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Kinematic aberration and some constructional parameters in camera for ultra-high speed cinematography with image commutation and mirror secondary objectives

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The formulae for parameters which define the kinematic aberration considered in the earlier works of the author have been derived. These formulae concern, in particular, the cameras of Miller type with the secondary mirror objectives. Simplified formulae found for aberration considerably facilitate its calculations. In order to verify the simplifying assumption, the checking calculations have been made for several constructional variants of the cameras. It has been shown how the designer, starting with an acceptable value of the kinematic aberration, may define the important design parameters of camera.

1. Introduction

Figure 1 presents an optical scheme of the camera for ultra-high speed cinematography with the image commutation. The principal objective (*PO*) images the examined object A in the place where there exists the rotating mirror (*RM*). This mirror directs the reflected light beam successively to the elements of an array of identical secondary objectives (*SO*) which surround it. Each of those objectives produces its final image A''' on the film tape F spread concentrically with the gallery of *SO* objectives. This image – apart from small rotation – is immovable, if the rotation axis of *RM* lies simultaneously in the reflecting plane of this mirror and in the plane of the intermediate image A' produced by the principal objective (*PO*). Therefore, the film tape in such a camera is also immovable and the role of *RM* is reduced to realize the commutation of the light beam to particular secondary objectives (*SO*).

Such a system offers high advantages from both the designer's and technological points of view, especially when compared to the other constructions used in ultra-high speed cinematography. For instance, the cameras based on optical compensation of the continuous movement of the film tape (realized by applying the rotating multiangle prisms or sets of mirror or lenses) are more difficult to design. The only moving element in the Miller type camera is a light mirror of small sizes, which may be driven to very high rotation frequencies. This allows to attain very high filming frequencies with relatively good quality of pictures.

In camera described in papers [1, 2] the secondary lens objectives are replaced by concave mirrors (fig. 2), which considerably simplifies the design, technology as well as adjustment of the camera system if compared with classical systems. Due to crossed mirrors *RM* with a common axis of rotation a great number of pictures (on two parallel film tapes) have been achieved and simultaneously the so-called "waiting system" has been obtained, i.e., one in which no synchronisation of the initial moment of the recording process with a definite position of the *RM* mirror is required.

The condition, mentioned above, that the reflecting plane of *RM* should contain the axis of rotation is technologically very difficult to fulfil, especially at very high rotation frequencies. In practice, the reflecting surface is always located at some distance r from the axis. Consequently, this leads to some residual movement of the final image A''' with respect to the film tape. The segment travelled by an image point on the film tape during the exposure is the measure of the unsharpness of the image recorded (when the object A is immobile) and has been called the kinematic aberration A_k.

The intermediate image, which, in turn, is the object of imaging for the secondary objective, draws during the exposure of one picture a small segment of the curve, being the Pascal leaf [3]. In fig. 3 this segment is presented as a straight line segment $\overline{A_1'' A'' A_2''}$ which, in order to determine the kinematic aberrations, has been decomposed into the components $2g$ and $2e$, respectively, perpendicular and parallel to the optical axis of the secondary objective. The basic formulae determining the kinematic aberration are the following [4]:

