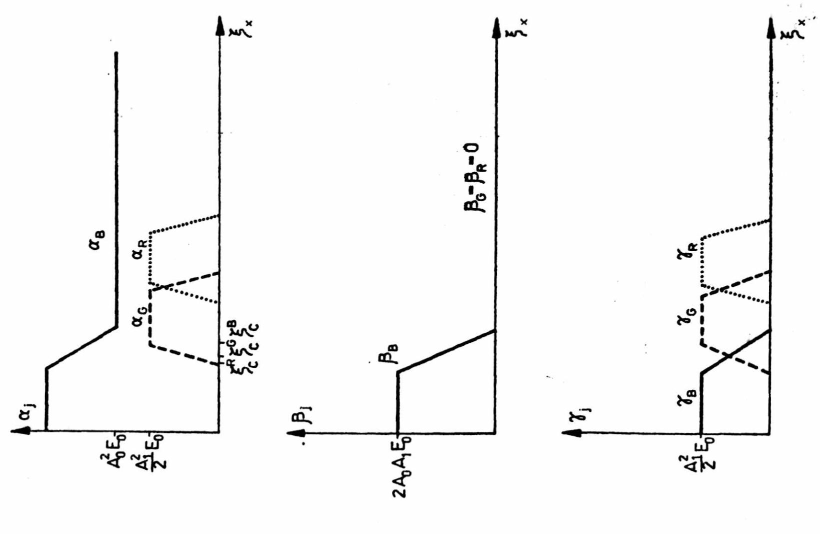
a. for limiting case $x_s \to 0$ (coherent illumination)



b. for limiting case $x_s \to \infty$ (incoherent illumination)