$$A_{k,0} = |2g'|+|\sigma|, \tag{1}$$

$$2g' = z\left(\frac{r'}{r}\cos da - \cos a\right), \tag{2}$$

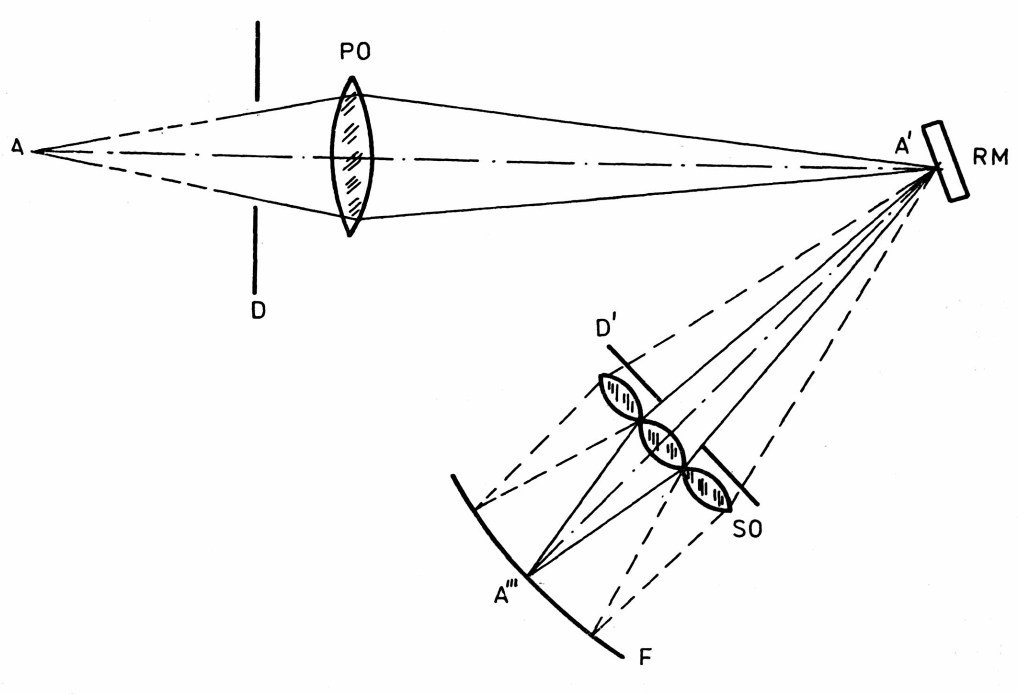

Fig. 1

$$\sigma = \frac{\beta e h}{a\cos\gamma - \beta e}, \tag{3}$$

$$z = \frac{4\beta r \sin da}{\cos\gamma}, \tag{4}$$

$$e = 2r\sin(a-\gamma)\sin da + r'\sin\gamma\sin 2da, \tag{5}$$

where r — distance of the reflecting plane RM from the axis of rotation,
r' — distance of the intermediate image A' from the axis of rotation,
a — angle defining the instantaneous position of RM (i.e. angle between the normal to RM and the principal ray of the incident light beam),
γ — angle between the principal ray of the light beam reflected from RM and the optical axis of SO,
β — magnification of the final image A''' with respect to the intermediate image A'' (figs. 3 and 4),

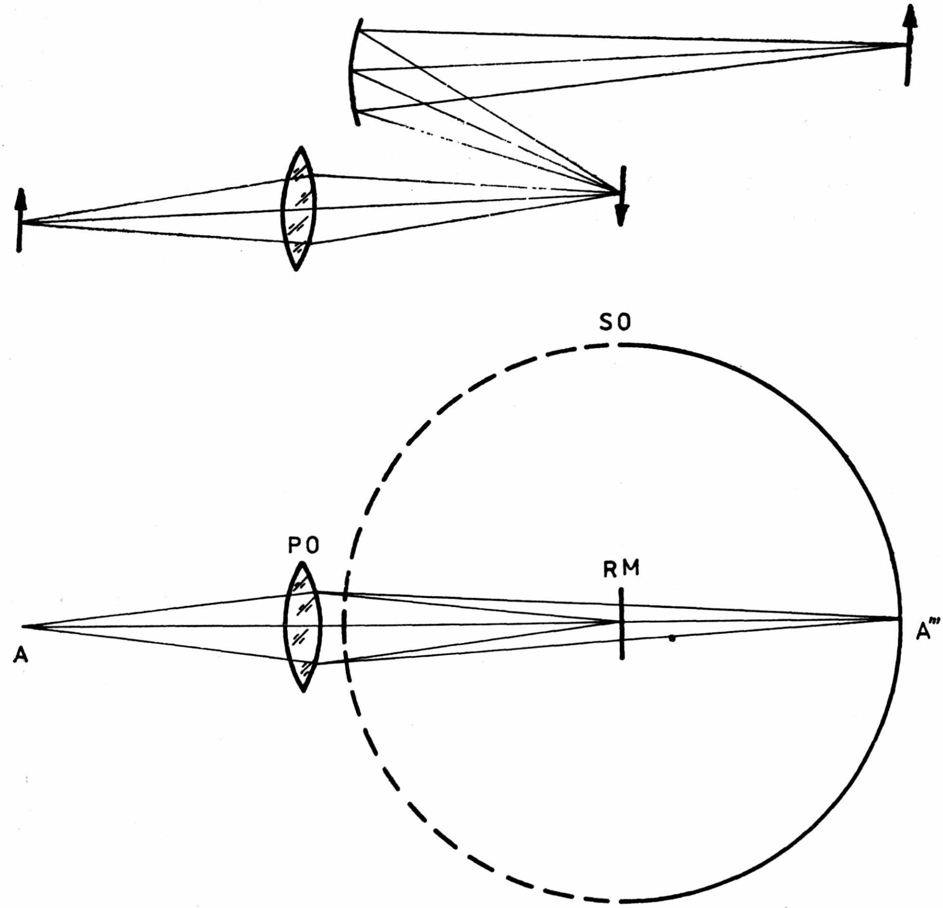

Fig. 2

$2da$ — angle of rotation of RM which causes the shift of the commuting beam across single SO,

h — diameter of SO (width in the direction of the light beam movement,

$a = \overline{A''O}$ (figs. 3, 4).

In the paper [5] it has been pointed out that for the working sector of the camera defined by the initial (a_b) and final (a_e) values of the angle a, when the light beam reflected from the RM falls respectively on the first and the last SO, there exists some optimal distribution of the kinematic aberration. It appears when

$$r' = \frac{r}{\cos da} \frac{\cos a_b + \cos a_e}{2}. \tag{6}$$

The diaphragm D (fig. 1) defining the aperture of the principle objective is located in front of it at such a distance that its image D' be positioned at the place where there exists the secondary objective. The width p' of this image is often assumed to be equal to the width h of the secondary objective. It should be noticed that due to very small amount of light (the apertures of cameras are of order of one tenths and less) and to the limited sensitivity of the light sensitive material, reduced additionally by very short exposure time, the light from the initial phase of exposure, when the commuting light beam first starts to cover the secondary objective does not participate in the process of image recording. In the case when $p' = h$ the components σ in eq. (1) is by one half less than it follows from eq. (3) [6]. In practice, p' greater than h or $2h$ is very rarery used, since, this means, that the exposure time of one picture is considerably longer than the period of change of pictures equal to the reciprocity of the filming frequency.

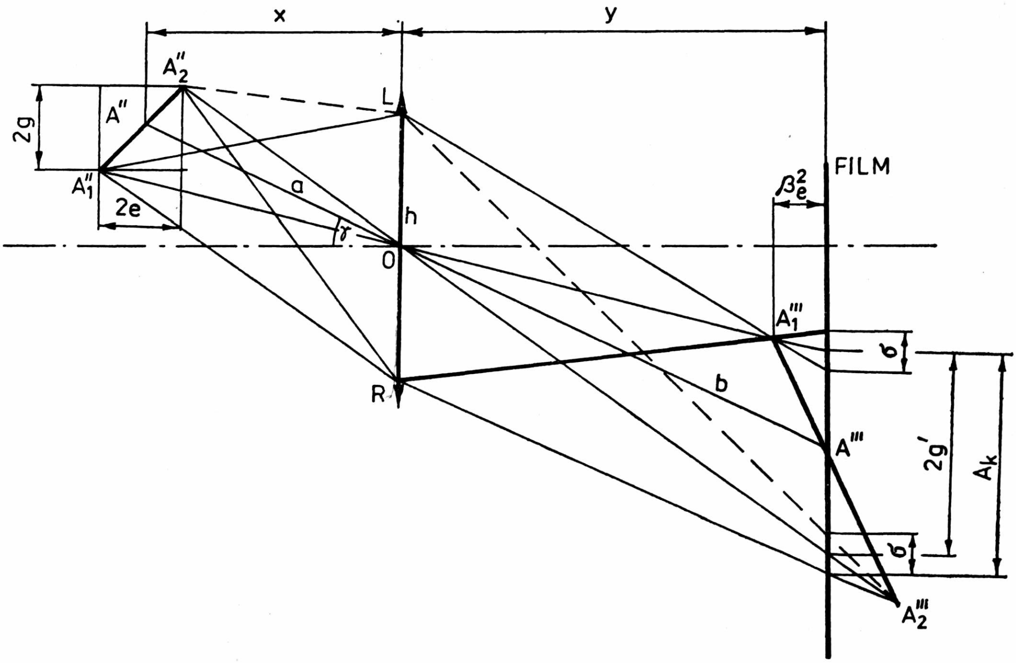

Fig. 3

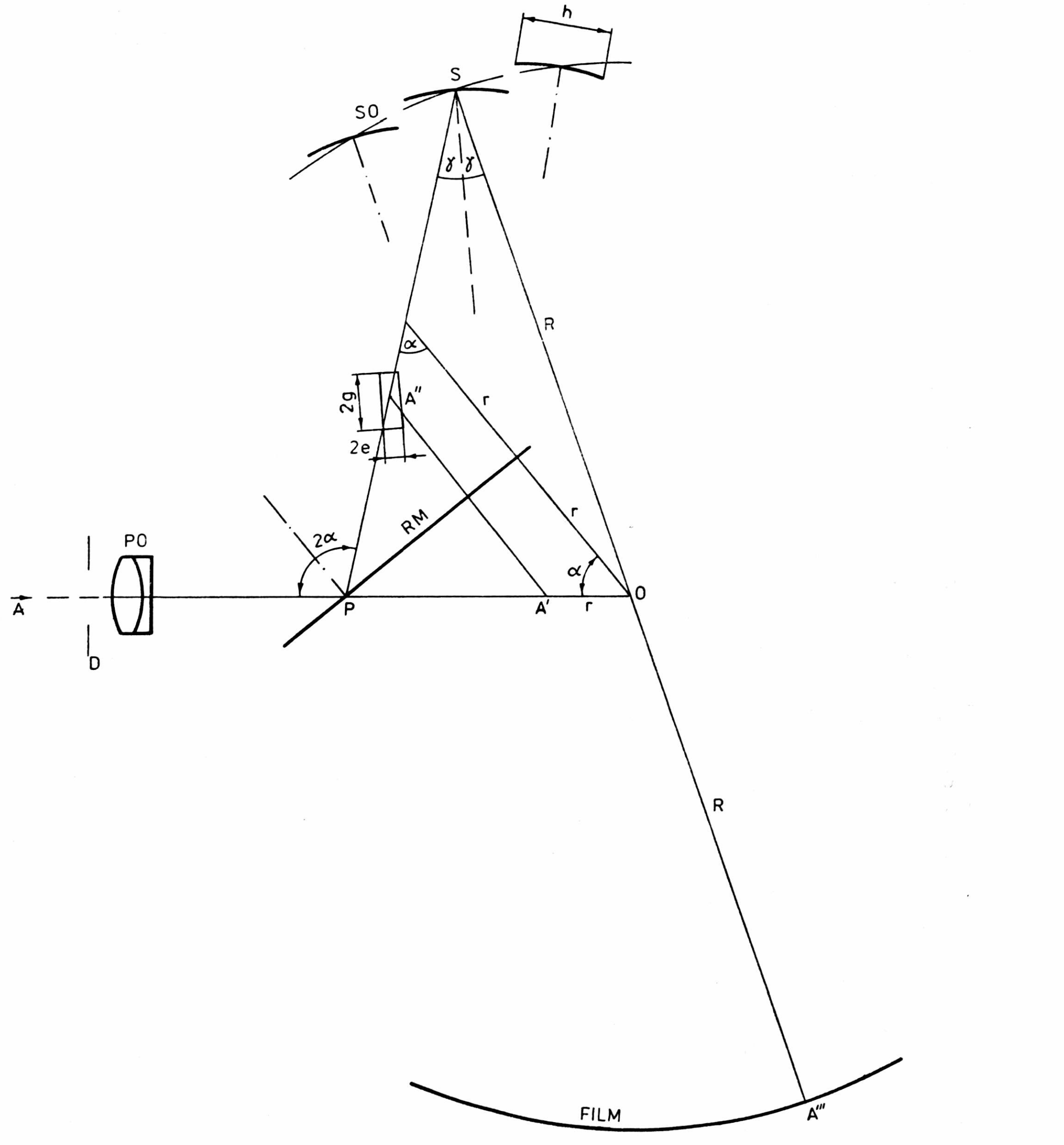

Fig. 4

2. Aberration parameters

In order to calculate the kinematic aberration according to formulae (1)–(6) the parameters h, r, a, γ, β, and da should be first established or calculated. The parameter h is fixed by the designer on the base of general demands to be fulfilled by the camera, like the required frequency and the number of pictures taken. The parameter r follows most frequently from the technological possibilities of the rotating mirror production. The other parameters will be determined for the cameras with secondary mirror objectives as indicated in [1]. For such cameras, by analysing the course of principal ray it is obtained that (fig. 4):