d. for $x_s = \frac{1}{2} x_c$



c. for $x_s = \frac{1}{4} x_c$

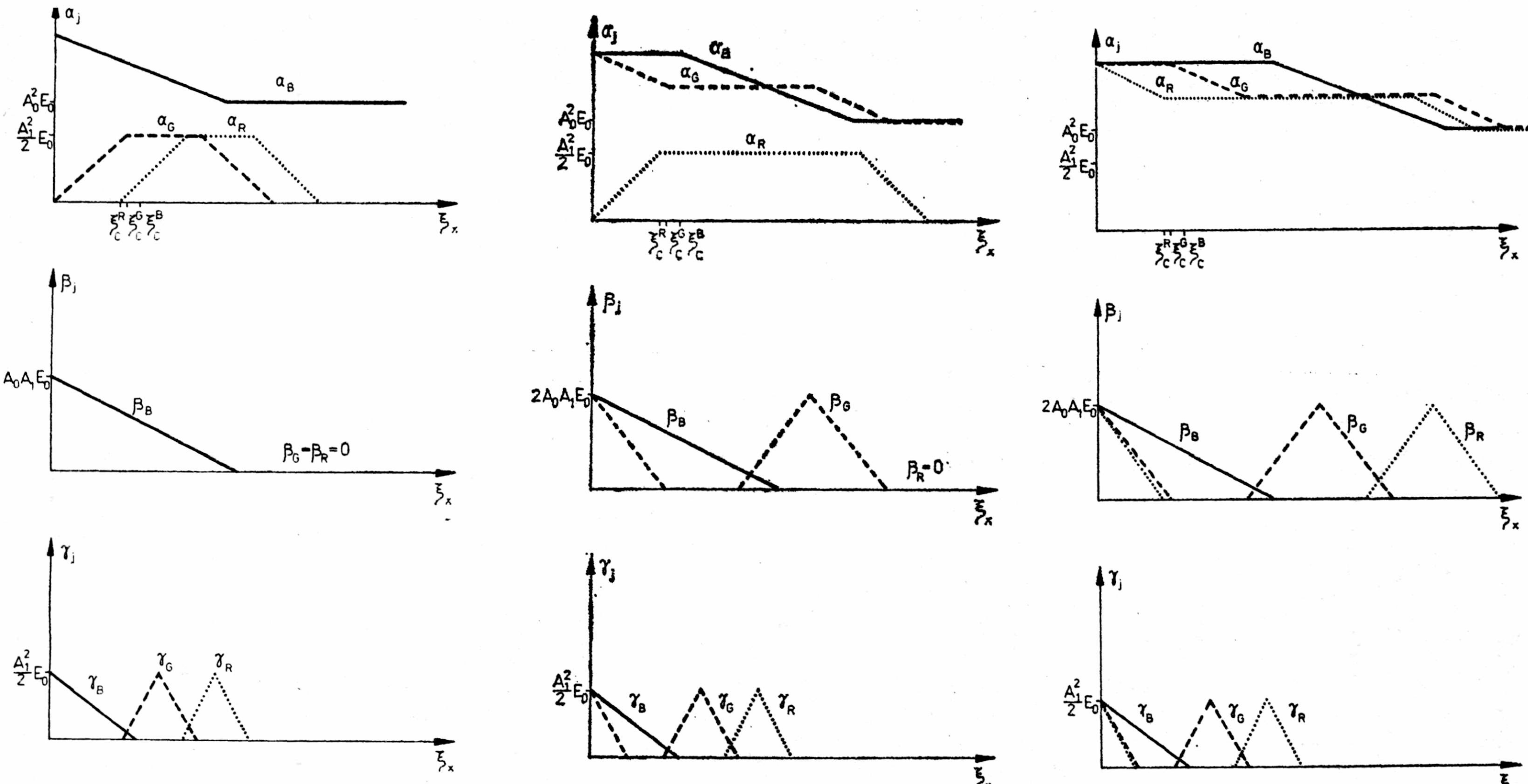


Fig. 2. Pseudo-transfer functions:
e. for $x_s = x$

f. for $x_s = 2x_c$

g. for $x_s = 3x_c$

3. The influence of the position of the image point on its colour

The above interpretation of the coefficients $a_j(\bar{\xi})$, $\beta_j(\bar{\xi})$, $\gamma_j(\bar{\xi})$ allows to relate certain colours in the image to respective sinusoidal objects of spatial frequencies $\bar{\xi}$. In order to quantitatively describe this correspondence let us use the trichromatic coordinates [9] as a colour measure. Let us first notice that if a three-colour light of luminances E_j, $j = B, G, R$ is used in the filtering system characterized by the coefficients $a_j(\bar{\xi})$, $\beta_j(\bar{\xi})$, $\gamma_j(\bar{\xi})$ the image of the sinusoidal object may be written down as a sum of three sinusoidal distributions of light intensities of colours $j = B, G, R$ with the weighting factors E_j

$$I(\bar{P}_6, \bar{\xi}) = [E_B, E_G, E_R] \begin{bmatrix} a_B(\bar{\xi}) & \beta_B(\bar{\xi}) & \gamma_B(\bar{\xi}) \\ a_G(\bar{\xi}) & \beta_G(\bar{\xi}) & \gamma_G(\bar{\xi}) \\ a_R(\bar{\xi}) & \beta_R(\bar{\xi}) & \gamma_R(\bar{\xi}) \end{bmatrix} \begin{bmatrix} 1 \\ \cos(2\pi\bar{\xi}\bar{P}_6) \\ \cos(4\pi\bar{\xi}\bar{P}_6) \end{bmatrix} = [E_B\ E_G\ E_R] \begin{bmatrix} I_B(\bar{P}_6, \bar{\xi}) \\ I_G(\bar{P}_6, \bar{\xi}) \\ I_R(\bar{P}_6, \bar{\xi}) \end{bmatrix}. \quad (10)$$

The resultant colour in the image may be now determined by assuming that the trichromatic coordinates corresponding to the wavelengths λ_j are known. Let them be (x_j, y_j). The coordinates of the resultant colour are equal to

$$x = \frac{\sum_{j=B}^{G,R} I_j(\bar{P}_6, \bar{\xi}) E_j x_j}{I(\bar{P}_6, \bar{\xi})}, \quad (11a)$$

$$y = \frac{\sum_{j=B}^{G,R} I_j(\bar{P}_6, \bar{\xi}) E_j y_j}{I(\bar{P}_6, \bar{\xi})}. \quad (11b)$$

As it should be expected the colour in the image depends upon the spatial frequency $\bar{\xi}$ of the sinusoidal test. Besides, from the relations (11a) and (11b) it follows that the colour is not constant but changes periodically depending upon the position of the point P_6 in the image. This follows from different contributions from each of the colour components $j = B, G, R$ depending on the position of the point $\bar{P}_6$ in the image of the sinusoidal test. This means, that the sinusoidal test of a single spatial frequency does not correspond to one colour only but is rather related to some continuous set of colours. When displacing the observation point $\bar{P}_6$ in the image trichromatic coordinates of the changing colour travel along a closed curve in the colour triangle.

For instance, let us consider the one-dimensional case of partially coherent filtration, which occurs when the light source of width $x_s = 0.5\, x_c$ and of equal partition colours $j = B, G, R$ (i.e., $E_B = E_G = E_R = 1$) are used. The corresponding coefficients $a_j(\xi_x)$, $\beta_i(\xi_x)$, $\gamma_j(\xi_x)$ are presented in fig. 2d.