$$\frac{1}{a}+\frac{1}{b}=\frac{\cos\gamma}{f}, \tag{7}$$

$$b = R+R_F, \tag{8}$$

$$\sin 2\gamma = \frac{2r}{R}\sin\alpha, \tag{9}$$

$$a = R\cos 2\gamma - 2r\cos\alpha + r', \tag{10}$$

$$\beta = \frac{b}{a} = \frac{R+R_F}{a} = \frac{f}{a\cos\gamma - f}. \tag{11}$$

The meanings of the symbols a, b, R_F, R appearing in these formulas can be easily read out from fig. 4. The point O denotes the common centre of both the areas of the film tapes and the secondary objective gallery, while f is the focal length of the secondary objective.

The point A' in fig. 4 is, as denoted above, the image of the object point produced by PO. The point A'' is the image of the point A' produced by RM (A_1'' and A_2'' are the end positions of the point A'' during the exposure of one picture) the point A''' is the image of the point A'' produced by SO, thus, it is the final image of the point A.

For constructional reasons the assumption that all the secondary objectives should have the same focal length is quite obvious. (In the camera described in paper [1] the secondary objectives are concave mirrors of rectangular shape defined by 30 mm height and 7.6 mm width, the latter equal to the frame step on the 16 mm cinematographic film tape). It would be most convenient to assume the initial value $R = R_0$ for $\alpha = 0$. Fixing additionally $R_F = R_0$, which is another technological simplification in the camera design, the formulae (7)–(11) yield

$$\frac{1}{f} = \frac{1}{R_0 - 2r + r'} + \frac{1}{2R_0}. \tag{12}$$

(If the assumption $R_F = R_0$ is not justified R_F must be established by other reasons). The system of eqs. (7)–(11) has been solved by taking account of (6) and (12) for $R_F = 500$ and $h = 7.62$, by the numerical calculation for sixty values of α covering the whole working sector $\alpha_b = 0^\circ$, $\alpha_e = 45^\circ$. In table 1 R, γ and β are listed for 13 values of α (located more densely in the surrounding of the aberration minimum). It may be seen that these quantities depend only slightly on α. The angle γ remains small within the whole sector.

The angle $2d\alpha$ may be determined by analysing the course of the beam reflected from the rotating mirror in different phases of its rotation. In fig. 5 the symbols RM_1, RM_0, RM_2 denote the three successive positions of the rotating mirror. At the position RM_0 the principal ray of the reflected beam falls on the centre S of the secondary objective. In accordance with the considerations given above the initial position of RM has been assumed to be such a one at which, at least, a part c of the secondary objective is covered with light. The final position RM_2 has been accepted to be symmetrical to RM_1, i.e., when a part c of SO

is still filled with light. In the numerical calculations below c has been assumed to be equal to $h/2$.

In order to calculate the quantity $4d\alpha$ it is convenient to use the fig. 6, which is a transformed version of fig. 5 produced by a parallel shift of the principal rays of the reflected beam at its extreme positions RM_1 and RM_2 to the point P_0. In all the constructional solu-

Table 1. $R_F = R_O = 500$, $p' = h = 7.62$, $c = h/2$,

α [deg]	α [min]	R [mm]	γ [min]	$4d\alpha$ [min]	β	$2g'$ [mm]	$A_{k,0}$ [mm]	$A_{k,1}$ [mm]	$A_{k,2}$ [mm]
0	0.0	500.00	0.00	52.813	2.0185	0.0182	0.0181650	0.0181650	0.0179988
10	0.0	499.90	4.78	52.817	2.0182	0.0163	0.0164452	0.0164450	0.0162942
20	0.0	499.62	9.41	52.828	2.0174	0.0107	0.0110079	0.0110077	0.0109071
30	0.0	499.16	13.77	52.847	2.0159	0.00154	0.0020226	0.0020225	0.0020011
31	0.0	499.10	14.19	52.849	2.0158	0.000449	0.0009388	0.0009387	0.0009265
31	23.0	499.08	14.349	52.8498	2.0157	0.0000185	0.0005147	0.0005147	0.0005060
31	23.9	499.0788	14.3547	52.84987	2.0157	0.0000016	0.0004980	0.0004980	0.0004895
31	24.0	499.0787	14.3554	52.84987	2.0157	0.0000003	0.0004968	0.0004967	0.0004883
31	24.1	499.0786	14.3561	52.84988	2.0157	0.0000022	0.0004987	0.0004986	0.0004901
31	25.0	499.078	14.362	52.8499	2.0157	0.0000191	0.0005158	0.0005158	0.0005071
32	0.0	499.04	14.60	52.851	2.0156	0.000682	0.0011873	0.0011873	0.0011729
40	0.0	498.53	17.73	52.872	2.0140	0.01084	0.0114552	0.0114547	0.0113571
45	0.0	498.15	19.52	52.887	2.0129	0.0182	0.0188140	0.0188132	0.0186610

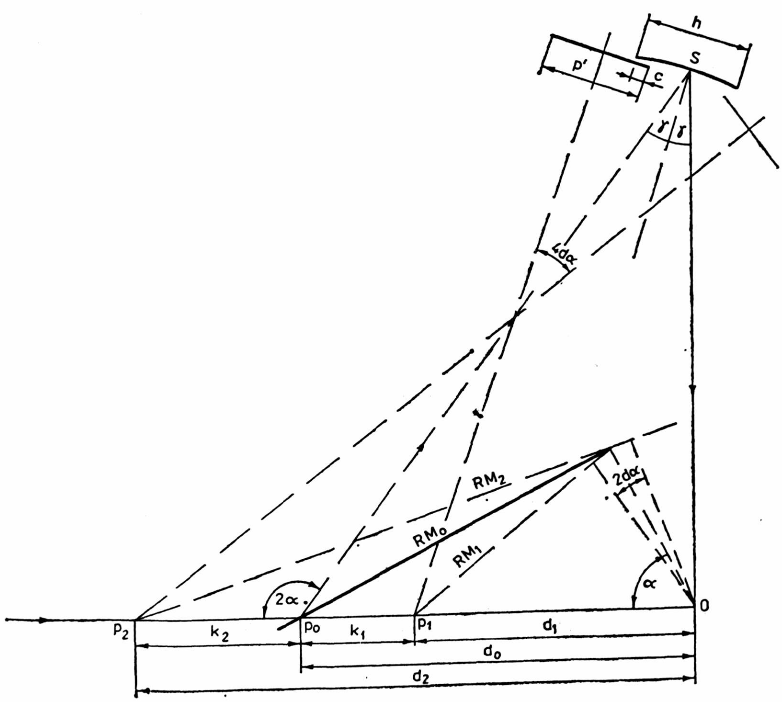

Fig. 5

tions giving the pictures of good quality the objective width h is much less than R (in the camera described in [1] – 7.62 and 500, respectively). This allows us to state that the angle $4da$ is small ($\sim 1°$ in [1]) and therefore on the base of fig. 6 it is possible to write with sufficiently good approximation:

$$\tan 2da = \frac{t}{2l},$$

where

$$t = \overline{KL} = h' + p' \cos 2da + (k_1 + k_2)\cos\varphi,$$
$$h' = (h-2c)\cos\gamma, \quad \varphi = 2a - 90°,$$
$$l = R\cos 2\gamma + d_0 \cos(180° - 2a),$$
$$k_1 = d_0 - d_1, \quad k_2 = d_2 - d_0,$$
$$d_0 = r/\cos a, \quad d_1 = r/\cos(a-da), \quad d_2 = r/\cos(a+da),$$

so that

$$l = R\cos 2\gamma - r\frac{\cos 2a}{\cos a}, \tag{13}$$

$$t = p' \cos 2da + (h-2c)\cos\gamma + \frac{4r\sin a \sin 2a \sin da}{\cos 2a + \cos 2da}, \tag{14}$$

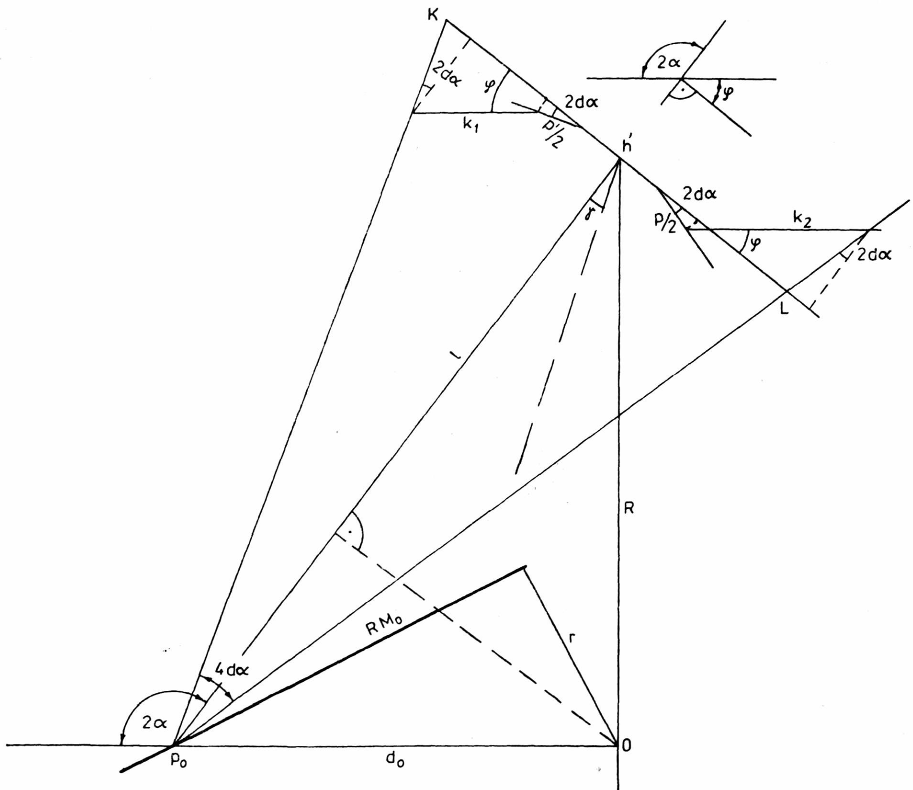

Fig. 6

and

$$\tan 2d = \frac{p'\cos 2da + (h-2c)\cos\gamma + \dfrac{4r\sin a\,\sin 2a\,\sin da}{\cos 2a + \cos 2da}}{2\left(R\cos 2\gamma - r\,\dfrac{\cos 2a}{\cos a}\right)}. \tag{15}$$