The following distribution of intensity in the image

$$I(x_6, \xi_1) = [1\ 1\ 1] \begin{bmatrix} \frac{A_1^2}{2} + A_0^2 & 2\, A_0 A_1 & \frac{A_1^2}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \cos(2\pi\xi_1 x_6) \\ \cos(4\pi\xi_1 x_6) \end{bmatrix} = I_B(x_6 \xi_x) \quad (12)$$

corresponds to the sinusoidal test of spatial frequency ξ_1 marked in fig. 2d. This intensity distribution is blue, only its brightness changes.

If the spatial frequency of the test is ξ_2 (see fig. 2d) the image is described by

$$I(x_6, \xi_2) = [1\ 1\ 1] \begin{bmatrix} A_0^2 & 0 & 0 \\ \frac{2}{5}\frac{A_1^2}{2} & 0 & 0 \\ \frac{A_1^2}{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \cos(2\pi\xi_2 x_6) \\ \cos(4\pi\xi_2 x_6) \end{bmatrix}. \quad (13)$$

The trichromatic coordinates of the colours in image are given by equations

$$x(x_6) = \frac{x_B + \frac{1}{5}\left(\frac{A_1}{A_0}\right)^2 x_G + \frac{1}{2}\left(\frac{A_1}{A_0}\right)^2 [1+\cos(4\pi\xi_2 x_6)]x_R}{1+\frac{1}{5}\left(\frac{A_1}{A_0}\right)^2 + \frac{1}{2}\left(\frac{A_1}{A_0}\right)^2 [1+\cos(4\pi\xi_2 x_6)]}.$$

$$y(x_6) = \frac{y_B + \frac{1}{5}\left(\frac{A_1}{A_0}\right)^2 y_G + \frac{1}{2}\left(\frac{A_1}{A_0}\right)^2 [1+\cos(4\pi\xi_2 x_6)]y_R}{1+\frac{1}{5}\left(\frac{A_1}{A_0}\right)^2 + \frac{1}{2}\left(\frac{A_1}{A_0}\right)^2 [1+\cos(4\pi\xi_2 x_6)]} \tag{14b}$$

If the wavelengths λ_j are equal to: $\lambda_B = 450$ nm, $\lambda_G = 550$ nm, $\lambda_R = 650$ nm, respectivey, and the greatest modulation depth is $A_1/A_0 = 1$, then

$$x(x_6) = \frac{1.2+0.73\cos(4\pi\xi_2 x_6)}{3.6+\cos(4\pi\xi_2 x_6)}, \tag{15a}$$

$$y(x_6) = \frac{0.6+0.27\cos(4\pi\xi_2 x_6)}{3.6+\cos(4\pi\xi_2 x_6)}. \tag{15b}$$

The colour in the image of this test changes within the colour triangle depending upon the position of the observation poi fig. 3.

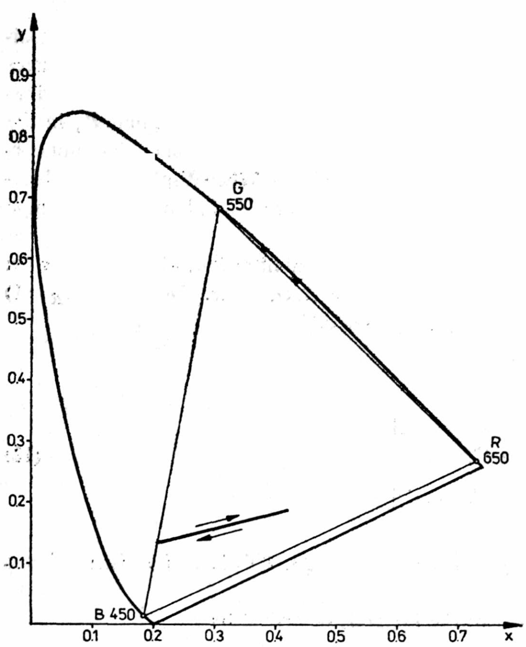


Fig. 3. Colour change in image of sinusoidal test-object

4. Conclusions

The application in the filtration process of an incoherent and spatially limited source of white light causes that the result of optical filtration (pseudocolour encoding) is influenced also by the spectral distribution in the light source and its shape. Consequently, the image colour depends on the position of the point $\bar{P}_0$

and the local spatial frequencies $\bar{\xi}$ of the filtered object. The colour saturation depends also upon the source width P_s.

For these reasons, a complete description of this phenomenon may be first obtained when considering the intensity distribution in the detection plane as a function of both spatial coordinates and spatial frequencies of the object plane.

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