The values of the angles $4da$, listed in table 1, are calculated on the basis of the above formula for $p = h = 7.62$, $c = h/2$ by using the recurrence method $\mathrm{tg}\,2da_n = f(2da_{n-1})$, assuming a final value of $2da_n$ for a_{k-1} as the initial value of $2da_1$ for given a_k. For $a = 0$ it has been assumed that $2da_1 = 0$. As the final value of $2da_n$ such value was accepted which differed from the previous one $2da_{n-1}$ by less than 10^{-5}.

The formula (15) refers not only to the secondary mirror objectives, but also to secondary objectives of lens type. In the other case it should be taken $\cos\gamma$ instead of $\cos 2\gamma$ in the denominator of (15), since, then the optical axes of the objectives are directed to the point O.

Having da, and R, a, γ, β, f it is possible to calculate the kinematic aberration $A_{k,0}$ with the help of (5)–(1), by taking account of (6). The value estimated in this way will be considered as an accurate value $A_{k,0}$. The values of $2g'$, $A_{k,0}$ and approximated values of $A_{k,1}$ and $A_{k,2}$ are collected in table 1 while the latter two were obtained from the formulae derived below.

3. An approximation of the kinematic aberration

By accepting certain simplifying assumptions it is possible to obtain the approximate expressions for kinematic aberration, the calculation of which is significantly less time and labour consuming. To verify the correctness of these assumptions the control calculation of the exact and approximate kinematic aberrations have been performed for twelve variants of the camera of the type described in [1], covering with some excess the region of probable realizations (due to the possible values of A_k) for the margin of the working sector, $a_e = 45°$, where A_k takes the maximum value.

By neglecting βe in eq. (3) with respect to $a\cos\gamma$, since always $a\cos\gamma \gg \beta e$, and omitting the component containing the product of the small values $\sin\gamma\sin da$ we obtain

$$e = 2r\cos\gamma\sin a\sin da,$$

$$\sigma_1 = \frac{\beta e h}{a\cos\gamma} = \frac{2\beta r h\sin a\sin da}{a},$$

$$A_{k,1} = z\left(\left|\frac{r'}{r}\cos da - \cos a\right| + \frac{h}{2a}\cos\gamma\sin|a|\right). \tag{16}$$

Assuming further for the whole working sector $\cos\gamma = 1$, $\cos da = 1$, $\tan 2da = \sin 2da = 2da_0$, $\sin da = da_0$, $a = R_0 = R_F$, ($da_0 = da$, for $a = 0$) from (4), (11), (15), (6) and

(16) it is obtained

$$z = \frac{2rp'}{R_0 - r}, \ \beta = 2, \ da_0 = \frac{p'}{4(R_0 - r)}, \tag{17}$$

$$A_{k2} = \frac{rp'}{R_0 - r}\left(|\cos a_b + \cos a_e - 2\cos a| + \frac{h}{R_0}\sin|a|\right), \tag{18}$$

for $c = h/2$.

In table 2 the values of $A_{k,0}$, $A_{k,1}$, $A_{k,2}$ are collected together with the relative errors of these quantities for all mentioned variants, for $a = a_e = 45°$, $p' = h = 7.62$. From these results it is clear that the deviation of $A_{k,1}$ from $A_{k,0}$ never exceeds -0.01%, while the second approximation (which is especially simple as it does not require the numerical solution of the system of equations in order to determine a, γ and da), differs usually from $A_{k,0}$ by less than -1%. From the further calculations it follows that for all the variants in the whole sector 0–45° the parameters R, $2da$, β and z are practically constant.

Table 2. $R_F = R_0$, $p' = h = 7.62$, $a = 45°$, $c = h/2$

W	R_0 [mm]	r [mm]	$A_{k,0}$ [mm]	$A_{k,1}$ [mm]	$\frac{A_{k,1}-A_{k,0}}{A_{k,0}}$ [%]	$A_{k,2}$ [mm]	$\frac{A_{k,2}-A_{k,0}}{A_{k,0}}$ [%]
I	500	4.0	0.0188140	0.0188132	−0.0048	0.0186610	−0.81
II	500	3.5	0.0164287	0.0164281	−0.0037	0.0163119	−0.71
III	500	3.0	0.0140531	0.0140526	−0.0036	0.0139676	−0.61
IV	450	4.0	0.0210258	0.0210246	−0.0057	0.0208348	−0.91
V	450	3.5	0.0183558	0.0183549	−0.0049	0.0182101	−0.79
VI	450	3.0	0.0156978	0.0156971	−0.0045	0.0155912	−0.68
VII	400	4.0	0.0238256	0.0238239	−0.0071	0.0235807	−1.03
VIII	400	3.5	0.0207939	0.0207926	−0.0063	0.0206071	−0.90
IX	400	3.0	0.0177776	0.0177767	−0.0051	0.0176410	−0.77
X	350	4.0	0.0274832	0.0274806	−0.0095	0.0271579	−1.18
XI	350	3.5	0.0239768	0.0239748	−0.0083	0.0237288	−1.03
XII	350	3.0	0.0204910	0.0204895	−0.0068	0.0203097	−0.88

4. The choice of constructional parameters from given values of kinematic aberration

Figure 7 presents the graphs of $A_{k,0}$ and $2g$ for the variants I and X. It may be seen that $A_{k,0}$ has a minimum at the position, at which the function $2g'$ takes the zero value. By a suitable choice of the parameter r' this minimum may be situated at an arbitrary position of the sector (or outside of it). By fixing it, for instance, at the origin of the sector the twice greater value of aberration is obtained at the end of it. The optimal distribution of aberrations is found to be such for which the value of A_k is the same at the origin and the end of the sector. For this purpose the parameter r' should be selected according to (6). The mechanical construction must assure both the proper adjusting movement of the principal objective (for instance) and the control of the r', by offering a possibility of checking (with the help of the special optical adjusting device) that there is no kinematic aberration for the angle α, for which $2g' = 0$.

After having accepted the optimal position of the intermediate image A' according to eq. (6), the designer is interested only in determining the maximum value of aberration at the edges of the sector. For this purpose we may substitute $\alpha = \alpha_e$ in eq. (18) to obtain

$$A_{k,s} = \frac{rp'}{R_0 - r}\left(|\cos\alpha_p - \cos\alpha_k| + \frac{h}{R_0}\sin|\alpha_k|\right). \tag{19}$$

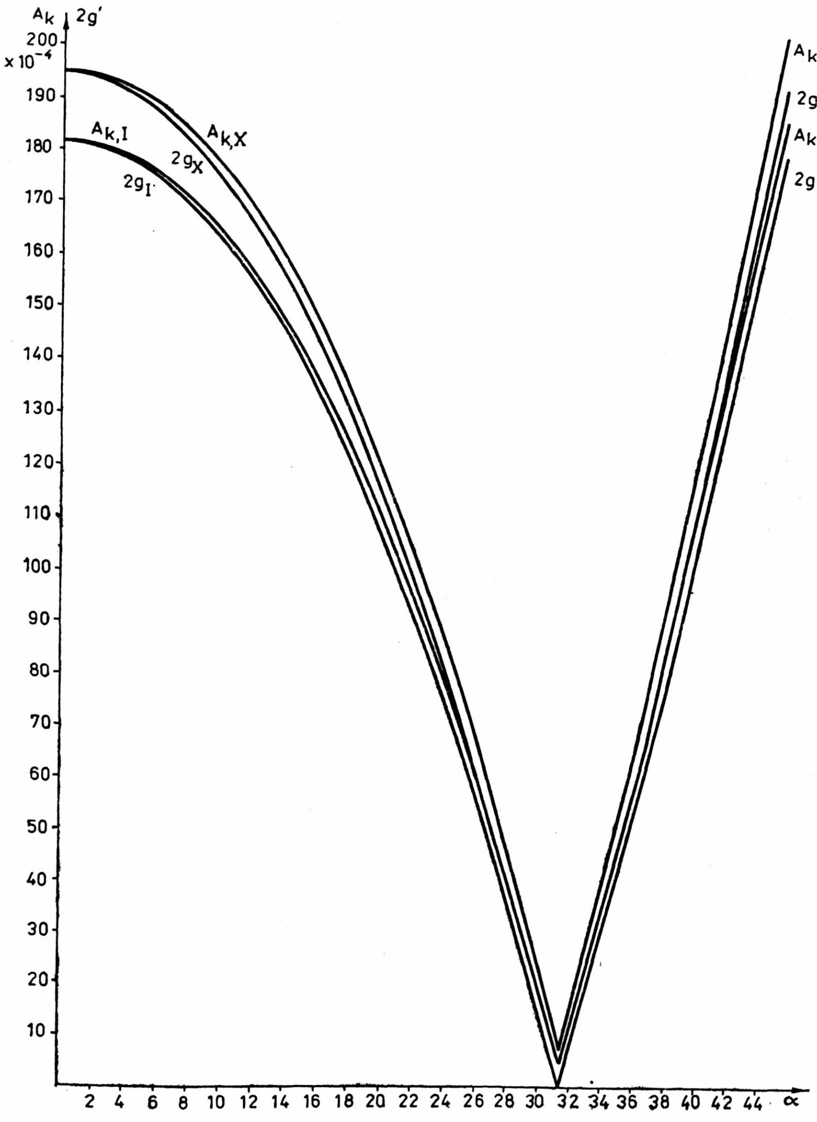

Fig. 7

The formula (19) allows to choose the values of parameters r and R_0 so that the value of A_k accepted as admissible be not exceeded. The parameter r, which should be chosen as small as possible, is defined by technological possibilities of the producer as well as by strength of the material of which the rotating mirror is made. In such case the formula (19) allows to determine the minimal value of R_0. If the parameter R_0 is limited, for instance, by the assumed number or frequency of pictures, then by solving eq. (19) with respect to r it may be verified whether or not the obtained value meets the existing technological conditions.

For example, for the camera type described in [1], for $\alpha_b = 0$, $\alpha_e = 45°$, $p' = h$ we obtain from eq. (19):

$$A_{k,s} = \frac{rh}{R_0 - r}\left[1 - \frac{1}{\sqrt{2}}\left(1 - \frac{h}{R_0}\right)\right], \tag{19a}$$

and hence the value

$$R_0 = l + \sqrt{l^2 + mh}, \tag{20}$$

where $m = \dfrac{rh}{\sqrt{2}A_{k,s}}$, $l = \dfrac{(\sqrt{2}-1)m + r}{2}$,

or

$$r = \frac{A_{k,s} R_0}{A_{k,s} + \dfrac{h^2}{\sqrt{2}R_0} + \left(1 - \dfrac{1}{\sqrt{2}}\right)h}. \tag{21}$$

Figures 8, 9, and 10 show the graphs of $R_0 = f(r)$, $R_0 = f(A_{k,s})$ and $r = f(A_{k,s})$ made on the base of (20) and (21).

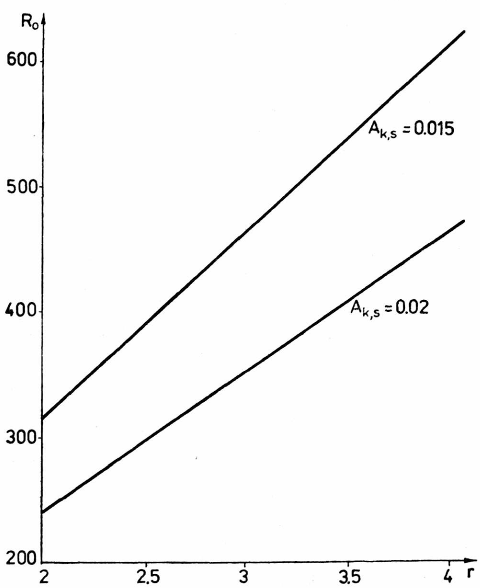

Fig. 8

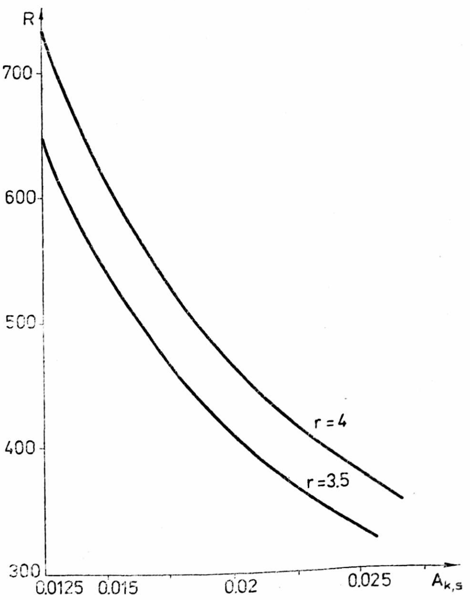

Fig. 9

Table 3. $p' = h = 7.62$, $R_F = R_0$, $a = 45°$, $c = h/2$,

W	$\bar{r}$ [mm]	$A_{k,s}$ [mm]	$\bar{R}_0$ [mm]	R_0 [mm]	$\frac{R_0 - \bar{R}_0}{\bar{R}_0}$ [%]	r [mm]	$\frac{r - \bar{r}}{\bar{r}}$ [%]
I	4.0	0.018814	500	496.10	−0.78	4.033	0.81
II	3.5	0.016429	500	496.58	−0.68	3.52	0.71
III	3.0	0.014053	500	497.08	−0.58	3.02	0.61
IV	4.0	0.021026	450	446.10	−0.87	4.04	0.91
V	3.5	0.018356	450	446.58	−0.76	3.53	0.80
VI	3.0	0.015698	450	447.07	−0.65	3.02	0.68
VII	4.0	0.023826	400	396.09	−0.98	4.04	1.03
VIII	3.5	0.020794	400	396.59	−0.85	3.53	0.90
IX	3.0	0.017778	400	397.07	−0.73	3.02	0.77
X	4.0	0.027483	350	346.11	−1.11	4.04	1.18
XI	3.5	0.023977	350	346.58	−0.98	3.54	1.04
XII	3.0	0.020491	350	347.07	−0.84	3.03	0.09

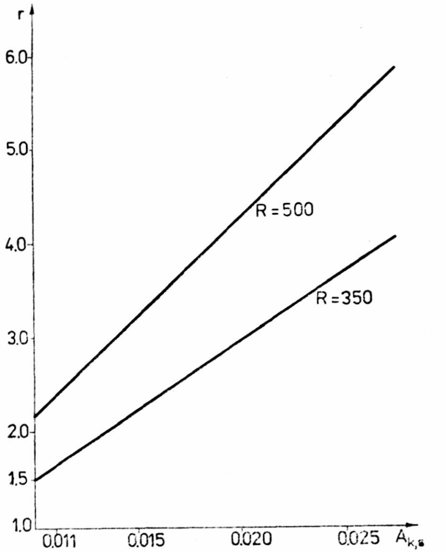

Fig.10

In table 3 the values of R_0 and r obtained from (20) and (21) are listed for all the variants with the starting data (denoted as $\bar{R}_0$ and $\bar{r}$) for which $A_{k,s} = A_{k,0}$ according to (1). From the given relative errors, kept usually below 1%, it follows that the approximation achieved is very good.

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Кинематическая аберрация, а также некоторые конструктивные параметры в камерах для скоростной кинематографии с коммутацией изображения, с вторичными зеркальными объективами

Введены формулы, определяющие параметры, от которых зависит обсуждаемая в более ранних работах автора кинематическая аберрация. В частности, эти формулы относятся к камерам типа Миллера с вторичными зеркальными объективами. Выведены упрощённые формулы для аберрации, значительно облегчающие её расчёт. Для верификации упрощающих предположений были произведены контрольные расчёты для свыше десяти конструктивных вариантов камеры. Показано, как конструктор, исходя из принятой величины кинематической абберации, может определить важные конструктивные параметры камеры.